Wednesday, May 8th

Plan For Today:

1. Go over Test 1. Any questions or concerns?
2. Any questions about material from last class? (Translations)
3. Continue working through transformations in Chapter 2
   - ✓ 2.0 Graphing Review
   - ✓ 2.4 Horizontal and Vertical Translations
   - **2.4 Reflections**
   - **2.4 Stretches**
   - 2.5 Inverse of a Relation
   - **2.6 Combining Transformations**

$$f(x) = af(b(x-c)) + d$$

4. Work on practice questions in workbook and practice questions handout.
5. Work on Ch2 Transformations Desmos project.

   Project for Chapter 2 is **online**.
   Join my PC12 Class in Desmos: https://tinyurl.com/PC12-Spring2024
   Start the Ch2 Transformations Project in Desmos: https://tinyurl.com/Ch1-Project-Spring2024

Plan Going Forward:

1. Work on practice questions from 2.4 & 2.6 in the workbook. Start working on Ch1 Project (in Desmos).

   - **2.4 & 2.6 COMBINING TRANSFORMATIONS CHECK-IN QUIZ ON THURSDAY, MAY 9TH**

2. We will finish Ch2 with 2.5 Inverse Functions on Thursday and review Ch1-2 for the U1 Exam.

   - **UNIT 1 EXAM ON CH1&2 ON MONDAY, MAY 13TH**
     - 12 Multiple Choice & 20 marks on the Written
     - ~1.5 hours - please prepare so you are not "learning" while doing the test
     - Closed-book - no notes, formula sheet provided
     - I'll try to mark it for Tuesday

3. We will start Chapter 3 Polynomials on Monday after the Unit 1 Exam (depending on time)

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.
Anurita Dhiman = adhiman@sd35.bc.ca

Translation Practice

p.79 #2abc #1ace #9d p81

Reflections.

Base

$y=f(x)$ → Reflection in y-axis is a horizontal reflection ∴ x becomes $-x$

$y=f(-x)$

$y=f(x)$ → Reflection in x-axis is a vertical reflection ∴ y becomes $-y$ or $-f(x)$

$y=-f(x)$

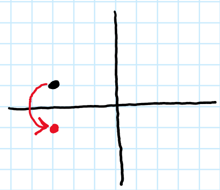
#2 d e f p.79

$(-3,1)$ (x,y)

2d) $y=-f(x)$

reflection in x-axis → multiply y-coordinate by (-1)

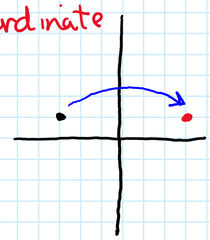
$(-3,1) \rightarrow (-3,-1)$



2e) $y=f(-x)$

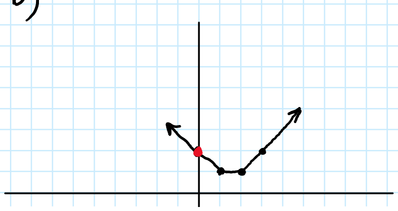
reflection in y-axis → multiply x-coordinate by (-1)

$(-3,1) \rightarrow (3,1)$



#9 p.81 a) b)

x	y
0	2
1	1
2	1



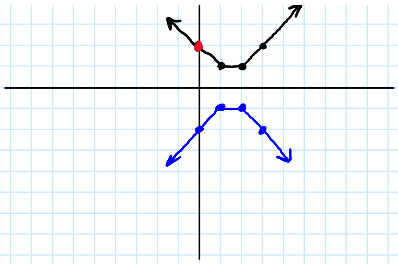
9a) $y=-f(x)$

ref. in x-axis

$(x,-y)$

x	$-y$

0	2
1	1
2	1
3	2



$(x, -y)$

x	$-y$
0	-2
1	-1
2	-1
3	-2

9b) $y = f(-x)$

$-x$	y
0	2

invariant = point remains the same after transformation

finish ↗

Try #10.

Compressions + Expansions (Stretches)

Base Function

$y = f(x) \longrightarrow y = a f(bx)$

a = vertical stretch by a factor of a

$y = 2f(x) \Rightarrow$ vert. expansion of 2 (VE of 2)

$(x, 2y)$

$y = \frac{1}{3}f(x) \Rightarrow$ vert. compression of $\frac{1}{3}$ (VC of $\frac{1}{3}$)

$(x, \frac{1}{3}y)$

b = reciprocal to $\frac{1}{b}$

horizontal stretch by a factor of $\frac{1}{b}$

$y = f(3x) \Rightarrow$ horiz. compression of $\frac{1}{3}$ (HC of $\frac{1}{3}$)

$(\frac{1}{3}x, y)$

$y = f(\frac{2}{5}x) \Rightarrow$ horiz. expansion of $\frac{5}{2}$ (HE of $\frac{5}{2}$)

$(\frac{5}{2}x, y)$

2.6 Combining Transformations. p. 95

$y = f(x) \longrightarrow y = a f(b(x - c)) + d$

Base Function

(x, y)

- a: vert. stretch by a; if $-a$ = ref. in x-axis
- b: horiz. stretch by $\frac{1}{b}$; if $-b$ = ref. in y-axis.
- c: horiz. translation of h; $(x-5)$ right 5; $(x+5)$ left 5
- d: vert. translation; $y = f(x) - 2$ 2 down; $y = f(x) + 2$ 2 up.

ORDER ① do stretches + reflections.

$(\frac{1}{b}x, ay)$

② do translations

$(\frac{1}{b}x + c, ay + d)$

(2) do translations

$(bx + c,\ ay + d)$ — full mapping notation.

Ex: $y = \sqrt{x} \longrightarrow y = -\frac{1}{2}\sqrt{4(x-2)} + 6$

(1) ref. in x-axis + VC of $\frac{1}{2}$ and HC of $\frac{1}{4}$

(2) 2 right + 6 up.

TRY #18-19 p.83-84 (2.4)

#7 p.99.

Graphing Review

Basic Graphing Review – Know these base functions and their graphs so you are able to apply transformations on them in the course.

- Label the x- and y- axis
- Make a table of values
- Plot the point on your grid
- Draw a line or smooth curve
- Domain: the set of x values valid in the equation
- Range: the set of y values valid in the equation

Use Set Notation for writing domain and range:
$\{x \mid x \in R\}$ means x is in the set of real numbers $\{x \in \mathbb{R}\}$
$\{y \mid y \in R\}$ means y is in the set of real numbers
Use the following symbols:
$\leq$ for less than and equal to; $<$ for only less than
$\geq$ for greater than and equal to: $>$ for only greater than
$\neq$ for not equal to

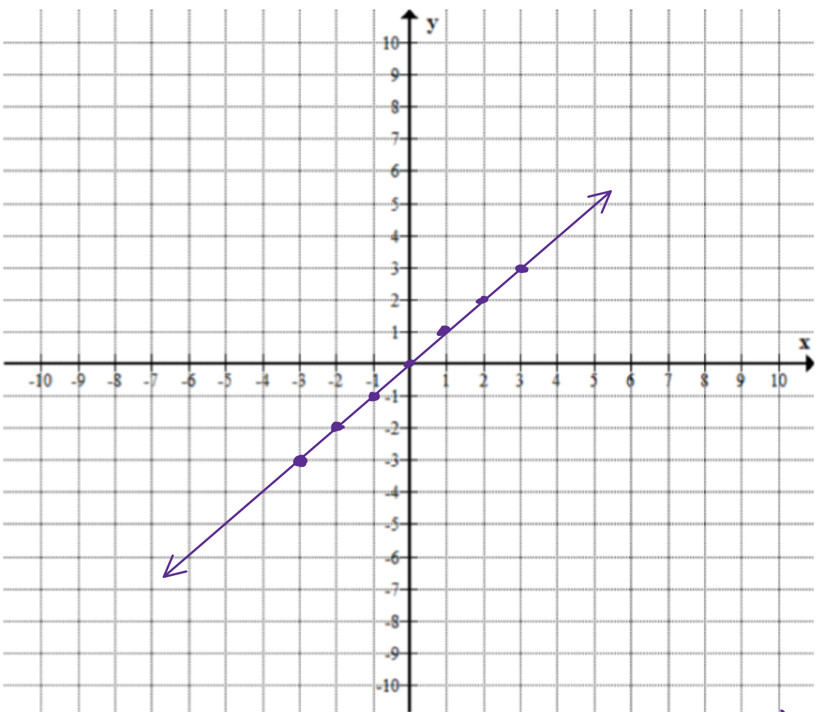
When graphing, start with a table of values. Look at restrictions and use your graphing calc to verify.

1. Graph: $y = x$

x: input, y: output

x	y
-3	-3
-2	-2
-1	-1
0	0
1	1
2	2
3	3

coordinates (x, y)



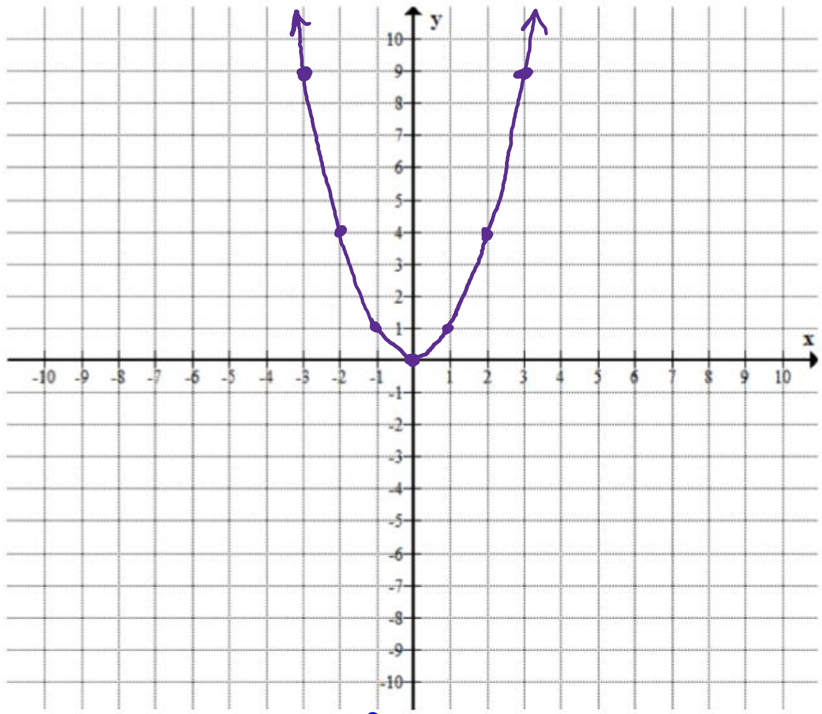
$\{x \in \mathbb{R}\}$
$\{y \in \mathbb{R}\}$

not in words "all real numbers"

2. Graph: $y = x^2$ ← called a quadratic function → graph is a parabola

input (x) → output (y)

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



$\{x \in \mathbb{R}\}$

$\{y \geq 0, y \in \mathbb{R}\}$

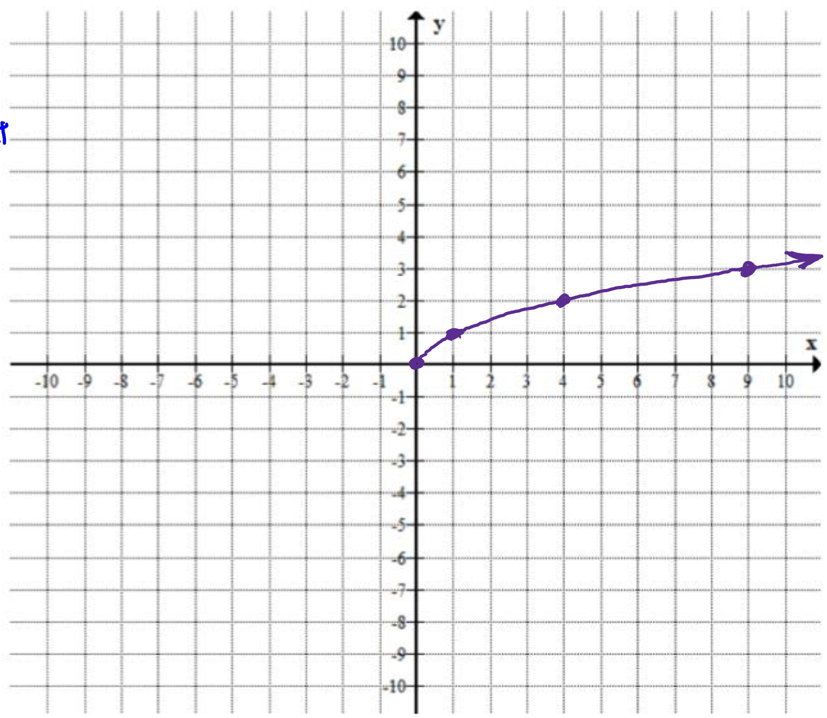
OR

$\{y \geq 0\}$.

3. Graph: $y = \sqrt{x}$ ← called a radical function

input (x) → output (y)

x	y
0	0
1	1
4	2
9	3
16	4



$\{x \geq 0\}$

$\{y \geq 0\}$.

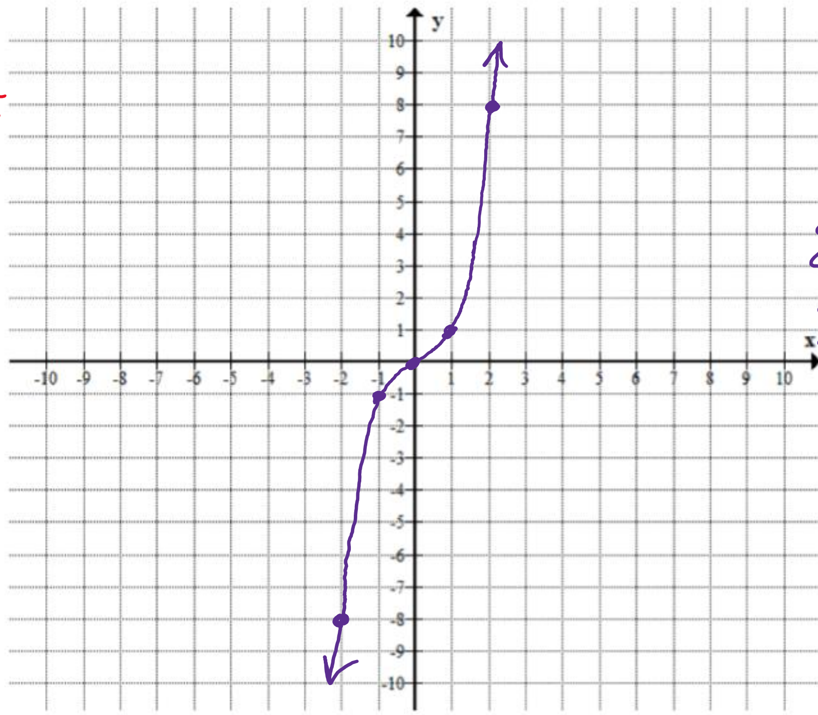
4. Graph: $y = x^3$ called a cubic function

$y=\sqrt[3]{x}$

x
0
1
8
27

input → output

x	y
−3	−27
−2	−8
−1	−1
0	0
1	1
2	8
3	27

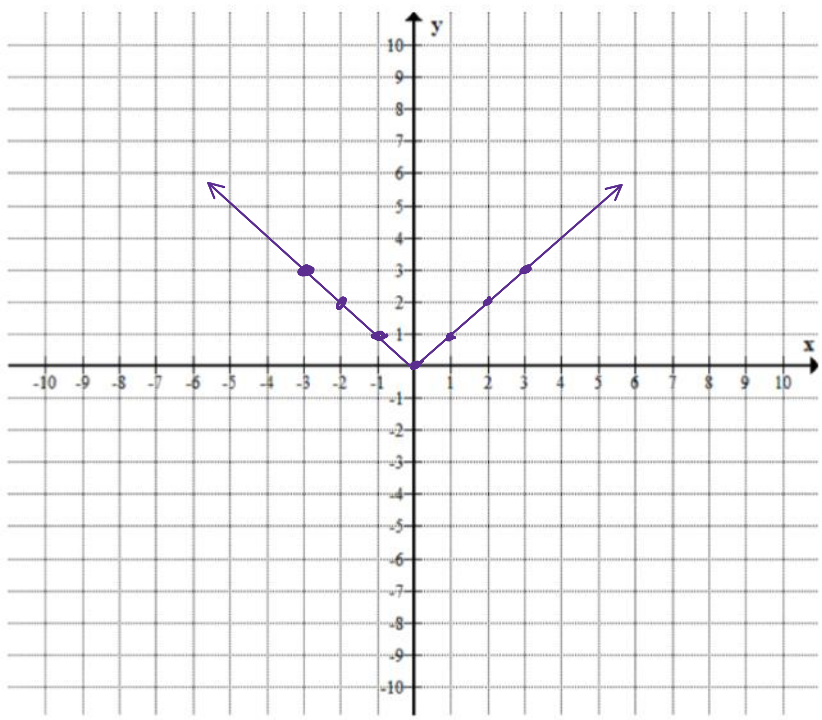


$\{x \mid x \in \mathbb{R}\}$

$\{y \mid y \in \mathbb{R}\}$

5. Graph: $y = |x|$ ← called an absolute value function

x	y
−3	3
−2	2
−1	1
0	0
1	1
2	2
3	3



$\{x \in \mathbb{R}\}$

$\{y \geq 0\}$.

Circle: $x^2+y^2=r^2$

$y^2=r^2-x^2$

$y=\pm\sqrt{r^2-x^2}$

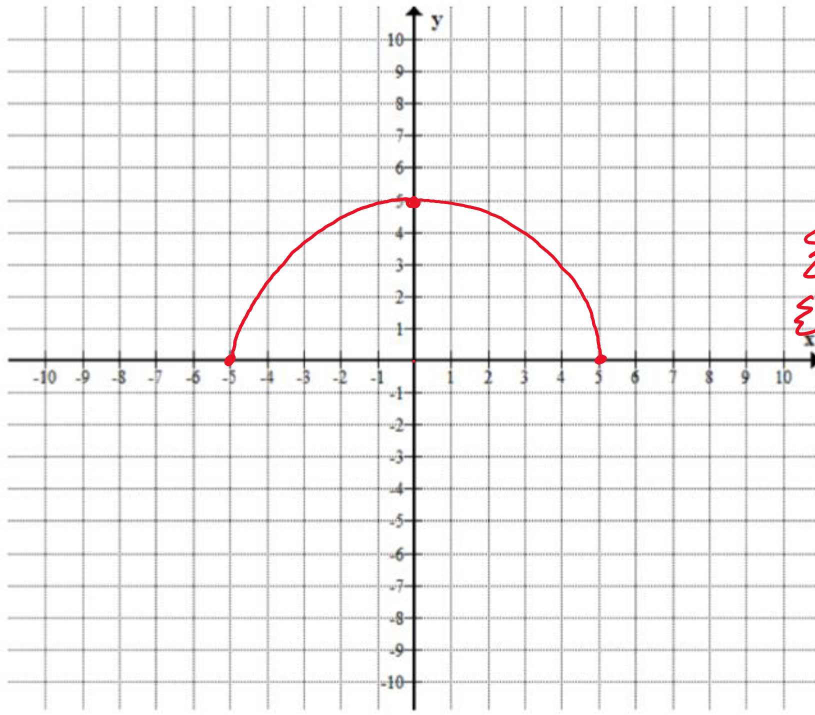
* $y=\sqrt{r^2-x^2}$

$y=-\sqrt{r^2-x^2}$

6. Graph: $y=\sqrt{25-x^2}$ (general: $y=\sqrt{r^2-x^2}$)

$r^2=25$ (not $r=-5$) $r=5$

x	y
-5	0
0	5
5	0

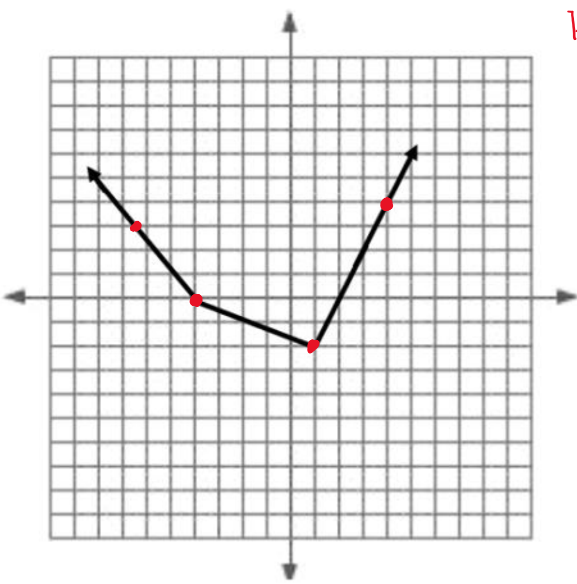


$\{x \mid -5 \le x \le 5\}$

$\{y \mid 0 \le y \le 5\}$

7. This is the graph of $y=f(x)$. List 4 or 5 points on this graph in the table of values.

left to right



x	y
-6.5	3
-4	0
1	-2
4	4

2.4 Reflections

reflection
- a transformation where each point of the original graph has an image point resulting from a reflection in a line
- may result in a change of orientation of a graph while preserving its shape

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

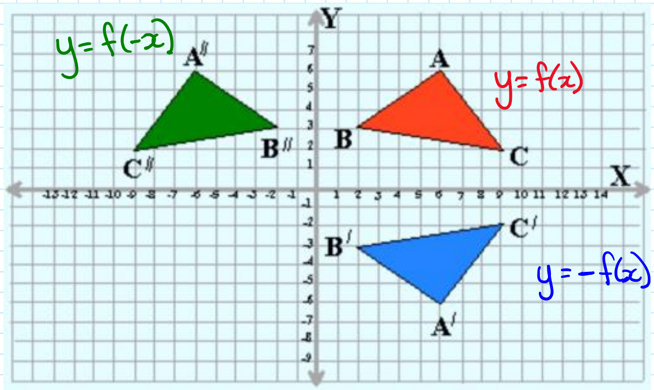
- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

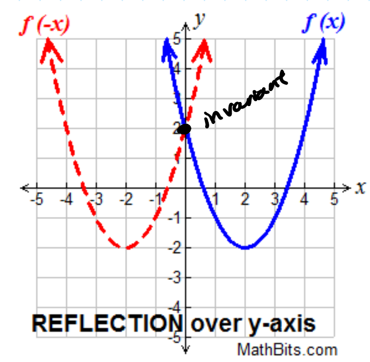
invariant point
- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

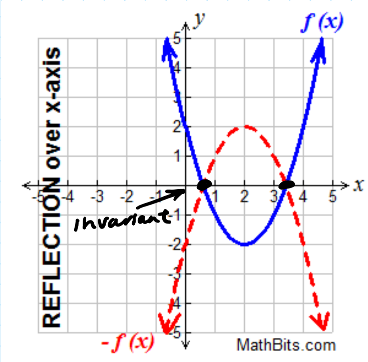
A GENERAL NOTE: REFLECTIONS

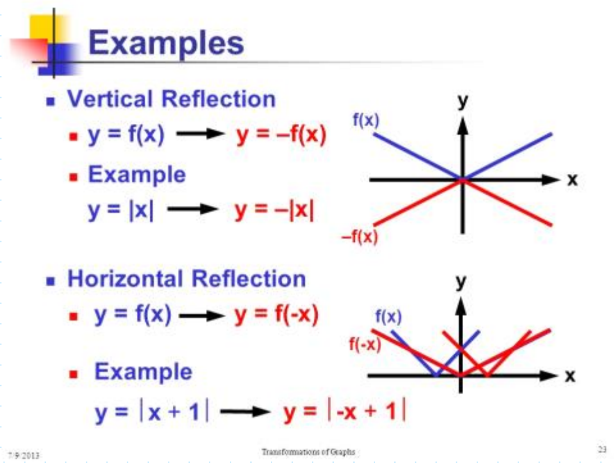
Given a function $f(x)$, a new function $g(x) = -f(x)$ is a **vertical reflection** of the function $f(x)$, sometimes called a reflection about (or over, or through) the x-axis.

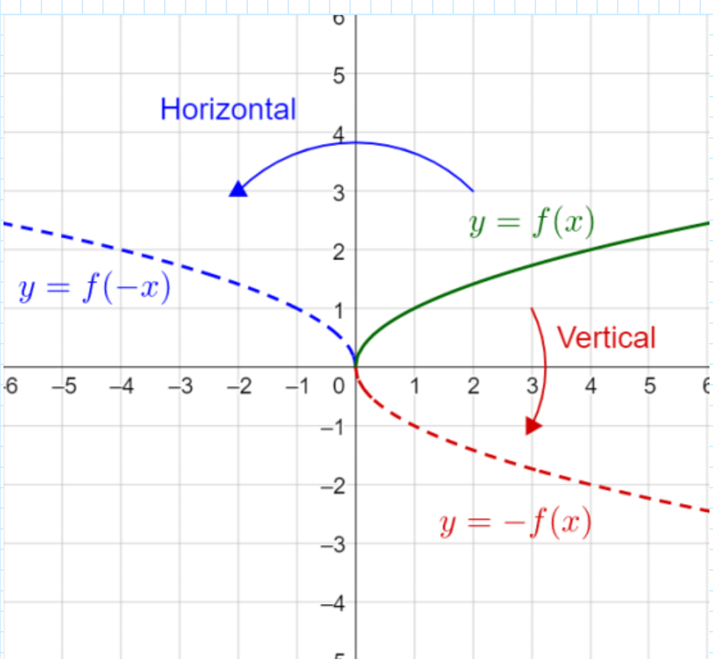
Given a function $f(x)$, a new function $g(x) = f(-x)$ is a **horizontal reflection** of the function $f(x)$, sometimes called a reflection about the y-axis.

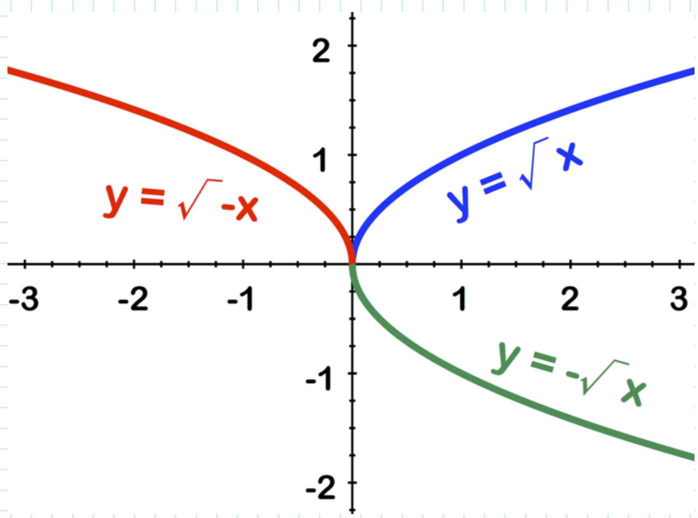


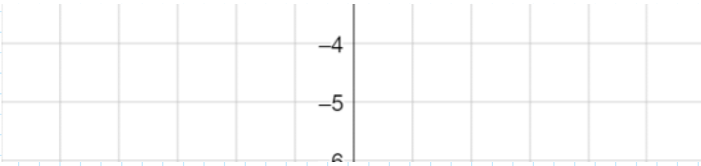


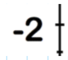


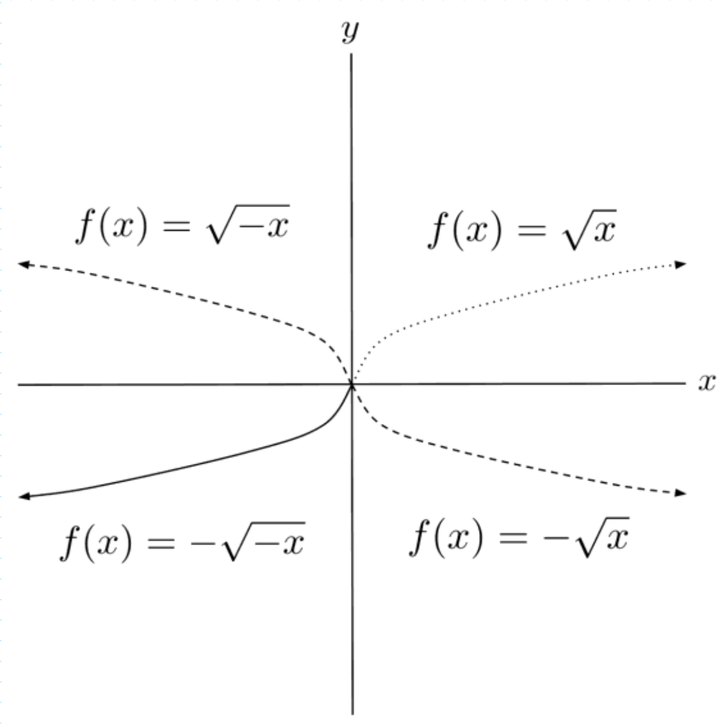












Reflection in x-axis	$(x, y) \rightarrow (x, -y)$
Reflection in y-axis	$(x, y) \rightarrow (-x, y)$
Reflection in $y = x$	$(x, y) \rightarrow (y, x)$
Reflection in $y = -x$	$(x, y) \rightarrow (-y, -x)$

Calcworkshop.com

2.4 Compressions/Expansions

Vertical and Horizontal Stretches

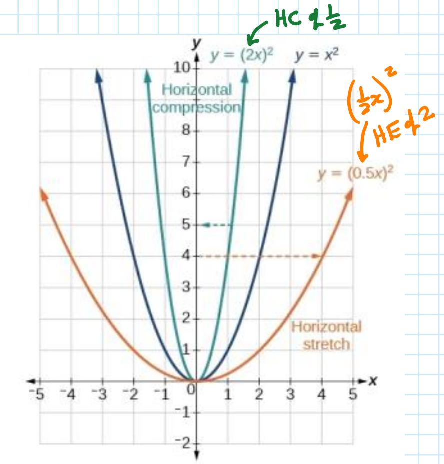
stretch

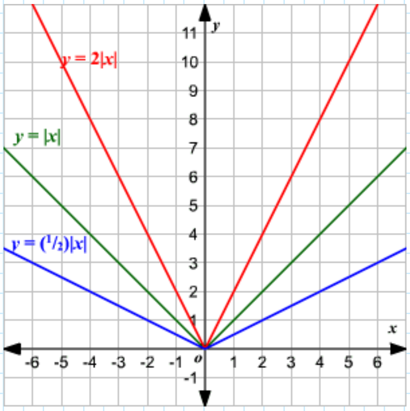
- a transformation in which the distance of each x-coordinate or y-coordinate from the line of reflection is multiplied by some scale factor
- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

$$y = f(x) \longrightarrow y = af(bx)$$





Transformation Rules for Functions		
Function Notation	**Type of Transformation**	**Change to Coordinate Point**
f(x) + d	Vertical translation up d units	(x, y) → (x, y + d)
f(x) − d	Vertical translation down d units	(x, y) → (x, y − d)
f(x + c)	Horizontal translation left c units	(x, y) → (x − c, y)
f(x − c)	Horizontal translation right c units	(x, y) → (x + c, y)
−f(x)	Reflection over x-axis	(x, y) → (x, −y)
f(−x)	Reflection over y-axis	(x, y) → (−x, y)
af(x)	Vertical stretch for |a| > 1	(x, y) → (x, ay)
	Vertical compression for 0 < |a| < 1	
f(bx)	Horizontal compression for |b| > 1	$(x, y) \rightarrow \left(\frac{x}{b}, y\right)$
	Horizontal stretch for 0 < |b| < 1	

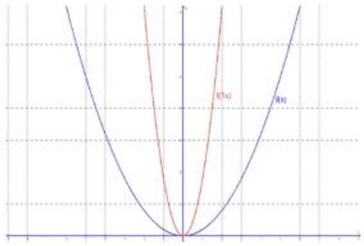
Horizontally Compressed

Example 1

The values and graph of the function $f(x)$ are shown in blue.
Make a table and a graph of the function $g(x) = f(3x)$.

Solution

x	$f(x)$	x	$g(x)$
-6	36	-2	36
-3	9	-1	9
-1	1	-1/3	1
0	0	0	0
1	1	1/3	1
3	9	1	9
6	36	2	36



2

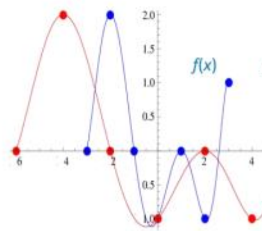
Horizontally Stretched

Example 1

The values and graph of the function $f(x)$ are shown in blue.
Make a table and a graph of the function $g(x) = f(½\, x)$.

Solution

x	$f(x)$	x	$g(x)$
-3	0	-6	0
-2	2	-4	2
-1	0	-2	0
0	-1	0	-1
1	0	2	0
2	-1	4	-1
3	1	6	1



3

Vertical Stretches $y - k = af(x - h)$

In general, for any function $y = f(x)$, the graph of the function $y = a\,f(x)$ has been vertically stretched about the x-axis by a factor of $|a|$.

The point $(x, y) \rightarrow (x, ay)$. Only the y coordinates are affected.

Invariant points are on the line of stretch, the x-axis.
are the x-intercepts.

When $|a| > 1$, the points on the graph move farther away from the x-axis. $y = 3f(x)$ Vertical stretch by a factor of 3

When $|a| < 1$, the points on the graph move closer to the x-axis. $y = \frac{1}{3}f(x)$ Vertical stretch by a factor of ⅓

Vertical Stretching $y = af(x)$, $|a| > 1$

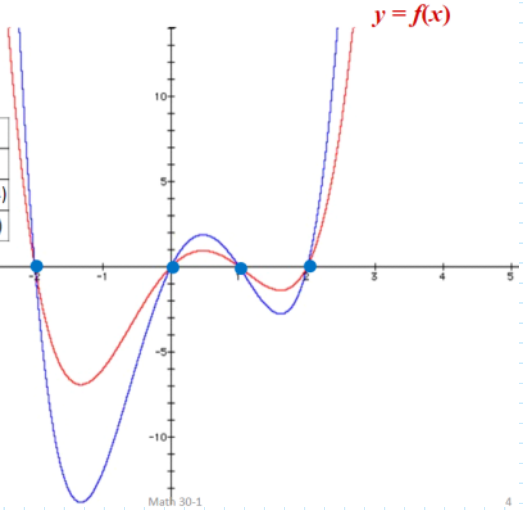
$y = 2f(x)$

A vertical stretch about the x-axis by a factor of 2.

$y = f(x)$

Key Points

(x, y)	→	(x, 2y)
(-2, 0)	→	(-2, 0)
(-1, -7)	→	(-1, -14)
(1/2, 1)	→	(1/2, 2)



Invariant Points

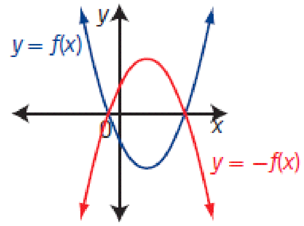
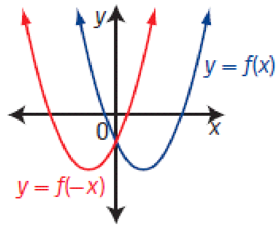
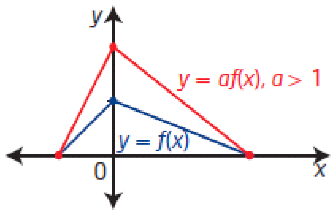
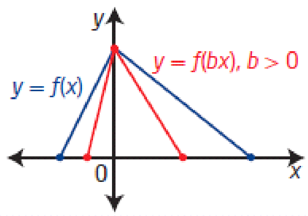
(-2, 0)
(0, 0)
(1, 0)
(2, 0)

Math 30-1

4

Key Ideas

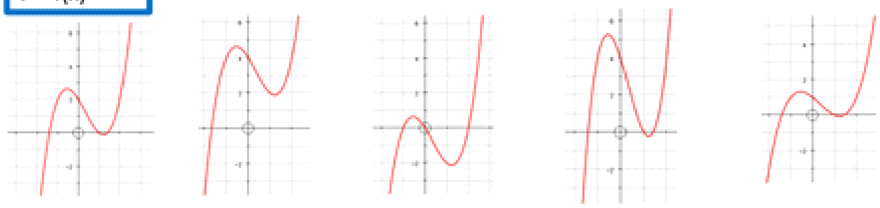
- Any point on a line of reflection is an invariant point.

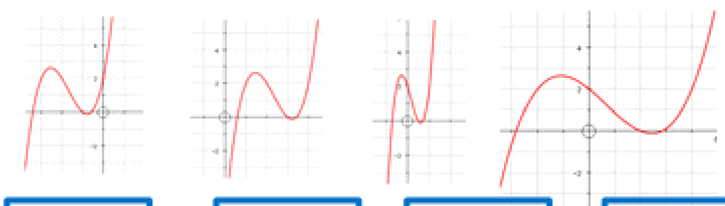
Function	Transformation from $y = f(x)$	Mapping	Example
$y = -f(x)$	A reflection in the x-axis	$(x, y) \rightarrow (x, -y)$	
$y = f(-x)$	A reflection in the y-axis	$(x, y) \rightarrow (-x, y)$	
$y = af(x)$	A vertical stretch about the x-axis by a factor of $\|a\|$; if $a < 0$, then the graph is also reflected in the x-axis	$(x, y) \rightarrow (x, ay)$	
$y = f(bx)$	A horizontal stretch about the y-axis by a factor of $\frac{1}{\|b\|}$; if $b < 0$, then the graph is also reflected in the y-axis	$(x, y) \rightarrow \left(\frac{x}{b}, y\right)$	

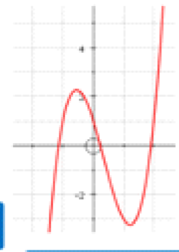
Transformations of Graphs

Using your knowledge of transformations of graphs match up the transformations of the function with the graph. The first one is done for you.

Y = f(x)







Y = f(x) + 2

Y = 2f(x)

Y = f(x+2)

Y = f(2x)

Y = 2f(x) – 3

This doubles in size and then moves down 3

Y = f(x) - 2

Y = ½ f(x)

Y = f(x-2)

Y = f(½ x)

Extension: The original graph has a peak at (-0.5, 2.5) Write the new location of this peak after the transformations for each graph. How has the peak moved and why has this happened?