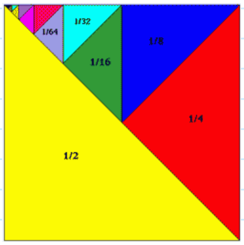
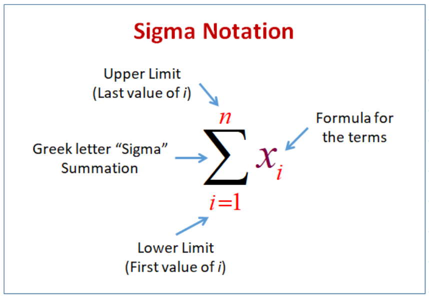
Monday, May 6th

Plan For Today:

WELCOME BACK TO PRE-CALCULUS 12 Spring 2024

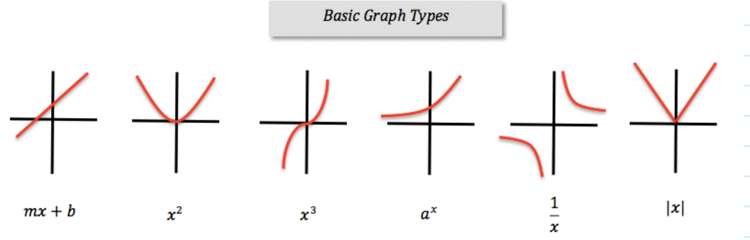
1. Any general questions about course?
2. Any questions about material from last class (1.3-1.4)?
   - Do 1.3-1.4 Geometric Sequences & Series Check-in Quiz
3. Finish Arithmetic & Geometric Sequences & Series
   - ✓ 1.1 Arithmetic Sequences
   - ✓ 1.2 Arithmetic Series
   - ✓ 1.3 Geometric Sequences
   - ✓ 1.4 Geometric Series
   - ✓ **Sigma Notation**
   - ✓ **1.5 Infinite Geometric Series**





$$\sum_{i=1}^{n} x_i$$

4. Work on practice questions and project.
5. Review basic graphing for Chapter 2
   - 2.0 Graphing Review
   - 2.1 Horizontal and Vertical Translations
   - 2.2 Reflections and Stretches
   - 2.3 Combining Transformations
   - 2.4 Inverse of a Relation



6. Work on Graphing Review Handout.

Plan Going Forward:

1. Finish going through practice question in Chapter 1 and finish the Chapter 1 Practice Questions Handout to prepare for the test next class. The KEY for this handout will be posted on my website after class today.
2. Finish working on the Ch1 Geometric Sequences and Series Project.

- **CH1 PROJECT DUE TOMORROW (TUESDAY, MAY 7TH)**
- **CH1 TEST ON TOMORROW (TUESDAY, MAY 7TH)**

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.
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Monday, May 6th In-Class Notes

Name: KEY **TOTAL =** ____ **/ 8 marks**

Check-in Quiz Section 1.3-1.4 – Geometric Sequences & Series

Complete the following questions SHOWING ALL WORK and steps where applicable.

1. Determine if the following sequence IS or IS NOT a geometric sequence.
   CIRCLE THE CORRECT ANSWER

   0.25 each = 1 mark

   a) $2, 6, 18, 54, \ldots$ $\frac{6}{2}=3 \quad \frac{18}{6}=3$ ✓ — IS GEOMETRIC — NOT GEOMETRIC

   b) $8, -2, \frac{1}{2}, -\frac{1}{8}, \ldots$ $=-\frac{1}{4}$ — IS GEOMETRIC — NOT GEOMETRIC

   c) $1, \frac{3}{2}, \frac{12}{4}, \frac{60}{8}, \ldots$ — IS GEOMETRIC — NOT GEOMETRIC

   d) $\frac{1}{x}, \frac{2}{x^2}, \frac{4}{x^3}, \ldots$ — IS GEOMETRIC — NOT GEOMETRIC

2. Algebraically determine the common ratio, r, in each of the following geometric sequences:

   1 mark

   a) $-48, 192, -768, 3072, \ldots$ $\quad r = \frac{192}{-48} \rightarrow \boxed{r = -4}$

   b) $2x, 5x^3, \frac{25}{2}x^5, \ldots$ $\quad r = \frac{5x^3}{2x} \rightarrow \boxed{r = \frac{5x^2}{2}}$ or $\frac{5}{2}x^2$

3. Algebraically determine the 10^{th} term, t_{10}, of the geometric sequence given the following information: 1 mark

   $a = -32, r = \frac{3}{2}$

   $$t_{10} = -32\left(\frac{3}{2}\right)^9$$
   $$= -\cancel{32}^{\,1}\left(\frac{19683}{\cancel{512}_{\,16}}\right)$$
   $$\boxed{t_{10} = -\frac{19683}{16}}$$

4. Algebraically determine which term of the following geometric sequence is **14348907**:

   1 mark

   $9, 27, 81, \ldots$

   $a = 9$
   $r = 3$

   $$\frac{9(3)^{n-1}}{9} = \frac{14348907}{9}$$
   $$3^{n-1} = 1594323$$
   $$\cancel{3}^{\,n-1} = \cancel{3}^{\,13}$$
   $$n-1 = 13 \quad +1$$
   $$\boxed{n = 14}$$

5. If the third term of a geometric sequence is $\frac{8}{5}$ and the sixth term is $\frac{64}{5}$. Determine the ratio and the first term.

2 marks

$\underbrace{a_3 \; - \; - \; a_6}_{r^3}$

① $a_3 r^3 = a_6$

$\frac{8}{5} r^3 = \frac{64}{5} \times \frac{5}{8}$

$\sqrt[3]{r^3} = \sqrt[3]{8}$

$\boxed{r = 2}$

② $a(2)^2 = \frac{8}{5}$

$4a = \frac{8}{5} \times \frac{1}{4}$

$\boxed{a = \frac{2}{5}}$

6. Algebraically determine the sum of the first 6 terms, S_6, of the following geometric series:

1 mark

$-48 + 192 - 768 + 3072, \ldots$

$$S_6 = \frac{-48(1-(-4)^6)}{\underbrace{1-(-4)}_{5}}$$

$$S_6 = 39312$$

7. If Shayne earns \$15.75 per hour and works 60 hours each month, how much would they have earned in total after five months if the hourly wage is increased by 0.5% per month?

1 mark

$a_1 = \$15.75/\text{hr} \times 60\text{hr} = \$945$

$r = 1.005$

$0.5\% = 0.005$

$$S_5 = \frac{945(1-1.005^5)}{\underbrace{1-1.005}_{-0.005}}$$

1.0253

$= -189000(1-1.005^5)$

$S_5 = \$4772.49$

p.14 1.2

Sigma Notation

end terms when you get to upper limit

$$\sum_{k=1}^{n} x_i \longleftarrow$$ expression to represent each term of sequence

- if it's arithmetic use $t_n = a + (n-1)d$
- if it's geometric use $t_n = ar^{n-1}$

Start terms where $k=1$ (start here)

☆ Sigma means SUM ∴ add terms of sequence or use sum formula.

Ex 4 p.14 $\sum_{k=1}^{100} (2k+1)$ Evaluate:

Book method is good ✓

Alternate Method.

$a =$ where $k=1$
$= 2(1)+1$
$a = 3$

$a_2 =$ make $k=2$.
$= 2(2)+1$
$a_2 = 5$

☆ notice this is arithmetic

$d = a_2 - a_1$
$= 5-3$
$d = 2$

n = number of terms = upper limit.

☆ if the bottom does not start at $k=1$, then use:

$n-k+1$

ex: $\sum_{k=5}^{10} x_i \rightarrow n = 10-5+1$
$n = 6$ terms.

$n=100$

$$S_{100} = \frac{100}{2}\left[2(3) + (100-1)2\right]$$

$$= 50\left[6 + 99(2)\right]$$

$$= 50(6 + 198)$$

$$= 50(204)$$

$$\boxed{S_{100} = 10200}$$

Ex:5 series into sigma notation.

$$\sum_{k=1}^{n} a + (k-1)d$$

term formula for arithmetic series.

$5 + 9 + 13 + \ldots + 137.$

solve for 'n'

$a = 5 \quad d = 4 \quad n = ? \rightarrow$ $5 + (n-1)(4) = 137$

$n = 34$ $\qquad n = 34$

$$\sum_{k=1}^{34} 5 + (k-1)4$$

$$\boxed{\sum_{k=1}^{34} 4k + 1}$$

Try #3 p. 18 a - d

Sigma for Geometric p. 27

Ex 3 + Ex 4.

Geometric $\longrightarrow$ $\underbrace{\sum_{k=1}^{n} ar^{k-1}}_{\text{evaluate.}}$ $\leftarrow$ term formula for geometric sequences.

use sum $\rightarrow$ $S_n = \frac{a(1-r^n)}{1-r}$

evaluate. use sum formula $\rightarrow S_n = \dfrac{a(1-r^n)}{1-r}$.

Ex 3 p.27

$$\sum_{k=1}^{10} 3(-2)^{k-1}$$

$a_1 = 3(-2)^{1-1}$

$= 3(-2)^0$

$= 3(1)$

$a_1 = 3$

$a_2 = 3(-2)^{2-1}$

$= 3(-2)$

$a_2 = -6$

ratio: $\dfrac{-6}{3}$

$r = -2$

IF $d = -6-3$

$d = -9$?

check $a_3 = 3(-2)^{3-1}$

$= 3(-2)^2$

$= 3(4)$

$a_3 = 12$. not arithmetic

b/c $-6 \times -2 = 12$

$-6 - 9 \neq 12$

$n = 10$

$$S_{10} = \frac{3(1-(-2)^{10})}{1-(-2)}$$

$(-2)^{10}$

$= 1024$

$1 - 1024 = -1023$

$$= \frac{\cancel{3}(-1023)}{\cancel{3}}$$

$$\boxed{S_{10} = -1023}$$

Ex4 $6 + 18 + 54 + 162 + 486$

$$\sum_{k=1}^{?} ar^{k-1} \rightarrow \boxed{\sum_{k=1}^{5} 6(3)^{k-1}}$$

Try #2-4 p. 28-29.

3f) $\sum_{i=3}^{11} \frac{2^i}{3^{i-1}}$

① $a_1 = \frac{2^3}{3^{3-1}}$ $(i=3)$

$a_1 = \frac{8}{9}$

② $a_2 = \frac{2^4}{3^{4-1}}$ $(i=4)$

$a_2 = \frac{16}{27}$

check

a_3 $\quad a_2 \times r = a_3$

③ $r = \frac{\frac{16}{27}}{\frac{8}{9}}$

$= \frac{\cancel{16}^{2}}{\cancel{27}_{3}} \times \frac{\cancel{9}^{1}}{\cancel{8}_{1}}$

$r = \frac{2}{3}$

④ $n = ?$

$n = 11 - 3 + 1$

$n = 9$

⑤ $S_9 = \frac{\frac{8}{9}\left(1 - \left(\frac{2}{3}\right)^9\right)}{\left(\frac{3}{3}\right) 1 - \frac{2}{3}}$

$= \frac{\frac{8}{9}\left(1 - \frac{512}{19683}\right)}{\frac{1}{3}}$ $\quad \frac{19683}{19683} - \frac{512}{19683}$

$= \frac{8}{\cancel{9}_{3}}\left(\frac{19171}{19683}\right) \cdot \frac{\cancel{3}}{1}$

$S_9 = \frac{153368}{59049}$ ✓ ≈ 2.5973

≈ 2.60

Infinite Series.

$S_\infty = \frac{a}{1-r}$ $\quad$ sum to infinity # of terms.

☆ $0 < |r| < 1$

ex: $r = \frac{1}{2}$

$r = \frac{2}{3}$

p.35

2 f) $-6 + 3 - \frac{3}{2} + \frac{3}{4} - \cdots$

$n = \infty$

$a = -6$

$r = \frac{3}{-6} \rightarrow \underbrace{r = -\frac{1}{2}}_{0 < |r| < 1}$

$S_\infty = \frac{-6}{1 - \left(-\frac{1}{2}\right)}$ $\quad \frac{2}{2} + \frac{1}{2}$

-6

$r = \frac{3}{-6} \rightarrow \underbrace{r = -\frac{1}{2}}_{0<|r|<1}$ $S_\infty = \frac{-6}{1-(-\frac{1}{2})}$ $\frac{2}{2}+\frac{1}{2}$

$= \frac{-6}{\frac{3}{2}}$

$= \cancel{-6}^{-2} \times \frac{2}{\cancel{3}}$

$\boxed{S_\infty = -4}$

p.36

3h) $\sum_{n=3}^{\infty} \frac{6}{3^{n-1}}$

(1) $a = \frac{6}{3^{3-1}}$

$= \frac{6}{9}$

$a = \frac{2}{3}$

(3) $r = \frac{\frac{2}{9}}{\frac{2}{3}}$

$= \frac{\cancel{2}}{\cancel{9}_3} \times \frac{\cancel{3}}{\cancel{2}}$

$r = \frac{1}{3}$

(2) $a_2 = \frac{6}{3^{4-1}}$

$= \frac{6}{27}$

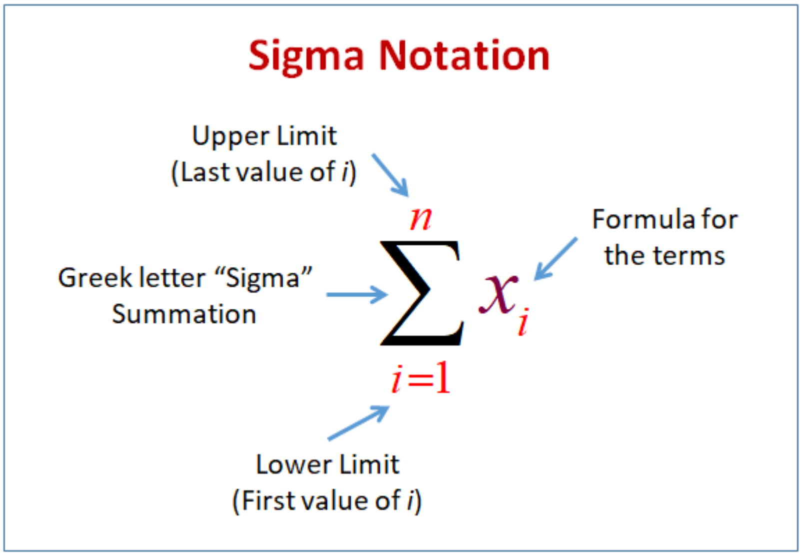
$a_2 = \frac{2}{9}$

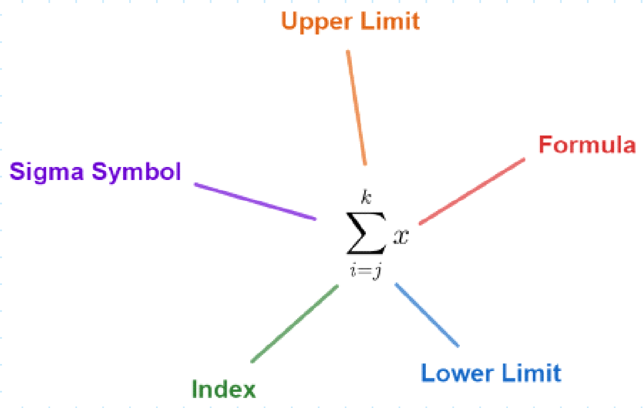
(4) $S_\infty = \frac{\frac{2}{3}}{1-\frac{1}{3}}$

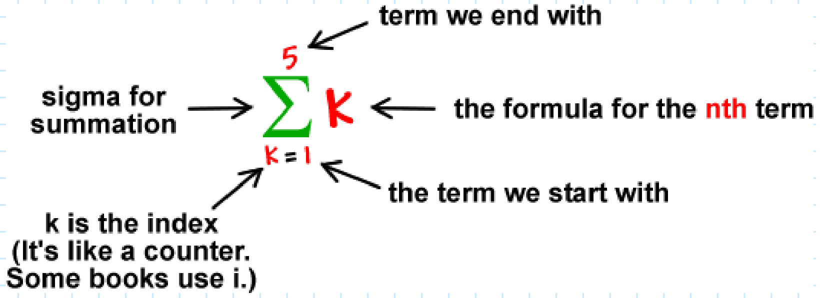
$= \frac{\frac{2}{3}}{\frac{2}{3}}$

$\boxed{S_\infty = 1}$

Sigma Notation







Sigma Notation

There is a special notation that is used to represent a series. For example, the geometric series $3 + 6 + 12 + 24 + 48 + 96$ has 6 terms, with first term 3 and common ratio 2. The general term is $t_n = 3(2)^{n-1}$.

Each term in the series can be expressed in this form.

$t_1 = 3(2)^{1-1}$ $\quad t_2 = 3(2)^{2-1}$ $\quad t_3 = 3(2)^{3-1}$

$t_4 = 3(2)^{4-1}$ $\quad t_5 = 3(2)^{5-1}$ $\quad t_6 = 3(2)^{6-1}$

The series is the sum of all these terms, and is represented as shown.

The sum of … $\longrightarrow \displaystyle\sum_{k=1}^{6} 3(2)^{k-1} \longleftarrow$ … all numbers of the form $3(2)^{k-1}$ …

… for integral values of k from 1 to 6.

The symbol Σ is the capital Greek letter sigma, which corresponds to S, the first letter of the word "sum." When Σ is used as shown above, it is called *sigma notation*. In sigma notation, k is frequently used as the variable under the Σ sign and in the expression following it. Any letter can be used, as long as it is not used elsewhere.

Example 1:

Finite geometric sequence: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots, \frac{1}{32768}$

Related finite geometric series: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots + \frac{1}{32768}$

Written in sigma notation: $\displaystyle\sum_{k=1}^{15} \frac{1}{2^k}$

Example 2:

Infinite geometric sequence: $2, 6, 18, 54, \ldots$

Related infinite geometric series: $2 + 6 + 18 + 54 + \ldots$

Written in sigma notation: $\displaystyle\sum_{n=1}^{\infty} \left(2 \cdot 3^{n-1}\right)$

1) Find the sum of the infinite series:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$$

As first term equals a_1, $a_1 = \frac{1}{2}$

Using $a_n = r a_{n-1}$ and first two terms, $r = \frac{1}{2}$

$$\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{k-1} = \frac{a_1}{1-r} = \frac{(1/2)}{1-(1/2)} = 1$$

2) Find the sum of $\sum_{k=1}^{\infty} 4\left(-\frac{1}{3}\right)^{k-1}$

$$a_1 = 4$$

$$r = -\frac{1}{3}$$

$$\therefore$$

$$\sum_{k=1}^{\infty} 4\left(-\frac{1}{3}\right)^{k-1} = \frac{a_1}{1-r} = \frac{4}{1-(-1/3)} = \frac{4}{(4/3)} = 4 \cdot \frac{3}{4} = 3$$

1.5 Infinite Geometric Series

Sum to infinity of a geometric series

$$S_\infty = \frac{a_1}{1-r}$$

exists for $-1 < r < 1$

a_1 is the first term

r is the ratio

$r = a_n \div a_{n-1}$

2. Find the value of the infinite geometric series $4+2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$

$$a_1 = 4, \quad r = \frac{1}{2}$$

$$S_\infty = \frac{a_1}{1-r}$$

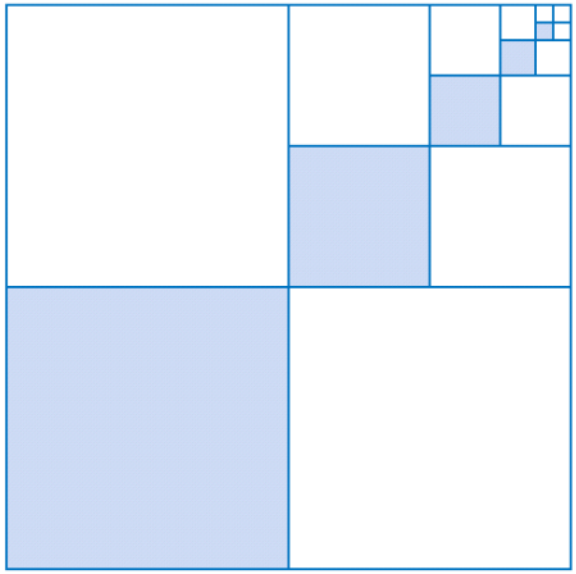
$$S_\infty = \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8$$

Apply the Sum of an Infinite Geometric Series

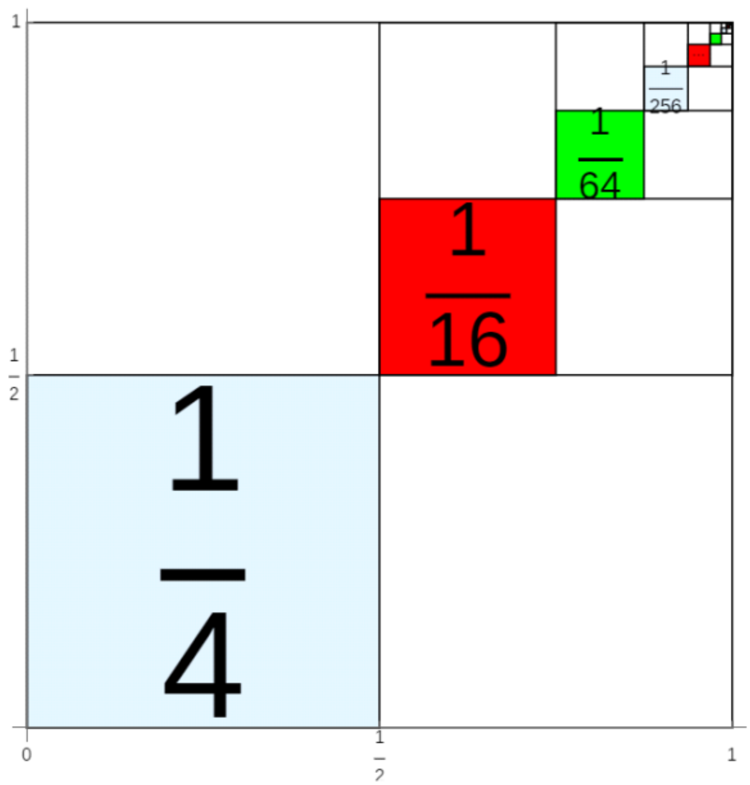
Assume that each shaded square represents $\frac{1}{4}$ of the area of the larger square bordering two of its adjacent sides and that the shading continues indefinitely in the indicated manner.

a) Write the series of terms that would represent this situation.

b) How much of the total area of the largest square is shaded?



Solution



$$S_\infty = \frac{\frac{1}{4}}{1-\frac{1}{4}}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$= \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{3}$$

$$\boxed{S_\infty = \frac{1}{3}}$$

Consider the infinite geometric series $4 - \frac{4}{5} + \frac{4}{25} - \ldots$

$r = -\frac{1}{5}$

a) Explain why the series has a sum to infinity.

b) Determine the sum to infinity.

The sum to infinity is $\frac{10}{3}$.

Which infinite geometric series has a sum? What is the sum?

a) $4 - 6 + 9 - 13.5 + \ldots$

b) $6 + 2 + \frac{2}{3} + \frac{2}{9} + \ldots$

Solution

a) $4 - 6 + 9 - 13.5 + \ldots$

For this series, $r = -1.5$; since $|r| > 1$, the series has no sum.

b) $6 + 2 + \frac{2}{3} + \frac{2}{9} + \ldots$

For this series, $r = \frac{1}{3}$; since $|r| < 1$, the series has a sum. Use the formula.

$$\begin{aligned} S &= \frac{a}{1-r} \\ &= \frac{6}{1 - \frac{1}{3}} \\ &= \frac{6}{\frac{2}{3}} \\ &= 9 \end{aligned}$$

The sum of the series is 9.

SUMMARY

Geometric Sequence and Geometric Series

A geometric sequence is a sequence of numbers in which the ratio between consecutive terms is constant.

The formula for the nth term of a geometric sequence is

$$a_n = ar^{n-1}$$

where a is the first term and r is the ratio

A geometric series results from adding the terms of a geometric sequence.

The formula for the sum of a finite geometric series is

$$S_n = \sum_{i=1}^{n} ar^{i-1} = \frac{a\left(1-r^n\right)}{1-r}, r \neq 1$$

where n is the number of terms, a is the first term and r is the ratio

The formula for the sum of an infinite geometric series is

$$S_n = \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}, |r| < 1$$

where a is the first term and r is the ratio

If |r| ≥ 1, then the infinite series does not have a sum