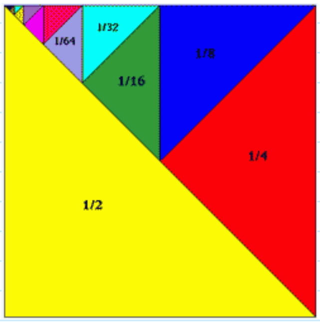
Thursday, May 2nd

Plan For Today:

WELCOME BACK TO PRE-CALCULUS 12 Spring 2024

1. Any general questions about course?
2. Any questions about material from last class (1.1-1.2)?
    - Do 1.1-1.2 Arithmetic Sequences & Series Check-in Quiz
3. Go over Arithmetic & Geometric Sequences & Series
    - ✓ 1.1 Arithmetic Sequences
    - ✓ 1.2 Arithmetic Series
    - **1.3 Geometric Sequences**
    - **1.4 Geometric Series**
    - 1.5 Infinite Geometric Series
    - Sigma Notation
4. Work on practice questions from Workbook and work on project.



Plan Going Forward:

1. Finish going through practice question from 1.3 (#4) - 1.4 (#1) in workbook. Also start working on Ch1 practice questions handout (#7-18)

1.3-1.4 Check-in Quiz at Start of Next Class

2. We will finish Chapter 1 on Monday and will do a graphing review lesson.

- **CH1 PROJECT DUE TUESDAY, MAY 7TH**
- **CH1 TEST ON TUESDAY, MAY 7TH**

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.
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Thursday, May 2nd In-Class Notes

Review

4. Write the first four terms of the recursive sequence.

a) $a = 4,\ t_n = 2 + t_{n-1}$

b) $a = 3,\ t_n = n - t_{n-1}$

c) $\underbrace{a = 2}_{a_1 = 2},\ a_2 = 3,\ a_n = a_{n-1} + a_{n-2}$

d) $a_1 = -1,\ a_2 = 1,\ a_n = na_{n-1} + a_{n-2}$

$\uparrow$ t_1 ✓

$\nearrow$ t_2 ✓

$a_3 = a_{3-1} + a_{3-2}$

$= a_2 + a_1$

$a_3 = 3 + 2$

$a_3 = 5$ ✓

t_3

t_4

$a_4 = a_{4-1} + a_{4-2}$

$= a_3 + a_2$

$= 5 + 3$

$a_4 = 8$ t_4 ✓

3

SKIP. **d)** 2, − 4, 6, − 8, …

$a=2$

$t_n = a + (n-1)d$

$d = -4 - 2$

$d = -6$

$\boxed{\#3 \text{ Skip. p. } 9}$

1.2
p.17

2) **f)** S_{62}, if $\underline{a_1 = 10}$, $\underline{d = 3}$

S_{62}

$\uparrow$

$n=62$

BEDMAS

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

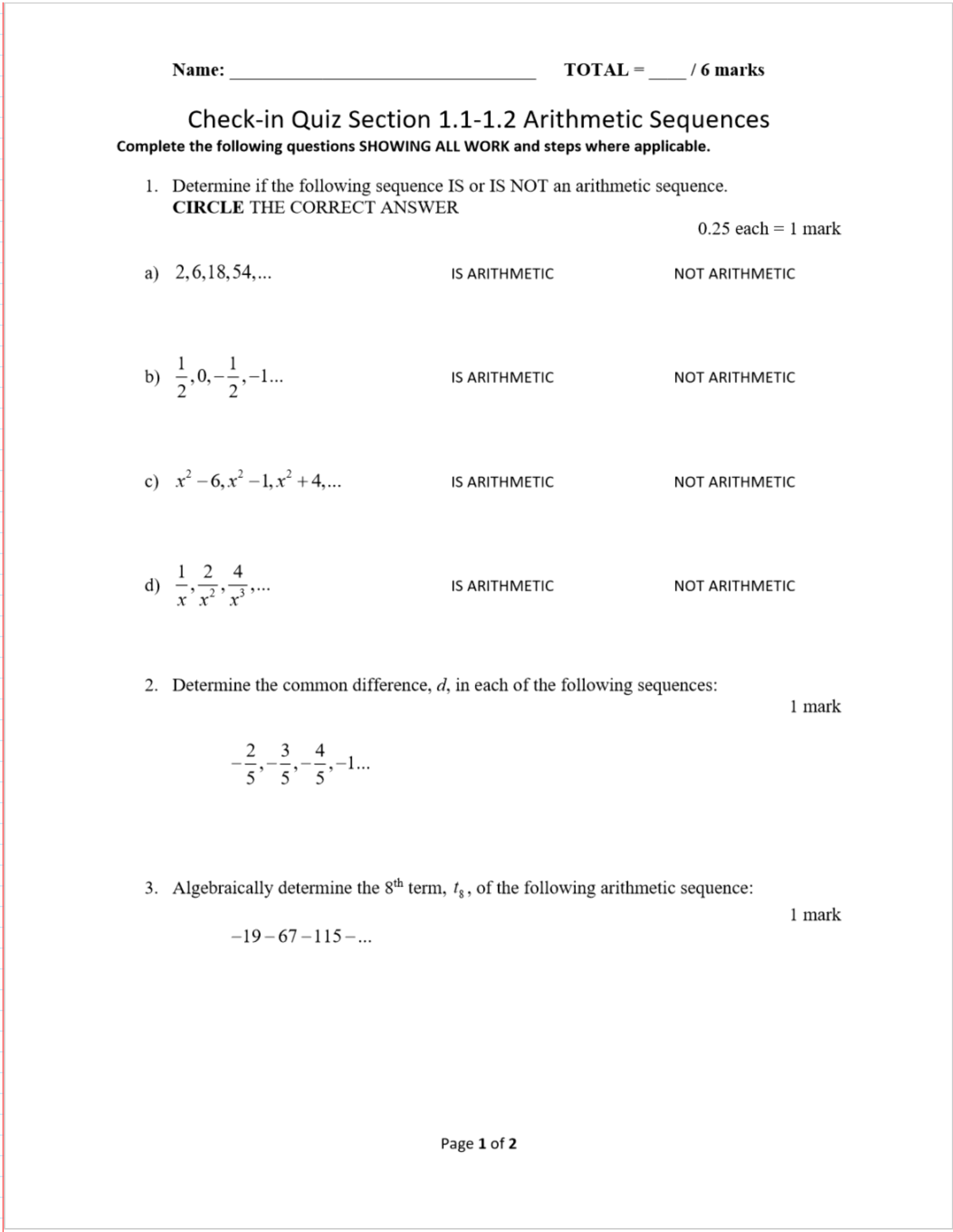
$$S_{62} = \underbrace{\frac{62}{2}}_{\textcircled{1}}\left[\underbrace{2(10)}_{\textcircled{1}} + \underbrace{(62-1)}_{\textcircled{1}}3\right]$$

$$= 31\left[20 + \underbrace{(61)(3)}_{\textcircled{2}}\right]$$

$$= 31\underbrace{\left[20 + 183\right]}_{\textcircled{3}}$$

$= 31\left[203\right]$

$\boxed{S_{62} = 6293}$



Name: ______________________________ **TOTAL =** ____ **/ 6 marks**

Check-in Quiz Section 1.1-1.2 Arithmetic Sequences

Complete the following questions SHOWING ALL WORK and steps where applicable.

1. Determine if the following sequence IS or IS NOT an arithmetic sequence.
   CIRCLE THE CORRECT ANSWER

0.25 each = 1 mark

a) $2, 6, 18, 54, \ldots$ IS ARITHMETIC NOT ARITHMETIC

b) $\frac{1}{2}, 0, -\frac{1}{2}, -1\ldots$ IS ARITHMETIC NOT ARITHMETIC

c) $x^2-6, x^2-1, x^2+4, \ldots$ IS ARITHMETIC NOT ARITHMETIC

d) $\frac{1}{x}, \frac{2}{x^2}, \frac{4}{x^3}, \ldots$ IS ARITHMETIC NOT ARITHMETIC

2. Determine the common difference, *d*, in each of the following sequences:

1 mark

$$-\frac{2}{5}, -\frac{3}{5}, -\frac{4}{5}, -1\ldots$$

3. Algebraically determine the 8th term, t_8, of the following arithmetic sequence:

1 mark

$$-19 - 67 - 115 - \ldots$$

4. Algebraically determine number of terms of the following arithmetic sequence:

1 mark

$$1.35+1.31+1.27+1.23+...+0.91$$

5. If the third term of an arithmetic sequence is 40 and the sixth term is 25, determine the first term and common difference:

1 mark

6. Determine the sum of the first 13 terms of the following arithmetic series:

1 mark

$$8+5+2-1-...$$

Name: KEY **TOTAL =** ____ **/ 6 marks**

Check-in Quiz Section 1.1-1.2 Arithmetic Sequences

Complete the following questions SHOWING ALL WORK and steps where applicable.

1. Determine if the following sequence IS or IS NOT an arithmetic sequence. **CIRCLE** THE CORRECT ANSWER

0.25 each = 1 mark

a) $2, 6, 18, 54, \ldots$ geometric

$+4 \quad \times$

IS ARITHMETIC [✓ NOT ARITHMETIC]

b) $\frac{1}{2}, 0, -\frac{1}{2}, -1 \ldots$

$+\frac{1}{2} \quad -\frac{1}{2} \quad -\frac{1}{2}$

[✓ IS ARITHMETIC] NOT ARITHMETIC

c) $x^2 - 6, x^2 - 1, x^2 + 4, \ldots$

$+5 \quad +5$

[✓ IS ARITHMETIC] NOT ARITHMETIC

d) $\frac{1}{x}, \frac{2}{x^2}, \frac{4}{x^3}, \ldots$ geometric

$\times \frac{2}{x} \quad \times \frac{2}{x}$

IS ARITHMETIC [✓ NOT ARITHMETIC]

2. Determine the common difference, *d*, in each of the following sequences:

1 mark

$-\frac{2}{5}, -\frac{3}{5}, -\frac{4}{5}, -1 \ldots$

$-\frac{3}{5} - \left(-\frac{2}{5}\right) = \frac{-3}{5} + \frac{2}{5} \rightarrow = -\frac{1}{5}$ $\quad d = -\frac{1}{5}$

3. Algebraically determine the 8th term, t_8, of the following arithmetic sequence:

1 mark

$-19 - 67 - 115 - \ldots$

$a = -19$

$d = -48$

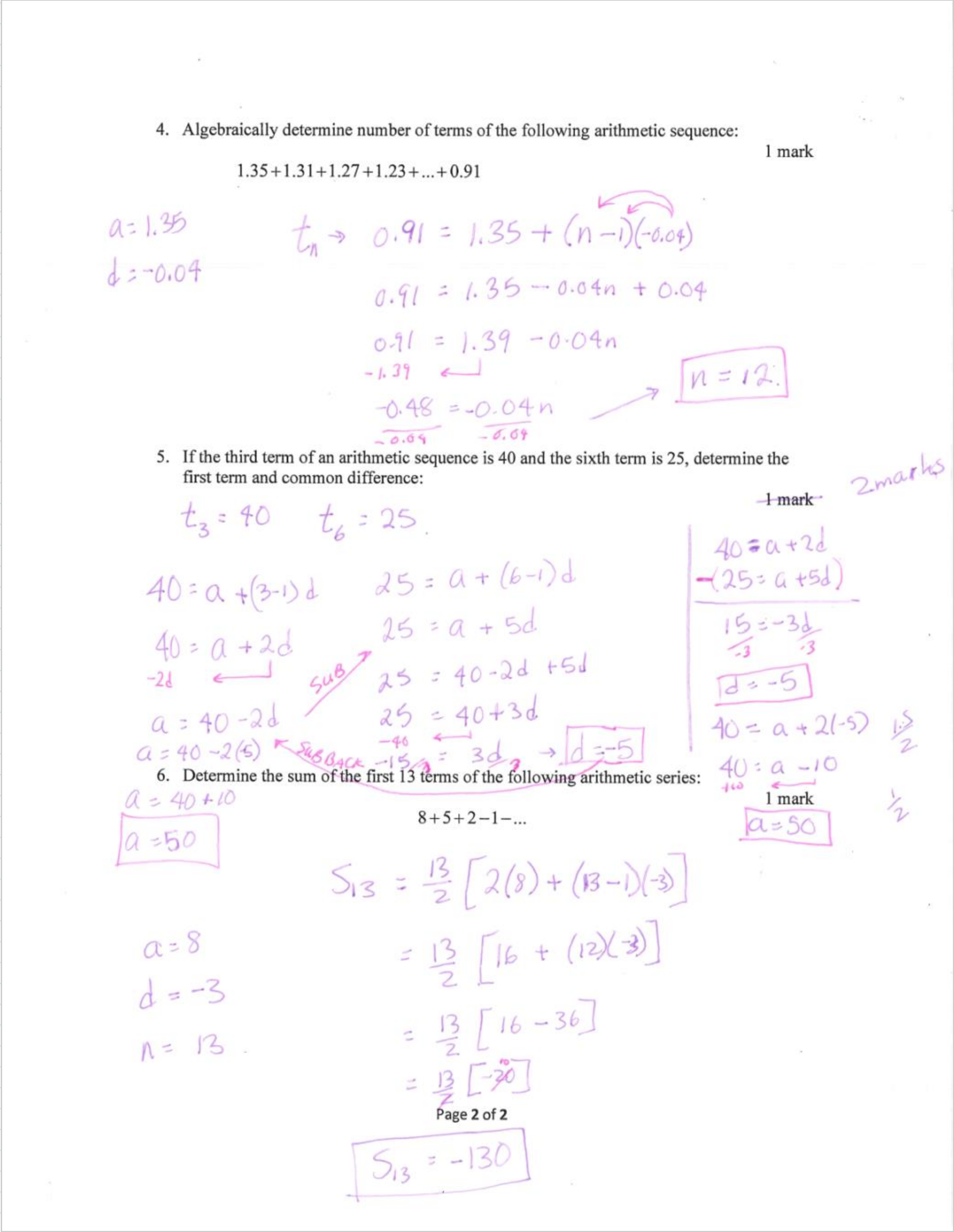
$n = 8$

$t_8 = -19 + (8-1)(-48)$

$= -19 + (7)(-48)$

$= -19 - 336$

$\boxed{t_8 = -355}$



4. Algebraically determine number of terms of the following arithmetic sequence:

1 mark

$1.35+1.31+1.27+1.23+...+0.91$

a = 1.35

d = −0.04

$t_n \rightarrow 0.91 = 1.35 + (n-1)(-0.04)$

$0.91 = 1.35 - 0.04n + 0.04$

$0.91 = 1.39 - 0.04n$

-1.39

$\frac{-0.48}{-0.04} = \frac{-0.04n}{-0.04}$

$\boxed{n = 12.}$

5. If the third term of an arithmetic sequence is 40 and the sixth term is 25, determine the first term and common difference:

~~1 mark~~ 2marks

$t_3 = 40 \quad t_6 = 25.$

$40 = a + (3-1)d \qquad 25 = a + (6-1)d$

$40 = a + 2d \qquad 25 = a + 5d$

$-2d$ SUB

$a = 40 - 2d \qquad 25 = 40 - 2d + 5d$

$25 = 40 + 3d$

-40

$a = 40 - 2(5)$ SUB BACK $\frac{-15}{3} = \frac{3d}{3} \rightarrow \boxed{d = -5}$

$a = 40 + 10$

$\boxed{a = 50}$

$40 = a + 2d$

$-(25 = a + 5d)$

$\frac{15}{-3} = \frac{-3d}{-3}$

$\boxed{d = -5}$

$40 = a + 2(-5)$ $\frac{1.5}{2}$

$40 = a - 10$

$+10$

$\boxed{a = 50}$ $\frac{1}{2}$

6. Determine the sum of the first 13 terms of the following arithmetic series:

1 mark

$8+5+2-1-...$

a = 8

d = −3

n = 13.

$S_{13} = \frac{13}{2}\left[2(8) + (13-1)(-3)\right]$

$= \frac{13}{2}\left[16 + (12)(-3)\right]$

$= \frac{13}{2}\left[16 - 36\right]$

$= \frac{13}{2}\left[\overset{20}{-\cancel{30}}\right]$

Page 2 of 2

$\boxed{S_{13} = -130}$

1.3 Geometric Sequences.

Recall: Arithmetic Sequences have a common difference

$$d = t_2 - t_1 \qquad (d = t_n - t_{n-1})$$

± b/w terms

Geometric → it is a common ratio which is determined by dividing

$$r = \frac{t_2}{t_1} \qquad \left(r = \frac{t_n}{t_{n-1}} \right)$$

× b/w terms.

p.22

Ex: 1a) $4, 12, 36, 72, \ldots$

$r = \frac{12}{4}$ $\quad r = \frac{36}{12}$ $\quad r = \frac{72}{36}$

$r = 3$ $\quad r = 3$ $\quad r = 2$ $\quad$ ∴ not Geometric.

i) $3x^2, 12x^4y^3, 48x^6y^6, \ldots$

$r = \frac{12x^4y^3}{3x^2}$ $\qquad r = \frac{48x^6y^6}{12x^4y^3}$ $\qquad$ ∴ is geometric

$r = 4x^2y^3$ $\qquad r = 4x^2y^3$

p.23

Geometric Sequences Formula

$$t_n = ar^{n-1}$$

4) a) a_{11} $\quad a_1 = \frac{1}{128}$ $\quad r = 2.$

$a = \frac{1}{128}$

$r = 2$

$n = 11$

$$t_{11} = \frac{1}{128}(2)^{\overset{①}{11-1}}$$

$$= \frac{1}{128}\underbrace{(2)^{10}}_{②}$$

$$= \underbrace{\frac{1}{128}(1024)}_{③}$$

$$\boxed{t_{11} = 8}$$

Recall: Ex3 p.21

$$\begin{tikzcd} t_1 \arrow[r, bend right, "\times r"'] & t_2 \arrow[r, bend right, "\times r"'] & t_3 \arrow[r, bend right, "\times r"'] & {} \end{tikzcd}$$

$$\begin{tikzcd} t_4 \arrow[r, bend right, "r \times"'] & t_5 \arrow[r, bend right, "r \times"'] & t_6 \arrow[r, bend right, "r \times"'] & t_7 \arrow[r, bend right, "r \times"'] & t_8 \arrow[r, bend right, "r"'] & t_9 \end{tikzcd}$$

$$= r^5$$

$$t_4 \cdot r^5 = t_9$$

p.23 4 d) a_9 $\quad a_4 = 5$, $a_6 = 20$

$$\begin{tikzcd} a_4 \arrow[r, bend right, "r"'] & a_5 \arrow[r, bend right, "r"'] & a_6 \end{tikzcd}$$

$$= r^2$$

$$a_4 \cdot r^2 = a_6$$

solve:

$$\frac{5r^2}{5} = \frac{20}{5}$$

$$\sqrt{r^2} = \sqrt{4}$$

$$r = \pm 2$$

★ recall:
always write
± when square-
rooting with even
index

$r^2 \rightarrow r = \pm$

$r^4 \rightarrow r = \pm$

★ odd index, answer
is same sign.

$r^3 \rightarrow r = +$

$r^3 \rightarrow r = -$

$$a_4 = ar^{n-1}$$

$$\downarrow$$

$$5 = a(\pm 2)^{4-1}$$

$$5 = a(\pm 2)^3$$

$$\frac{5}{\pm 8} = \frac{\pm 8a}{\pm 8}$$

$$\pm\frac{5}{8} = a$$

$$a_9 = ar^{9-1}$$

$$= \left(\pm\frac{5}{8}\right)\underbrace{(\pm 2)^8}$$

$$= \left(\pm\frac{5}{\cancel{8}_{1}}\right)(\cancel{256}^{32})$$

$$\boxed{a_9 = \pm 160}$$

Alternate: Method

$$a_9 = a_6 \cdot r^3$$

$$\begin{tikzcd} a_6 \arrow[r, bend right, "r \times"'] & \_ \arrow[r, bend right, "r \times"'] & \_ \arrow[r, bend right, "r"'] & a_9 \end{tikzcd}$$

$$= r^3$$

$$a_9 = 20r^3$$

$$= 20(\pm 2)^3$$

$a_9 = 20r^3$

$= 20(\pm 2)^3$

$= 20(\pm 8)$

$\boxed{a_9 = \pm 160}$

#4 p.23

e) n if $a_1 = 729$, $a_2 = 243$, $l = \frac{1}{9}$

?

$r = \frac{a_2}{a_1}$

$= \frac{243}{729}$

$r = \frac{1}{3}$

$a = 729$

l = last term

$t_n = \frac{1}{9}$

$t_n = ar^{n-1}$

$\frac{\frac{1}{9}}{729} = \frac{729\left(\frac{1}{3}\right)^{n-1}}{729}$

$\frac{1}{9} \times \frac{1}{729}$

make a common base →

$\frac{1}{6561} = \left(\frac{1}{3}\right)^{n-1}$

$\frac{1}{3^n} = \frac{1}{6561}$

$3^8 = 6561$

$\left(\frac{1}{3}\right)^{n=8} = \left(\frac{1}{3}\right)^{n-1}$

$\cancel{\left(\frac{1}{3}\right)}^8 = \cancel{\left(\frac{1}{3}\right)}^{n-1}$

$8 = n - 1$

$+1$

$\boxed{n = 9}$

1.4 Geometric Series

↳ = sum of terms.

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{or} \quad S_n = \frac{a - rl}{1-r}$$

a = first term

r = common ratio

n = number of terms

l = last term of series

#1 p.28

r = common ratio

n= number of terms

#1 p.28

Ex. 1b) S_9 = sum of nine terms.

$a=-6$, $r=2$, $n=9$

$S_n = \frac{a(1-r^n)}{1-r}$

1g) ↓

$S_9 = \frac{-6(1-\overset{①}{2^9})}{\underbrace{1-2}_{①}}$

★ BEDMAS

$= \frac{\underbrace{-6\,(1-\overset{②}{512})}_{②}}{-1}$

$\frac{-6}{-1} \times (1-512)$

$= \underbrace{6\,(-511)}_{④}$

$\boxed{S_9 = -3066}$

1g) $t_3 = ?$ $\quad S_5 = 93 \quad r = 2.$

n=3 a?

n=5

determine a

① $S_n = \frac{a(1-r^n)}{1-r}$

↓

$93 = \frac{a(1-\overset{①}{2^5})}{\underbrace{1-2}_{①}}$

$93 = \frac{a(\overset{②}{1-32})}{-1}$

$93 = \frac{a(-31)}{-1}$ ③

$\frac{93}{31} = \frac{31a}{31}$

$a = 3$

② $t_n = ar^{n-1}$

$t_3 = 3(2)^{3-1}$

$= 3(2)^2$

$= 3(4)$

$\boxed{t_3 = 12}$

h) r, if $S_3 = 39$, $a = 3$

n=3

a=3

$S_3=39$

r=?

$S_n = \frac{a(1-r^n)}{1-r}$

$39 = \frac{\cancel{3}(1-r^3)}{1-r}$

divide both sides by 3

$S_3 = 39$

$r = ?$

$$\frac{39}{3} = \frac{\frac{\cancel{3}(1-r^3)}{1-r}}{\cancel{3}}$$

divide both sides by 3

$$13 = \frac{1-r^3}{1-r}$$

factor out -1 to change signs

$$13 = \frac{\cancel{-}(r^3-1)}{\cancel{-}(r-1)}$$

$$13 = \frac{r^3-1}{r-1}$$

factor a difference of cubes (SOAP)

$$13 = \frac{\cancel{(r-1)}(r^2+r+1)}{\cancel{r-1}}$$

simplify

$$13 = r^2 + r + 1$$

$$\hookrightarrow \qquad -13$$

move terms to one side + make equation =0

$$0 = r^2 + r - 12$$

$AC = -12$

$4, -3 = 1$

trinomial factoring AC rule.

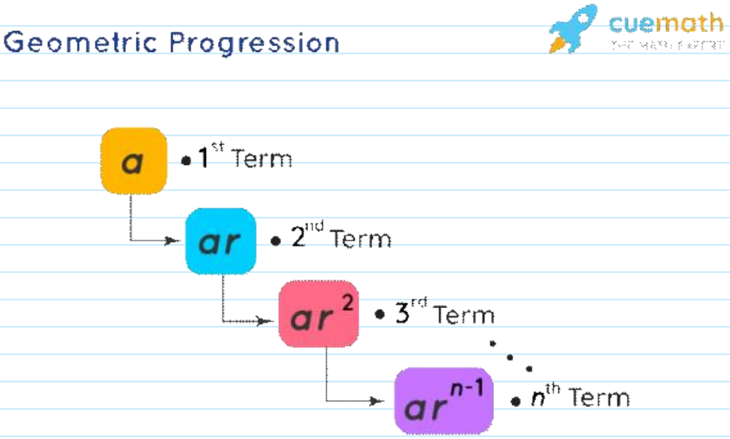
$$0 = (r+4)(r-3)$$

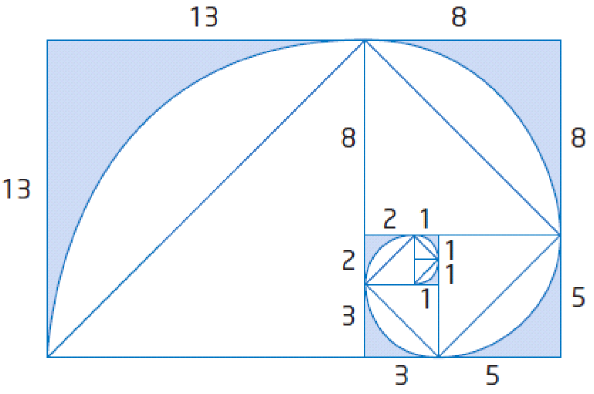
$$\downarrow \qquad \downarrow$$

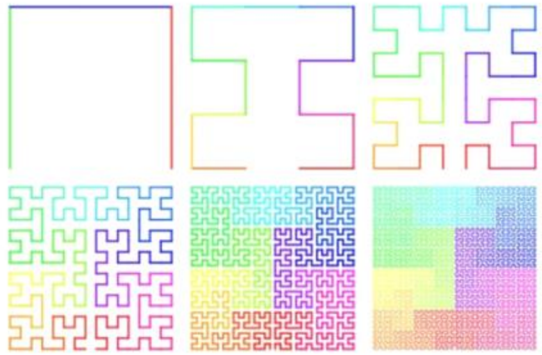
$$\boxed{r=-4 \qquad r=3}$$

solve for 'r'

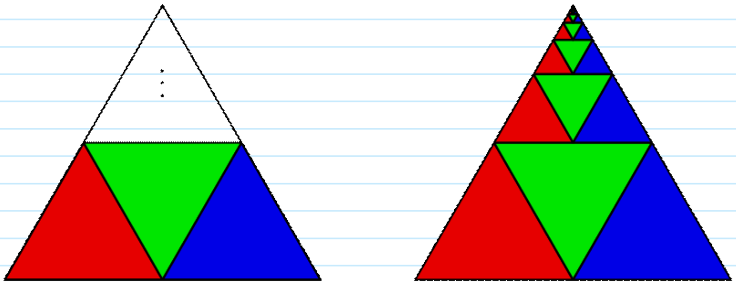
1.3 Geometric Sequences



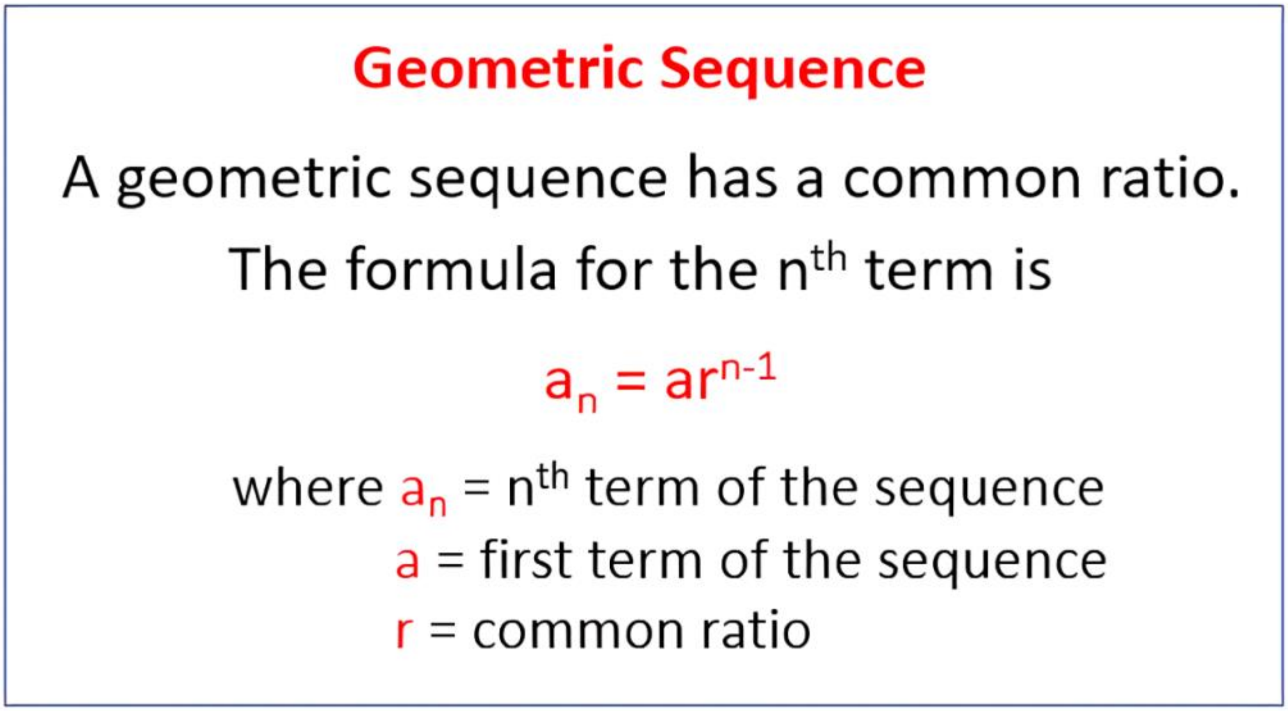




Fractal are self-similar patterns that repeat at all levels of scale.



Example: Area of the green triangle is reducing by 1/4 for each fractal level.



Geometric Sequence

A geometric sequence has a common ratio.

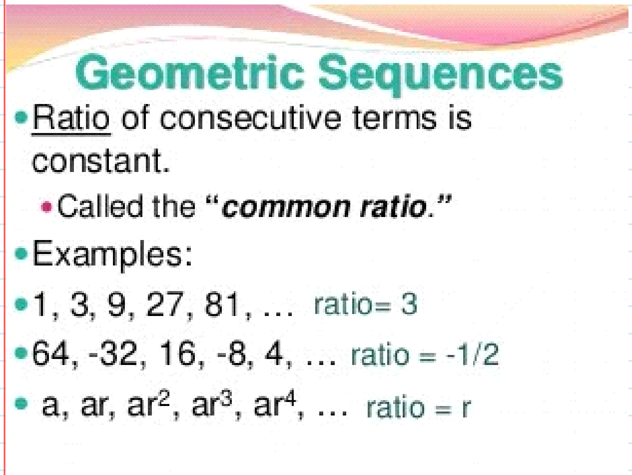
The formula for the nth term is

$$a_n = ar^{n-1}$$

where a_n = nth term of the sequence

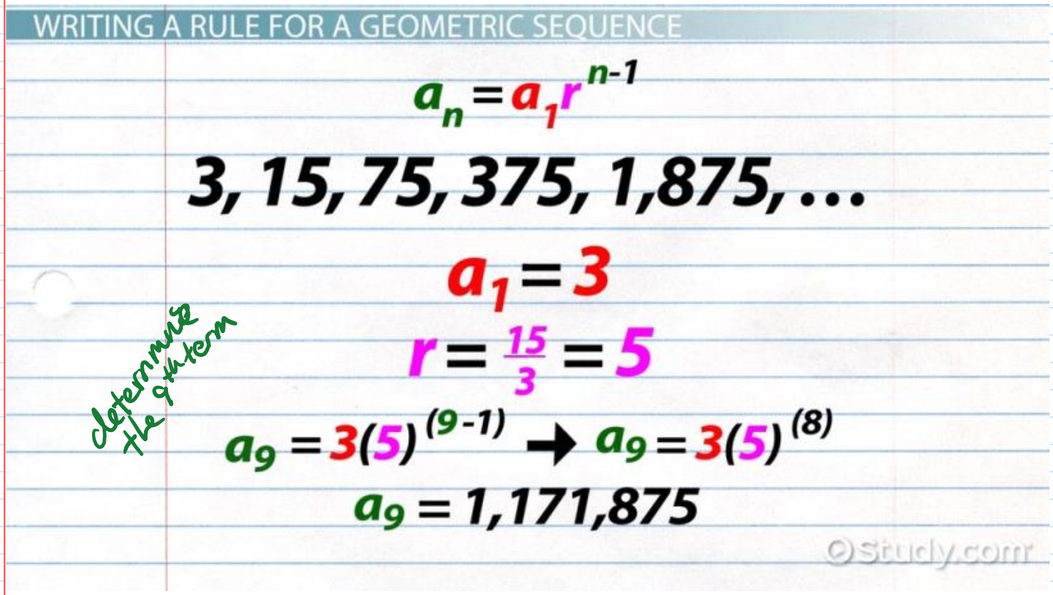
a = first term of the sequence

r = common ratio



Geometric Sequences

- Ratio of consecutive terms is constant.
  - Called the "***common ratio.***"
- Examples:
- 1, 3, 9, 27, 81, … ratio= 3
- 64, -32, 16, -8, 4, … ratio = -1/2
- a, ar, ar^2, ar^3, ar^4, … ratio = r



WRITING A RULE FOR A GEOMETRIC SEQUENCE

$$a_n = a_1 r^{n-1}$$

3, 15, 75, 375, 1,875, …

$$a_1 = 3$$

$$r = \frac{15}{3} = 5$$

determine the 9th term

$$a_9 = 3(5)^{(9-1)} \rightarrow a_9 = 3(5)^{(8)}$$

$$a_9 = 1,171,875$$

- Consider the geometric sequence:

  3, 6, 12, 24, 48, ...

  This sequence has $t_1 = 3$ and common ratio $r = 2$. Thus:

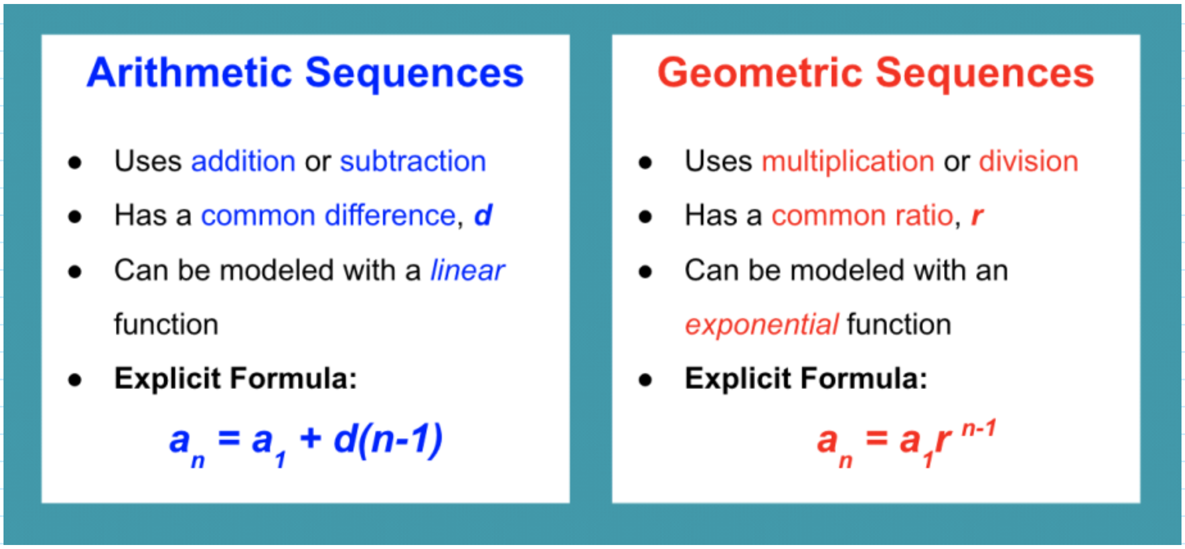
$$t_1 = 3$$

$$t_2 = 3 \cdot 2$$

$$t_3 = 3 \cdot 2 \cdot 2 = 3 \cdot 2^2$$

$$t_4 = 3 \cdot 2 \cdot 2 \cdot 2 = 3 \cdot 2^3$$

$$t_n = 3 \cdot 2^{n-1}$$



Arithmetic Sequences

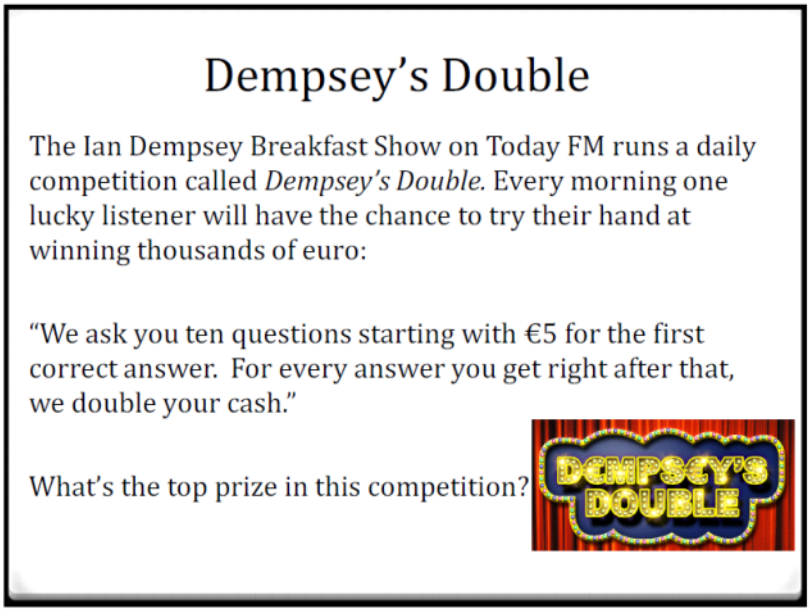
- Uses addition or subtraction
- Has a common difference, ***d***
- Can be modeled with a *linear* function
- **Explicit Formula:**

$$a_n = a_1 + d(n-1)$$

Geometric Sequences

- Uses multiplication or division
- Has a common ratio, ***r***
- Can be modeled with an *exponential* function
- **Explicit Formula:**

$$a_n = a_1 r^{n-1}$$



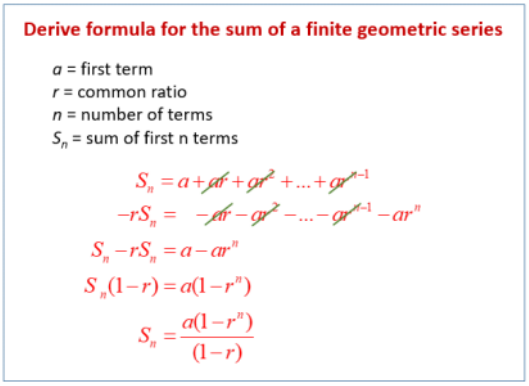
Dempsey's Double

The Ian Dempsey Breakfast Show on Today FM runs a daily competition called *Dempsey's Double*. Every morning one lucky listener will have the chance to try their hand at winning thousands of euro:

"We ask you ten questions starting with €5 for the first correct answer. For every answer you get right after that, we double your cash."

What's the top prize in this competition?

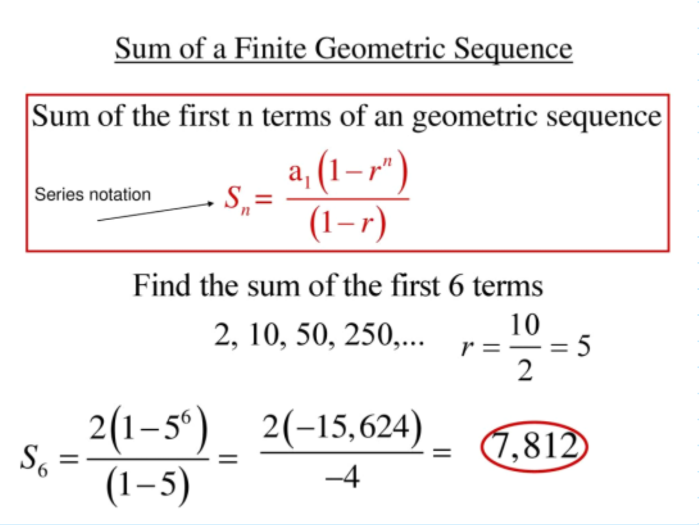
1.4 Geometric Series



Derive formula for the sum of a finite geometric series

a = first term
r = common ratio
n = number of terms
S_n = sum of first n terms

$$S_n = a + \cancel{ar} + \cancel{ar^2} + \ldots + \cancel{ar^{n-1}}$$
$$-rS_n = \quad -\cancel{ar} - \cancel{ar^2} - \ldots - \cancel{ar^{n-1}} - ar^n$$
$$S_n - rS_n = a - ar^n$$
$$S_n(1-r) = a(1-r^n)$$
$$S_n = \frac{a(1-r^n)}{(1-r)}$$



Sum of a Finite Geometric Sequence

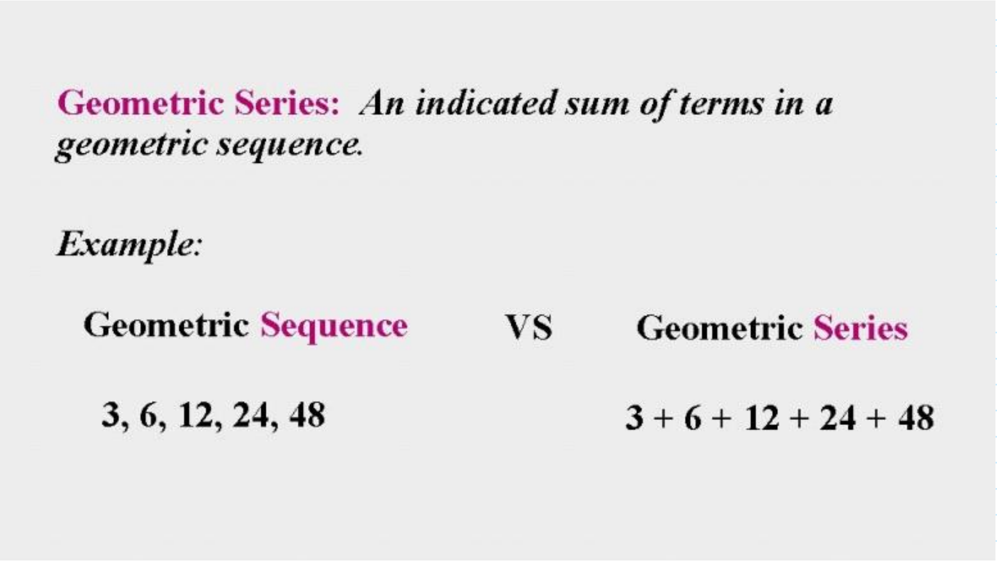
Sum of the first n terms of an geometric sequence

Series notation $\longrightarrow$ $S_n = \dfrac{a_1\left(1-r^n\right)}{\left(1-r\right)}$

Find the sum of the first 6 terms

2, 10, 50, 250,... $\quad r = \dfrac{10}{2} = 5$

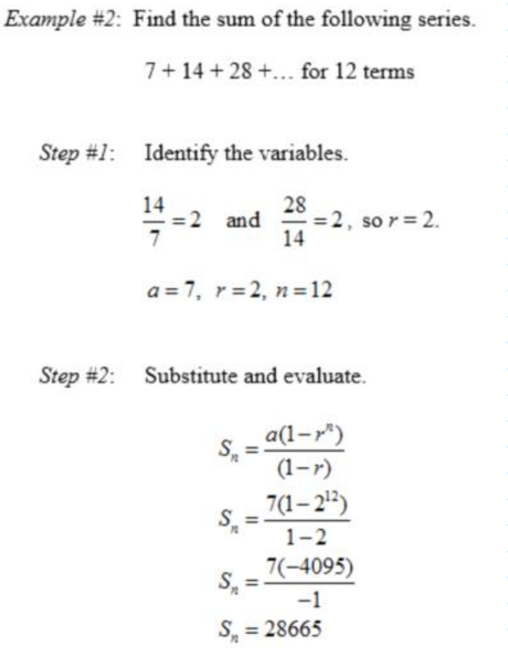
$$S_6 = \frac{2\left(1-5^6\right)}{\left(1-5\right)} = \frac{2(-15{,}624)}{-4} = \boxed{7{,}812}$$



Geometric Series: *An indicated sum of terms in a geometric sequence.*

Example:

Geometric Sequence	VS	Geometric Series
3, 6, 12, 24, 48		3 + 6 + 12 + 24 + 48



Example #2: Find the sum of the following series.

$7 + 14 + 28 + \ldots$ for 12 terms

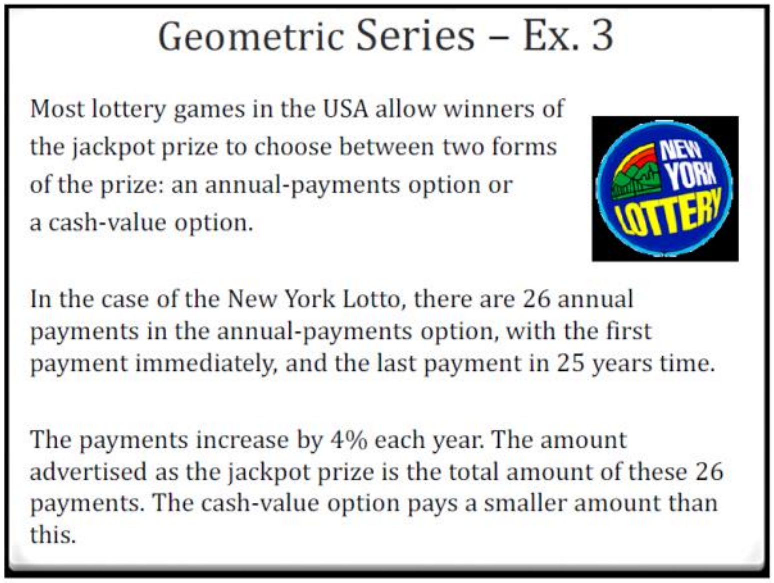
Step #1: Identify the variables.

$\frac{14}{7} = 2$ and $\frac{28}{14} = 2$, so $r = 2$.

$a = 7,\ r = 2,\ n = 12$

Step #2: Substitute and evaluate.

$$S_n = \frac{a(1-r^n)}{(1-r)}$$
$$S_n = \frac{7(1-2^{12})}{1-2}$$
$$S_n = \frac{7(-4095)}{-1}$$
$$S_n = 28665$$

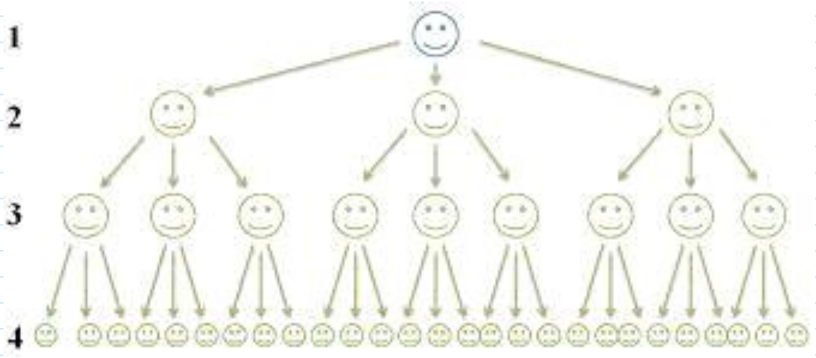


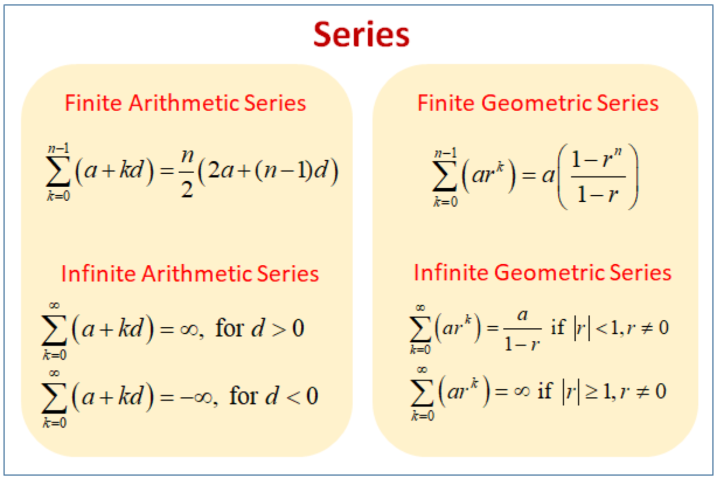
Geometric Series – Ex. 3

Most lottery games in the USA allow winners of the jackpot prize to choose between two forms of the prize: an annual-payments option or a cash-value option.

In the case of the New York Lotto, there are 26 annual payments in the annual-payments option, with the first payment immediately, and the last payment in 25 years time.

The payments increase by 4% each year. The amount advertised as the jackpot prize is the total amount of these 26 payments. The cash-value option pays a smaller amount than this.





Series

Finite Arithmetic Series

$$\sum_{k=0}^{n-1}(a+kd) = \frac{n}{2}(2a+(n-1)d)$$

Finite Geometric Series

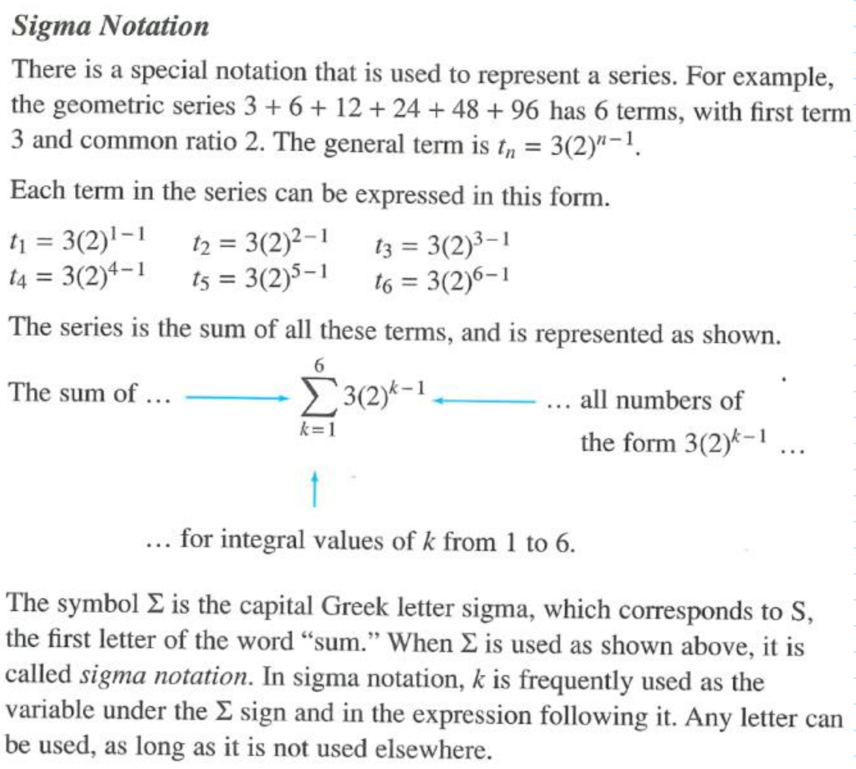
$$\sum_{k=0}^{n-1}(ar^k) = a\left(\frac{1-r^n}{1-r}\right)$$

Infinite Arithmetic Series

$$\sum_{k=0}^{\infty}(a+kd) = \infty, \text{ for } d > 0$$
$$\sum_{k=0}^{\infty}(a+kd) = -\infty, \text{ for } d < 0$$

Infinite Geometric Series

$$\sum_{k=0}^{\infty}(ar^k) = \frac{a}{1-r} \text{ if } |r| < 1, r \neq 0$$
$$\sum_{k=0}^{\infty}(ar^k) = \infty \text{ if } |r| \geq 1, r \neq 0$$



Sigma Notation

There is a special notation that is used to represent a series. For example, the geometric series $3 + 6 + 12 + 24 + 48 + 96$ has 6 terms, with first term 3 and common ratio 2. The general term is $t_n = 3(2)^{n-1}$.

Each term in the series can be expressed in this form.

$t_1 = 3(2)^{1-1}$ $t_2 = 3(2)^{2-1}$ $t_3 = 3(2)^{3-1}$

$t_4 = 3(2)^{4-1}$ $t_5 = 3(2)^{5-1}$ $t_6 = 3(2)^{6-1}$

The series is the sum of all these terms, and is represented as shown.

$$\text{The sum of} \ldots \longrightarrow \sum_{k=1}^{6} 3(2)^{k-1} \longleftarrow \ldots \text{ all numbers of the form } 3(2)^{k-1} \ldots$$

$\uparrow$

… for integral values of k from 1 to 6.

The symbol Σ is the capital Greek letter sigma, which corresponds to S, the first letter of the word "sum." When Σ is used as shown above, it is called *sigma notation.* In sigma notation, k is frequently used as the variable under the Σ sign and in the expression following it. Any letter can be used, as long as it is not used elsewhere.