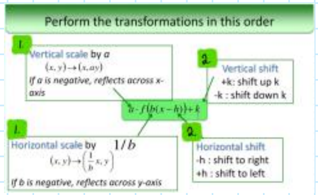
Thursday, May 9th

Plan For Today:

1. Review & any questions about material from last class? (Translations & Reflections)
   - **Do 2.4 & 2.6 Check-in Quiz**
2. Finish working through transformations in Chapter 2
   - ✓ 2.0 Graphing Review
   - ✓ 2.4 Horizontal and Vertical Translations
   - ✓ 2.4 Reflections
   - ✓ 2.4 Stretches
   - ✓ 2.6 Combining Transformations
   - ✸ **2.5 Inverse of a Relation**



$$f(x) = af(b(x-c)) + d$$

3. Work on practice questions in workbook and practice questions handout.
4. Work on Ch2 Transformations Desmos project.

Project for Chapter 2 is **online**.
Join my PC12 Class in Desmos (use your FULL NAME): https://tinyurl.com/PC12-Spring2024
Start the Ch2 Transformations Project in Desmos: https://tinyurl.com/Ch1-Project-Spring2024

5. Review for U1 Exam and Work on Practice Questions.

Plan Going Forward:

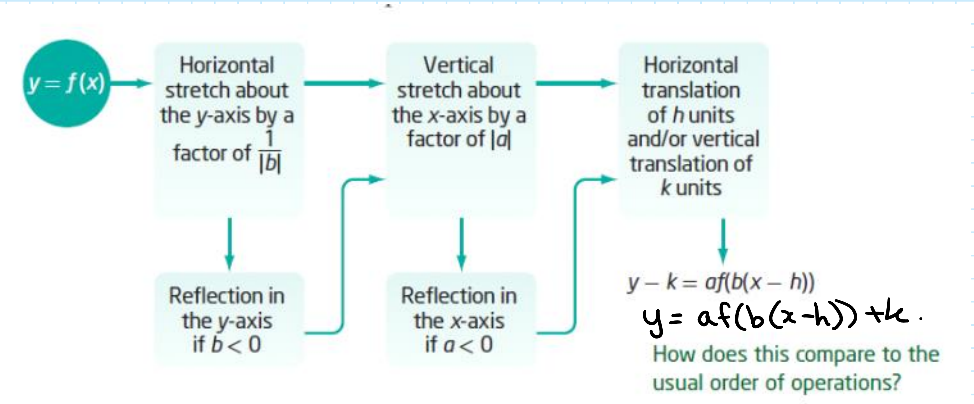
1. Work on practice questions from Ch1 & Ch2 (not 2.2-2.3) in the workbook. Finish the Desmos project online.

   - ✸ **Ch2 Desmos project due Monday, May 13th**
   - ✸ **UNIT 1 EXAM ON CH1&2 ON MONDAY, MAY 13TH**
     - 12 Multiple Choice & 20 marks on the Written
     - ~1.5 hours - please prepare so you are not "learning" while doing the test
     - Closed-book - no notes, formula sheet provided
     - I'll try to mark it for Tuesday
     - (rewrite is on last day of class Wednesday, June 19th)

2. We will start Chapter 3 Polynomials on Tuesday, May 14th

Please let me know if you have any questions or concerns about your progress in this course.
The notes from today will be posted at anurita.weebly.com after class.
Anurita Dhiman = adhiman@sd35.bc.ca

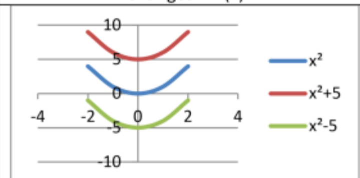
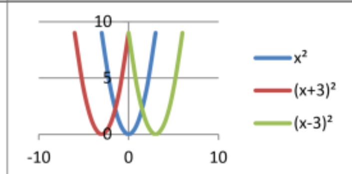
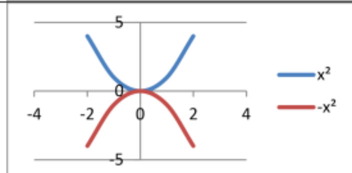
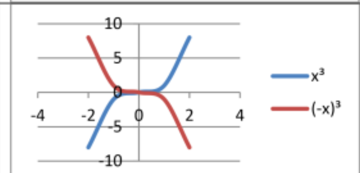
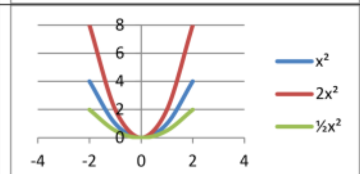
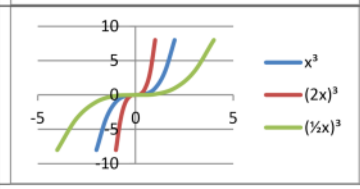
2.6 Combining Transformations



Transformation Rules for Functions		
Function Notation	**Type of Transformation**	**Change to Coordinate Point**
f(x) + d	Vertical translation up d units	(x, y) → (x, y + d)
f(x) − d	Vertical translation down d units	(x, y) → (x, y − d)
f(x + c)	Horizontal translation left c units	(x, y) → (x − c, y)
f(x − c)	Horizontal translation right c units	(x, y) → (x + c, y)
−f(x)	Reflection over x-axis	(x, y) → (x, −y)
f(−x)	Reflection over y-axis	(x, y) → (−x, y)
af(x)	Vertical stretch for |a| > 1	(x, y) → (x, ay)
	Vertical compression for 0 < |a| < 1	
f(bx)	Horizontal compression for |b| > 1	$(x, y) \rightarrow \left(\frac{x}{b}, y\right)$
	Horizontal stretch for 0 < |b| < 1	

Summary: standard form, mapping notation and order of performing transformations

Summary of Transformations

Graph	Draw the graph of f(x) and:	Changes in f(x)
Vertical shift $y = f(x) + c$ $y = f(x) - c$	Raise the graph of f(x) by c units -add c to y coordinate Lower the graph of f(x) by c units -subtract c from y coordinate	
Horizontal shift $y = f(x + c)$ $y = f(x - c)$	Shift the graph f(x) to the left c units -subtract c from x coordinate Shift the graph f(x) to the right c units -add c to x coordinate	
Reflection about the x-axis $y = -f(x)$	Reflect the graph of f(x) about the x-axis -multiply each y coordinate by -1	
Reflection about the y-axis $y = f(-x)$	Reflect the graph of f(x) about the y-axis -multiply each x coordinate by -1	
Vertical stretching and compression $y = cf(x), c > 1$ $y = cf(x), 0 < c < 1$	Vertically stretching the graph of f(x) $(c > 1)$ Vertically compressing the graph of f(x) $(0 < c < 1)$ -multiply each y coordinate by c	
Horizontal stretching and compression $y = f(cx), c > 1$ $y = f(cx), 0 < c < 1$	Horizontally compressing the graph of f(x) $(c > 1)$ Horizontally stretching the graph of f(x) $(0 < c < 1)$ -divide each x coordinate by c	
$y = \frac{1}{f(x)}$	Take the reciprocal of each y coordinate of f(x)	
Order of operations for transformations: 1) horizontal shifts 2) stretches/compressions 3) reflections 4) vertical shifts		

March 2017 MVCC Learning Commons Math Lab

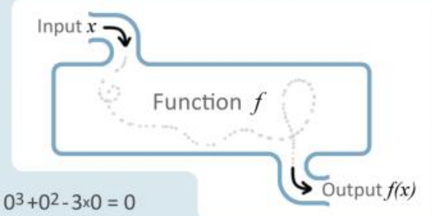
Functions & Graph Transformations

What is a Function?

A function describes a relation between two (or more) values. Each input value has one output.

For example: let $f(x) = x^3 + x^2 - 3x$

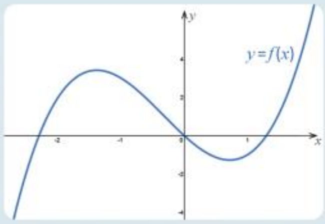
$f(2) = 2^3 + 2^2 - 3 \times 2 = 6$, $f(0) = 0^3 + 0^2 - 3 \times 0 = 0$



A function is usually called f but can, like a variable, be called any letter or symbol.

Graphing a Function

A function is graphed the same as the "y = ..." equations you're used to, we just use "$f(x)$ = ..." instead!



Transformations

Transformations cause functions to change in some way. Constants are used to either **translate** or **stretch** a function's graph.

Translate: a shift, the graph moves:

up & down		left & right
$f(x)+a$	or	$f(x+b)$

Stretch: (or squeeze) parallel to:

y axis		x axis
$c\,f(x)$	or	$f(dx)$

Top tip

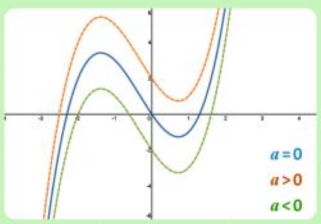
If the constant is **inside** the function, in with the x, then it causes transformations in the direction of the x axis.

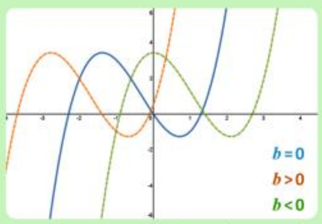
If it is **outside** the function, operating on $f(x)$, then it causes changes in the direction of the y axis.

$f(x)+a$

Shifts the graph **up for $a>0$** **down for $a<0$**

Think of moving the graph a units up the y axis





$f(x+b)$

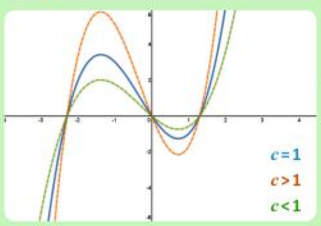
Shifts the graph **left for $b>0$** **right for $b<0$**

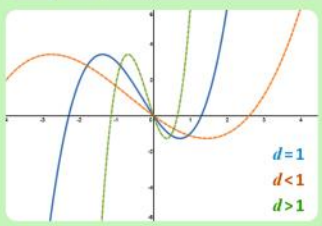
Think of moving the graph "$-b$" units along the x axis

$c\,f(x)$

Stretches the graph parallel to the y axis **out for $c>1$** (squeeze) **in for $c<1$**

The scale factor is c, think of points being c times further from $y=0$



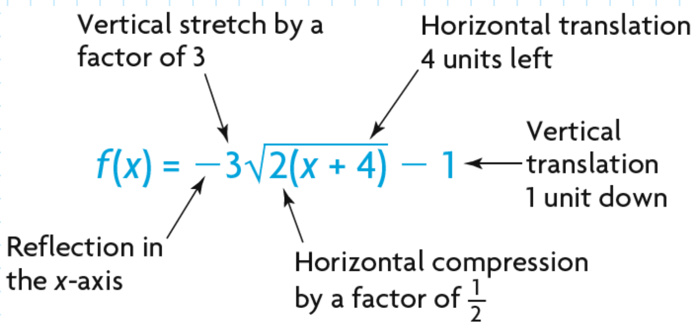


$f(dx)$

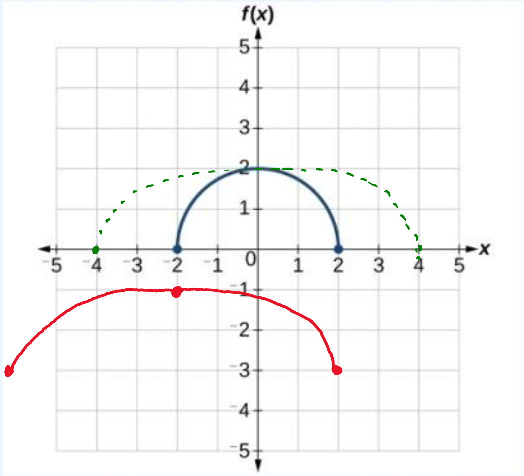
Stretches the graph parallel to the x axis **out for $d<1$** **in for $d>1$**

The scale factor is $1/d$, think of points being d times as close to $x=0$

These bottom two graphs only use positive constants. **Exercise:** can you sketch what they'd look like with negative constants?



Use the graph of $f(x)$ to sketch a graph of $k(x) = f\left(\frac{1}{2}x + 1\right) - 3$.



$y = f\left(\frac{1}{2}(x+2)\right) - 3$

HE of 2

2 left

3 down.

$y = f(x)$

x	y
-2	0
0	2
2	0

stretches

$2x$	y
-4	0
0	2
4	0

translations.

$2x-2$	$y-3$
-6	-3
-2	-1
2	-3

$\{x \mid -2 \le x \le 2\}$

$\{y \mid 0 \le y \le 2\}$

$\{x \mid -6 \le x \le 2\}$

$\{y \mid -3 \le y \le -1\}$.

Thursday, May 9th In-Class Notes

Name: ______________________________ **Date:** ____________ **TOTAL =** _____ **/ 7 marks**

Check-in Quiz Section 2.4 & 2.6: Transformations

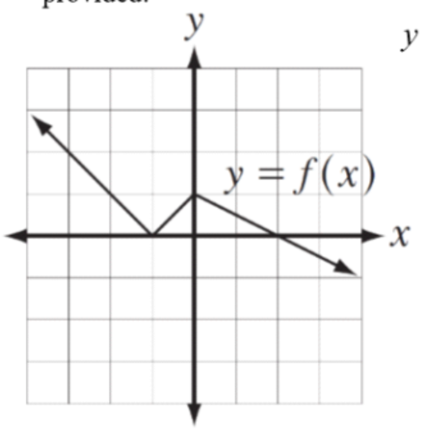
Complete the following questions SHOWING ALL WORK and steps where applicable.

1. Given the graph of $y = f(x)$,

a) Describe (in words) the transformation that can be applied to the graph of $y = f(x)$ to obtain the graph of the transformed function, $y = f(x-2)-1$.

(1 mark)

b) Sketch the graph of the transformed function. Show your work in the tables of values provided:



$y = f(x-2)-1$

(2 marks)

x	y

2. Graph the following function and answer the questions below:

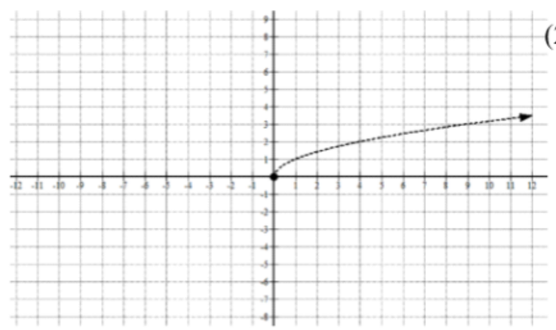
$$y = \sqrt{-x}$$

a) Describe/list the transformations on the base function.

(1 mark)

b) Sketch the graph of the transformed function. Show mapping notation. The base function is shown ($y = \sqrt{x}$)

x	y



(2 marks)

c) State the domain and range:

(1 mark)

3. Suppose that a function $y = f(x)$ contains the point $(12, -4)$. Find the coordinates of the ***image point*** after the following transformation.

$$y = \frac{1}{2} f(-2x - 4) - 5$$

4. Graph the following function and answer the questions below:

$$y = -2((x-4))^2 - 3$$

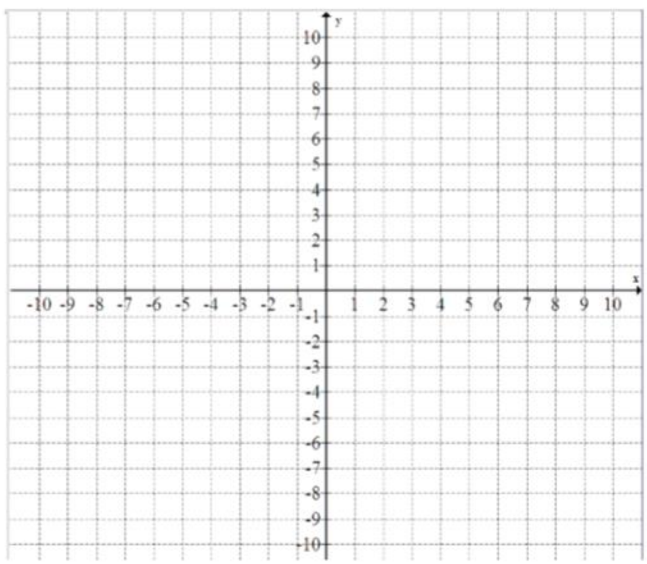
c) Describe/list the transformations on the base function.

(1 mark)

d) Sketch the graph of the transformed function. Show mapping notation.

(2 marks)

x	y



e) Determine the domain and range of the transformed function.

(1 mark)

Name: KEY Date: May 2024 TOTAL = ____ / 7 marks

Check-in Quiz Section 2.4 & 2.6: Transformations

Complete the following questions SHOWING ALL WORK and steps where applicable.

1. Given the graph of $y = f(x)$,

a) Describe (in words) the transformation that can be applied to the graph of $y = f(x)$ to obtain the graph of the transformed function, $y = f(x-2)-1$.

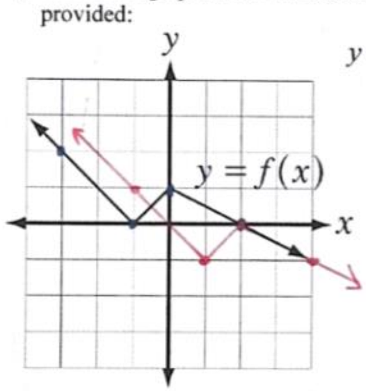
(1 mark)

2 right 1 down.

b) Sketch the graph of the transformed function. Show your work in the tables of values provided:

$y = f(x-2)-1$

(2 marks)



x	y
−3	2
−1	0
0	1
2	0

x+2	y−1
−1	1
1	−1
2	0
4	−1

2. Graph the following function and answer the questions below:

$$y = \sqrt{-x}$$

a) Describe/list the transformations on the base function.

reflection over y-axis

(1 mark)

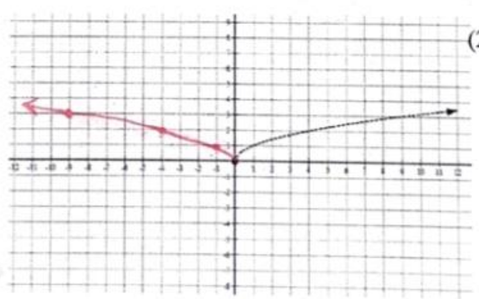
b) Sketch the graph of the transformed function. Show mapping notation. The base function is shown ($y = \sqrt{x}$)

(2 marks)

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

−x	y
0	0
−1	1
−4	2
−9	3



c) State the domain and range: $\{x \mid x \leq 0, x \in R\}$

(1 mark)

$\{y \mid y \geq 0, y \in R\}$.

3. Suppose that a function $y = f(x)$ contains the point $(12, -4)$. Find the coordinates of the ***image point*** after the following transformation.

$y = \frac{1}{2} f(-2x-4) - 5 \rightarrow y = \frac{1}{2} f(-2(x+2)) - 5$

$12, -4$

$x -\frac{1}{2}$ $\times \frac{1}{2}$

$-6, -2$

-2 -5

$(-8, -7)$

4. Graph the following function and answer the questions below:

$$y = -2\left((x-4)\right)^2 - 3$$

c) Describe/list the transformations on the base function. (1 mark)

① VE of 2 + ref. in x-axis

② 4 right, 3 down

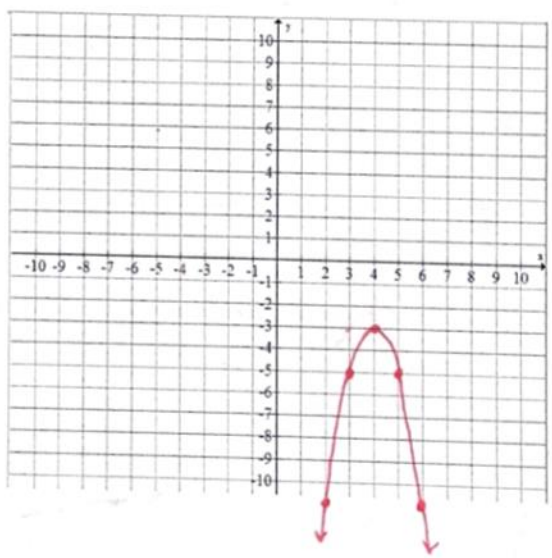
d) Sketch the graph of the transformed function. Show mapping notation. (2 marks)

$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

x	$-2y$
-2	-8
-1	-2
0	0
1	-2
2	-8

$x+4$	$-2y-3$
2	-11
3	-5
4	-3
5	-5
6	-11



e) Determine the domain and range of the transformed function. (1 mark)

$\{x \mid x \in R\}$, $\{y \mid y \leq -3, y \in R\}$

Inverse Functions Summary

Base Function		Inverse Function
$y=f(x)$		$y=f^{-1}(x)$
coordinates (x,y)	→	switch (y,x)
domain	→	range
range	→	domain
equation	→	switch x and y, then solve for y.

#2 p90 $f(x)$ is original → determine inverse function $f^{-1}(x)$
↳ this may or may not match $g(x)$ given

2 d) $f(x) = x^3 - 2$ → ① switch x + y

$y = x^2 - 2$

$x = y^3 - 2$
$+2 \qquad +2$

② solve for y

$\sqrt[3]{x+2} = \sqrt[3]{y^3}$

$y = \sqrt[3]{x+2}$

$f^{-1}(x) = \sqrt[3]{x+2}$

Do #4

$y = \sqrt[3]{x+2}$

Do #4

$$f^{-1}(x) = \sqrt[3]{x+2}$$

4 f) $f(x) = \dfrac{2x-1}{3x+2}$

① switch x + y

$$x = \frac{2y-1}{3y+2}$$

② solve for y

$$x(3y+2) = 2y-1$$

$$3xy + 2x = 2y - 1$$

$-2y$ $\quad$ $-2x$

GCF y

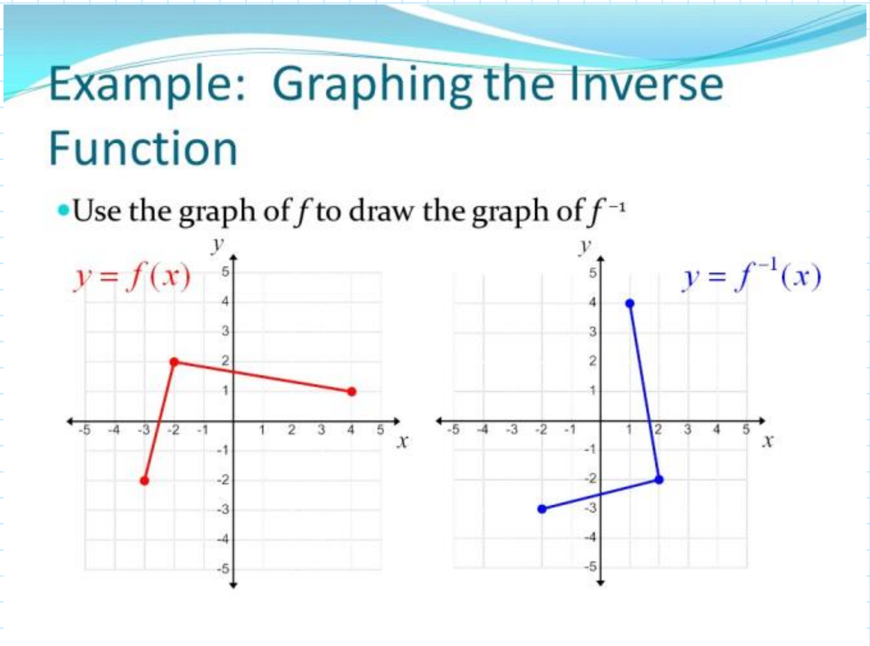
$$3xy - 2y = -2x - 1$$

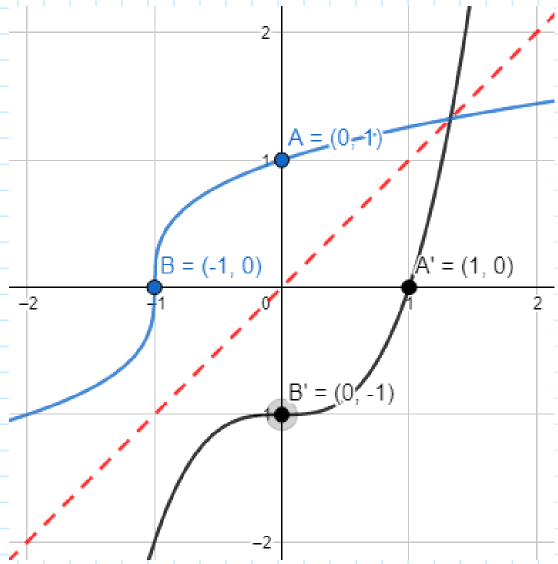
$$\frac{y(3x-2)}{(3x-2)} = \frac{-2x-1}{3x-2}$$

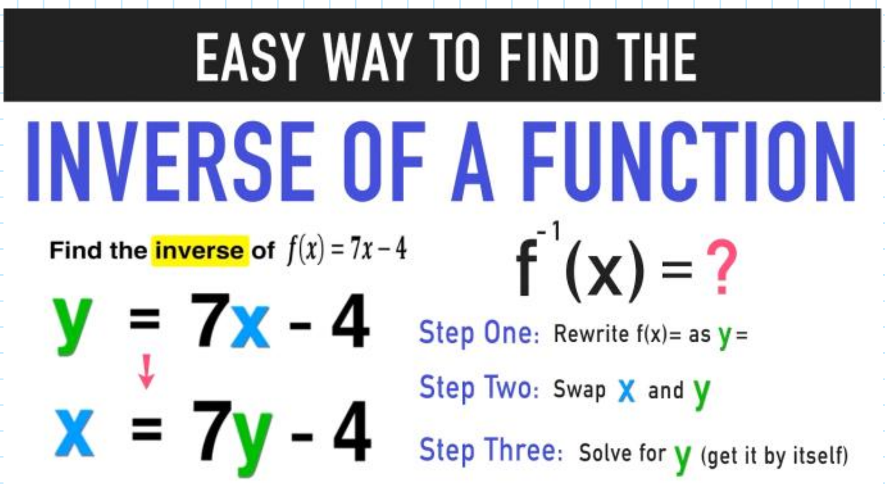
$$f^{-1}(x) = \frac{-2x-1}{3x-2}$$

2.5 Inverse

$$f^{-1}(x)$$



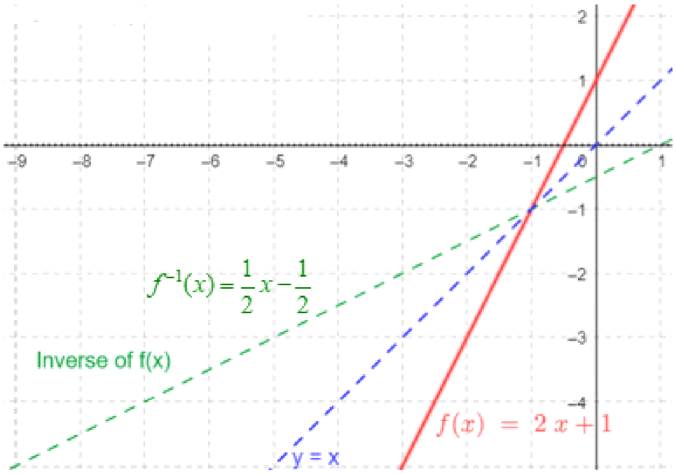




Inverse of a Function

Find the inverse of the function $f(x) = 2x + 1$

$$f(x) = 2x + 1$$
$$y = 2x + 1$$
$$y - 1 = 2x$$
$$\frac{y-1}{2} = x$$
$$y = \frac{x-1}{2}$$
$$y = \frac{1}{2}x - \frac{1}{2}$$
$$f^{-1}(x) = \frac{1}{2}x - \frac{1}{2}$$



$f(x)$ and $f^{-1}(x)$ are mirror images about the line $y = x$

Original Function $f(x)$

Inverse Function $f^{-1}(x)$

Domain: $x \geq 5$ → Range: $y \geq 5$

Range: $y \leq 0$ → Domain: $x \leq 0$

Find the Inverse of a Function

1. Replace f(x) with y
2. Interchange x and y
3. Solve the equation for y
4. Replace y with $f^{-1}(x)$

Example:

Given $f(x)=\frac{4x+2}{5}$ find the inverse of $f(x)$

$$f(x)=\frac{4x+2}{5}$$

$$y=\frac{4x+2}{5}$$ Replace f(x) with y

$$x=\frac{4y+2}{5}$$ Interchange x and y

$$5x=4y+2$$

$$5x-2=4y$$ Solve the equation for y

$$y=\frac{5x-2}{4}$$

$$f^{-1}(x)=\frac{5x-2}{4}$$ Replace y with $f^{-1}(x)$

Summary & Practice

Key Ideas

- Write the function in the form $y = af(b(x - h)) + k$ to better identify the transformations.
- Stretches and reflections may be performed in any order before translations.
- The parameters a, b, h, and k in the function $y = af(b(x - h)) + k$ correspond to the following transformations:
  - a corresponds to a vertical stretch about the x-axis by a factor of $|a|$. If $a < 0$, then the function is reflected in the x-axis.
  - b corresponds to a horizontal stretch about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the function is reflected in the y-axis.
  - h corresponds to a horizontal translation.
  - k corresponds to a vertical translation.

Perform the transformations in this order

$$a \cdot f\big(b(x-h)\big)+k$$

1. **Vertical scale** by *a*

$$(x, y) \to (x, ay)$$

If a is negative, reflects across x-axis

1. **Horizontal scale** by $1/b$

$$(x, y) \to \left(\frac{1}{b}x, y\right)$$

If b is negative, reflects across y-axis

2. **Vertical shift**

+k: shift up k

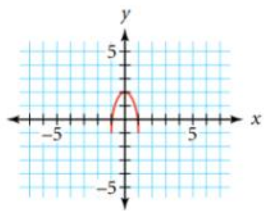
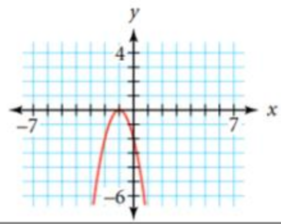
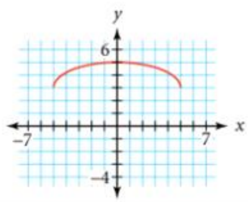
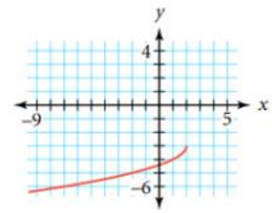
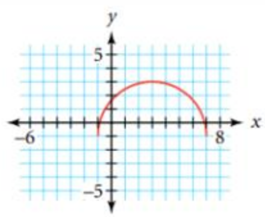
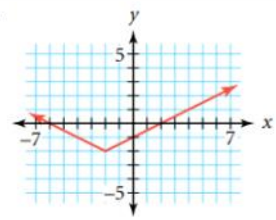
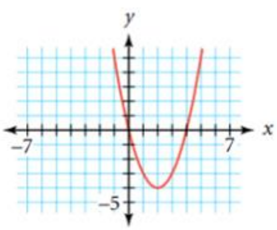
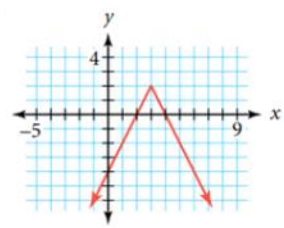
-k : shift down k

2. **Horizontal shift**

-h : shift to right

+h : shift to left

Try this matching:

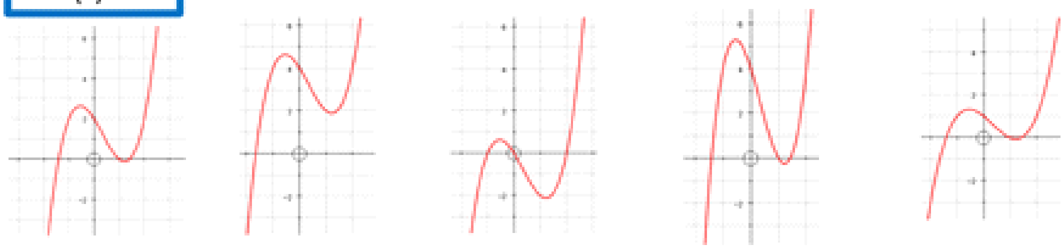
a. 	1. $y = (x-2)^2 - 4$ 2. $y = -\sqrt{-(x-2)} - 3$	e. 
b. 	3. $y = 0.5\lvert x+2\rvert - 2$ 4. $y = -2(x+1)^2$	f. 
c. 	5. $y = 3\sqrt{1-x^2} - 1$ 6. $y = 4\sqrt{1-\left(\frac{x-3}{4}\right)^2} - 1$	g. 
d. 	7. $y = -2\lvert x-3\rvert + 2$ 8. $y = 2\sqrt{1-\left(\frac{x}{5}\right)^2} + 3$	h. 

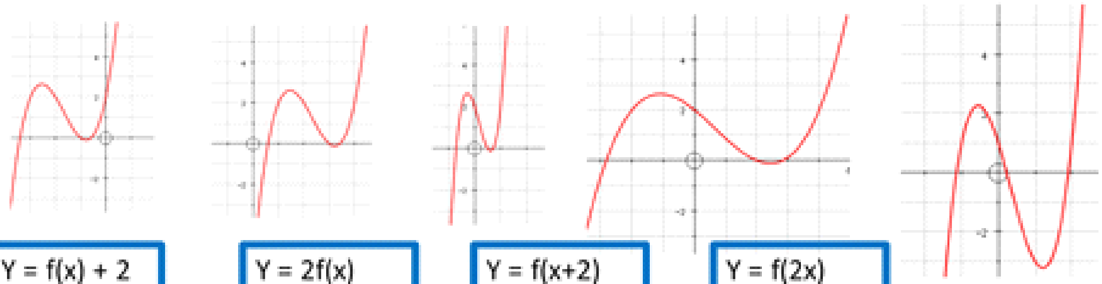
Answers : 1d, 2f, 3g, 4e, 5a, 6c, 7h, 8b

Transformations of Graphs

Using your knowledge of transformations of graphs match up the transformations of the function with the graph. The first one is done for you.

Y = f(x)





Y = f(x) + 2

Y = 2f(x)

Y = f(x+2)

Y = f(2x)

Y = f(x) - 2

Y = ½ f(x)

Y = f(x-2)

Y = f(½ x)

Y = 2f(x) – 3

This doubles in size and then moves down 3

Extension: The original graph has a peak at (-0.5, 2.5) Write the new location of this peak after the transformations for each graph. How has the peak moved and why has this happened?