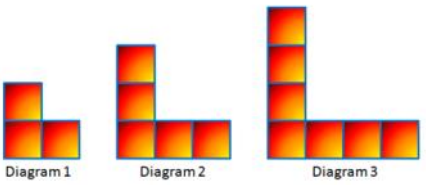
Wednesday, May 1st

Plan For Today:

WELCOME TO PRE-CALCULUS 12 Spring 2024

1. Intro to course: Course Outline & Calendar
2. Review Basic Algebra Handout - Complete by tomorrow
3. Go over Arithmetic & Geometric Sequences & Series
   - **1.1 Arithmetic Sequences**
   - **1.2 Arithmetic Series**
   - 1.3 Geometric Sequences
   - 1.4 Geometric Series
   - 1.5 Infinite Geometric Series
   - Sigma Notation



4. Work on practice questions from Textbook

Plan Going Forward:

1. Finish going through practice question from 1.1.-1.2 in workbook.
   1.1-1.2 Check-in Quiz at Start of Next Class
2. We will start Geometric Sequences & Series on Thursday.

- CH1 PROJECT DUE TUESDAY, MAY 7TH
- CH1 TEST ON TUESDAY, MAY 7TH

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.
Anurita Dhiman = adhiman@sd35.bc.ca

Arithmetic Sequences.

a = first term

d = common difference

↳ take term + subtract by the term before

ex: $t_3 - t_2$ or $t_5 - t_4$

n = the term number

Term Formula $t_n = a + (n-1)d$

Ex #8 b p. 10 $a = \frac{2}{3}$, $d = -\frac{1}{4}$, find t_9. ($n=9$)

BEDMAS

$t_n = \frac{2}{3} + (9-1)(-\frac{1}{4})$

$= \frac{2}{3} + (8)(-\frac{1}{4})$

$\frac{2}{\cancel{8}_1} \times -\frac{1}{\cancel{4}_1} = -\frac{2}{1}$

$= \frac{2}{3} + (-2)$

$= \frac{2}{3} - 2$

Common denominator

$\frac{2}{3} - \frac{2 \times 3}{1 \times 3}$

$\frac{2}{3} - \frac{6}{3} =$

$= \frac{2}{3} - \frac{6}{3}$

$t_9 = -\frac{4}{3}$ $(-\frac{4}{3})$

Ex #9 f p. 11

$-1, 4, 9 \ldots 159$ find # of terms → n

$a = -1$ $d = 4 - (-1)$ $t_n = 159$

$4+1$

$d = 5$

$t_n = a + (n-1)d$

$159 = -1 + (n-1)(5)$

$+1 \quad +1$

$160 = (n-1)5$

$\frac{160}{5} = \frac{(n-1)5}{5}$ (+1 on both sides)

$32 = n - 1$ (+1)

$n = 33$

* solving when there are two unknowns.

Example 9 The 7th term of an arithmetic sequence is 78, and the 18th term is 45. Find the first term.

▶ *Solution:* $t_n = a + (n-1)d$

elimination method

$t_7 = a + 6d = 78$

$t_{18} = a + 17d = 45$

$-11d = 33$

$d = -3$

$t_7 = a + 6d = 78$

$= a + 6(-3) = 78$

$a = 78 + 18$

$= 96$

The first term is 96.

Substitution method.

$t_7 \rightarrow a + (7-1)d = 78$

① $a + 6d = 78$ $-6d$

$t_{18} \rightarrow a + (18-1)d = 45$

② $a + 17d = 45$

① $a = 78 - 6d$

② $78 - 6d + 17d = 45$

$78 + 11d = 45$ -78

$11d = -33$ $\div 11$

$d = -3$

① $a = 78 - 6(-3)$

$a = 78 + 18$

$a = 96$

10. Find the first term in the arithmetic sequence.

a) 6th term is 10; 18th term is 46 (① ②)

$t_n = a + (n-1)d$

① $a + (6-1)d = 10$ ② $a + (18-1)d = 46$

$a + 5d = 10$ $a + 17d = 46$

Elimination Method to solve systems of equations (equations with Two unknowns)

Subtract equations

$a + 5d = 10$

$-\ (a + 17d = 46)$

$-12d = -36$ $\div -12$

$d = 3$

$a + 5(3) = 10$

$a + 15 = 10$ -15

$a = -5$

b) 4th term is 2; 18th term is 30 (① ②)

$a + (4-1)d = 2$ $a + (18-1)d = 30$

① $a + 3d = 2$ ② $a + 17d = 30$

$a + 3d = 2$

$a + 17d = 30$

$a+3d=2$

$-\ a+17d=30$

$-14d=-28$ (÷ −14)

$d=2$

$a+3(2)=2$

$a+6=2$ (−6)

$a=-4$

Arithmetic Series

↳ = sum of terms.

∴ use sum formula.

∴ therefore

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$$

S_n: sum; n = number of terms; a = first term; d = common difference.

Ex: 3 p. 14. $7+10+13+\ldots+100$ (Finite Series)

Sequence = terms

Series = sum of terms.

Find Sum

① n is unknown ∴ solve for n

$t_n=100$

$t_n=a+(n-1)d$

$100=7+(n-1)(3)$ (−7)

$\frac{93}{3}=\frac{3(n-1)}{3}$

$31=n-1$ (+1)

$n=32$.

$a=7$, $d=3$, $n=?$, $t_n=100$

IF $93=3n-3$ (+3)

Sum → $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$a=7$, $d=3$, $n=32$

$S_n=\frac{32}{2}\left[2(7)+(32-1)3\right]$

$=16\left[14+(31)3\right]$

$=16\left[14+93\right]$

$=16(107)$

$\boxed{S_n=1712}$

Ex: #2 e p. 17

e) $d = ?$ $\quad S_{40} = 680$, $a = 11$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = 680$

$n = 40$

$a = 11$

$d = ?$

$680 = \frac{40}{2}[2(11) + (40-1)d]$

$\frac{680}{20} = \frac{20[22 + 39d]}{20}$

$34 = 22 + 39d$

-22

$12 = 39d$

$\frac{12}{39} = \frac{39d}{39}$

$d = \frac{12 \div 3}{39 \div 3}$

$\boxed{d = \frac{4}{13}}$

#1 i)

i) $\frac{1}{2} + \frac{7}{8} + \frac{5}{4} + \cdots + \frac{55}{8}$

$a = \frac{1}{2}$

$d = \frac{7}{8} - \frac{1}{2} \xrightarrow{\times 4} d = \frac{7}{8} - \frac{4}{8}$

$d = \frac{3}{8}$

$n = ?$ $\quad t_n \rightarrow \frac{55}{8} = \frac{1}{2} + (n-1)\left(\frac{3}{8}\right)$

$-\frac{4}{8}$

$\frac{51}{8} = (n-1)\left(\frac{3}{8}\right)$

$\frac{3}{8}$ $\qquad \frac{3}{8}$

$\frac{51}{8} \times \frac{8}{3}$

$17 = n - 1$

$+1$

$n = 18$

$S_{18} = \frac{18}{2}\left[2\left(\frac{1}{2}\right) + (18-1)\left(\frac{3}{8}\right)\right]$

$= 9\left[1 + 17\left(\frac{3}{8}\right)\right]$

$= 9\left[1 + \frac{51}{8}\right]$ common denominator

$= 9\left[\frac{8}{8} + \frac{51}{8}\right]$

$= 9\left[\frac{59}{8}\right]$

$\boxed{S_{18} = \frac{531}{8}}$

1.1-1.2 Arithmetic Sequences & Series

Sequence

A Sequence is a set of things (usually numbers) that are in order.

Sequence:

3, 5, 7, 9, ...

1st term, 2nd term, 3rd term, 4th term

three dots means goes on forever (infinite)

("term", "element" or "member" mean the same thing)

Each number in the sequence is called a **term** (or sometimes "element" or "member"), read Sequences and Series for more details.

Arithmetic Sequence

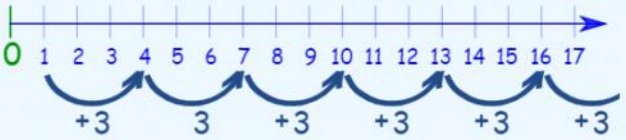
In an Arithmetic Sequence **the difference between one term and the next is a constant**.

In other words, we just add the same value each time ... infinitely.

Example:

1, 4, 7, 10, 13, 16, 19, 22, 25, ...

This sequence has a difference of 3 between each number.
The pattern is continued by **adding 3** to the last number each time, like this:



In General we could write an arithmetic sequence like this:

{a, a+d, a+2d, a+3d, ... }

where:

- **a** is the first term, and
- **d** is the difference between the terms (called the **"common difference"**)

Example: (continued)

$$1, 4, 7, 10, 13, 16, 19, 22, 25, \ldots$$

Has:

- a = 1 (the first term)
- d = 3 (the "common difference" between terms)

And we get:

$$\{a, a+d, a+2d, a+3d, \ldots\}$$

$$\{1, 1+3, 1+2\times3, 1+3\times3, \ldots\}$$

$$\{1, 4, 7, 10, \ldots\}$$

We can write an Arithmetic Sequence as a rule:

$$x_n = a + d(n-1)$$

(We use "n−1" because **d** is not used in the 1st term).

Example: Write a rule, and calculate the 9th term, for this Arithmetic Sequence:

3, 8, 13, 18, 23, 28, 33, 38, ...

This sequence has a difference of 5 between each number.

The values of **a** and **d** are:

- **a = 3** (the first term)
- **d = 5** (the "common difference")

Using the Arithmetic Sequence rule:

$$\mathbf{x_n} = a + d(n-1)$$

$$= 3 + 5(n-1)$$

$$= 3 + 5n - 5$$

$$= \mathbf{5n - 2}$$

So the 9th term is:

$$x_9 = 5 \times 9 - 2$$

$$= 43$$

Is that right? Check for yourself!

Example

Find the formula for the nth term of the arithmetic sequence

1 2, 5, 8, …

2 107, 98, 89, ….

Solution

1 Here $a = 2$ and $d = 3$, so

$$a_n = 2 + (n-1) \times 3 = 3n - 1.$$

2 Here $a = 107$ and $d = -9$, so

$$a_n = 107 + (n-1) \times -9 = 116 - 9n.$$

Arithmetic sequences

The difference between consecutive terms is always the same.

$a_n \rightarrow$ nth term of the sequence (formula)

$a_1, a_2, a_3, a_4, \ldots\ldots \rightarrow$ Sequence

$a_1 \rightarrow$ First term of the sequence

$d \rightarrow$ difference between consecutive terms

$n \rightarrow$ position number

Given an arithmetic sequence we can find the nth term of the sequence:

$$a_n = a_1 + d(n-1)$$

Arithmetic Sequence and Series

An arithmetic sequence is a sequence of numbers such that the difference d between each consecutive term is a constant.

$$a, a+d, a+2d, a+3d, \ldots$$

The nth term, $a_n = a+(n-1)d$

Sum of first n terms, $S_n = \frac{n}{2}[2a+(n-1)d]$

$$S_n = \frac{n}{2}[a+a_n]$$