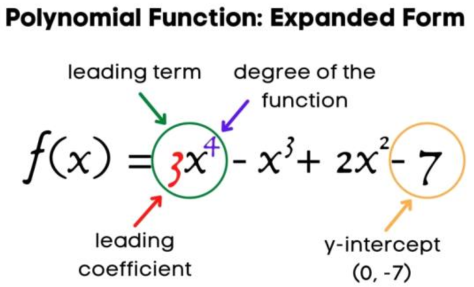
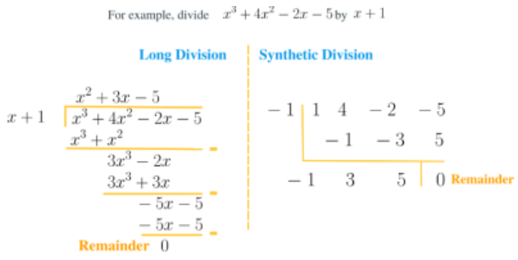
Tuesday, May 14th

Plan For Today:



1. Sorry, need more time to mark U1 Exam.
   We will go over it tomorrow...
2. Any Questions?
3. Start Chapter 3: Polynomial Functions
   - ✓ 3.1: Characteristics of Polynomial Functions
   - ✓ 3.2: Equations & Graphs of Polynomials Functions
   - ✸ 3.3: The Remainder Theorem
   - ✸ 3.4: Factoring Review & The Factor Theorem
   - ✸ 3.5: Applications & Word Problems
4. Work on practice questions from Workbook



Plan Going Forward:

1. Work on 3.1-3.2 practice from the workbook for tomorrow.

- ✸ **CHECK-IN QUIZ ON WEDNESDAY, MAY 15TH**

2. We will continue Chapter 3 Polynomials tomorrow and finish it on Thursday. We will do an intro to Ch4 on Thursday if time.

- ✸ **CHAPTER 3 PROJECT DUE TUESDAY, MAY 21ST**
- ✸ **CHAPTER 3 TEST ON TUESDAY, MAY 21ST**

VICTORIA DAY ON MONDAY, MAY 20TH - SCHOOL CLOSED

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class. Anurita Dhiman = adhiman@sd35.bc.ca

Polynomial Functions = whole # degrees

= not including radicals like $\sqrt{x}$

+ rationals like $\frac{1}{x}$

degree = highest degree

descending degree order.

$$\hookrightarrow \quad P(x) = ax^n + x^{n-1} + x^{n-2} + \ldots \underbrace{Cx^0}$$

leading coefficient

= C

constant term

= y-intercept

= if no C, then

C = 0

or $0x^0$

Characteristics of Graphs.

Even Degrees + (+) LC

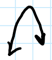
ex: $f(x) = x^2$

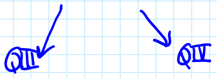
"up + up"

"up into QII + QI"

Even Degree + (−) LC

ex: $f(x) = -x^2$



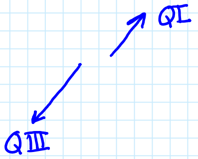


"down + down"

"down into QIII + QIV"

ODD Degree + (+) LC

ex: $f(x) = x$

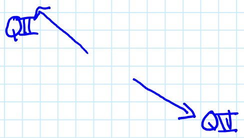


"starts down, ends up"

"down into QIII + up into QI"

ODD Degree + (−) LC

ex: $f(x) = -x$



"starts up + ends down"

"up into QII + down into QIV"

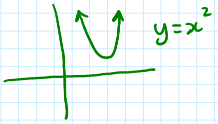
x-intercepts

↳ aka: zeros, roots, solutions

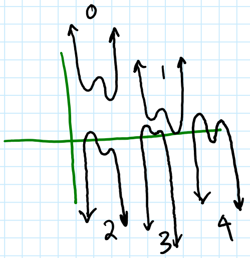
↳ maximum # of x-intercepts = n (degree of function)

Note: even degrees can have zero x-intercepts

ex:



∴ even-degrees have 0 − n x-intercepts.

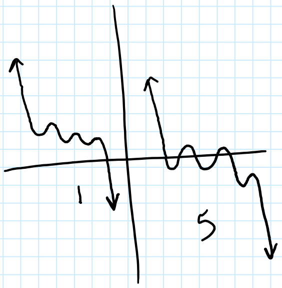


ex: $y = x^4 + 3x + 5$

$= 0 - 4$ possible x-intercepts.

Note: ODD degrees must have at least 1 x-int. b/c of end behaviour going down & up.

∴ ODD degrees have $1 - n$ possible x-intercepts



ex: $y = x^7 + x^2 + x + 10$

$= 1 - 7$ x-intercepts.

Multiplicity = how often an x-intercept occurs & it's based on the exponent of the x when factored.

ex: $f(x) = (x+1)(x+2)(x+3)$

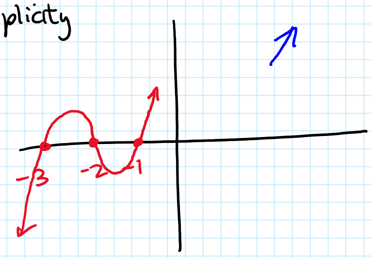
Notice x^3 = ODD & +LC

① Determine x-intercepts: -1, -2, -3

(x, y) ← $y=0$ for x-intercepts

$(-1,0), (-2,0), (-3,0)$

② Determine multiplicity: M1, M1, M1



∴ M1 means the line goes straight through the x-int.

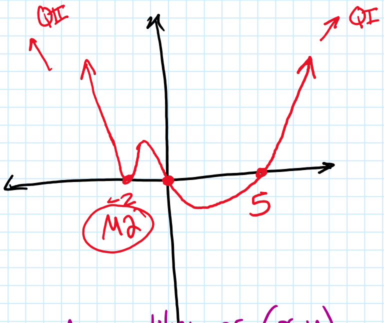
ex: $y = x(x+2)^2(x-5)$

x-int = 0, -2, 5

multiplicity = M1, M2, M1

degree = 4 ∴ even

LC = +

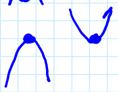


* Recall: y-int determined by making $x=0$ ★ written as (x,y)

★ Recall: y-int determined by making $x=0$ ★ written as (x,y) coordinate

y-int = $0(0+2)^2(0-5)^2 = 0$ $(0,0)$

M2 = draw graph touching x-intercept but not going through.



ex: $y=-(x+6)^2(x+2)(x-4)^3$

x-int =	-6	-2	4
multiplicity =	M2	M1	M3

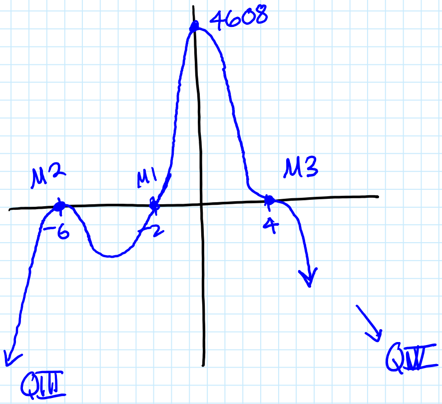
Degree = 6 = even } ↓ ↓

LC = (−)

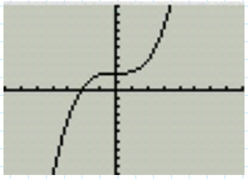
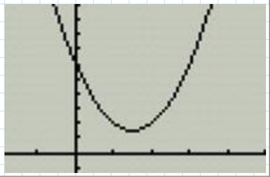
y-int = $(0, 4608)$

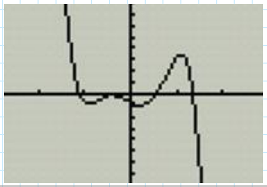
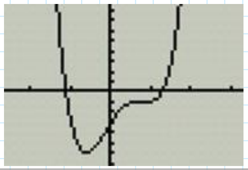
$-(0+6)^2(0+2)(0-4)^3$

$-(36)(2)(-64) = 4608$



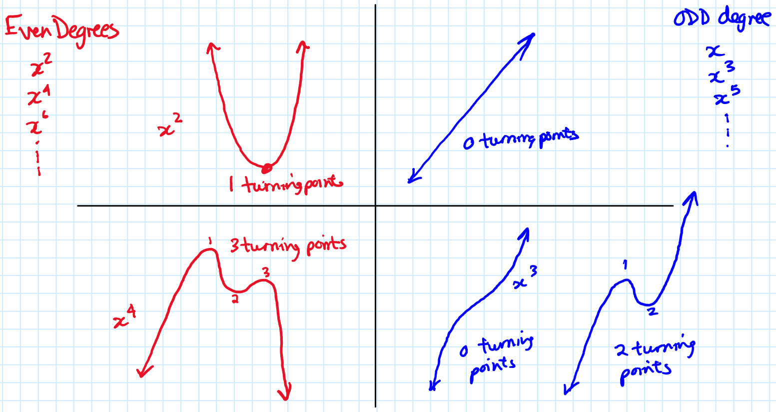
M3 = line goes through but flattens like a cubic function

1 $f(x)=x^6-2x^5-2x^4+4x+5$	**G** 
2 $f(x)=\frac{1}{5}x^3+2$	**D** 
3 $y=2x^2-6x+6$	**A** 
4 $y=5x-3$	**F** 
	H

5 $f(x)=\frac{1}{2}x^5-x^4-2x^3+3x^2+3x-2$	
6 $f(x)=-5x^5+9x^3+2x^2-2x-1$	**B** 
7 $y=-3x^4-2x^3+5x^2+4x+5$	**C** 
8 $y=4x^4-5x^3-3x^2+6x-3$	**E** 

3.1 p.118 Turning Points.

↳ changes in direction of the graph.



ex: $y=x$ = 0 turning points 0TP

$y=x^2$ = 1 TP

$y=x^3$ = 0, 2 TP (not 1 TP b/c its even degree)

$y=x^4$ = 1, 3 TP (not 0 or 2 b/c these are for odd degrees)

$y=x^5$ = 0, 2, 4 TP

$y=x^6$ = 1, 3, 5 TP

3.2 cont.

IF → you are given x-intercepts + Multiplicity
or a graph you can get the x-int. from;
use this to generate an equation
in this form ↓

multiplicity

$$y = a\underbrace{(x-p_1)^{n_1}(x-p_2)^{n_2}}_{x\text{-intercepts}} \text{ etc} \ldots$$

THEN → use another point given or on graph
to substitute for (x, y) + solve
for 'a'

(skip ex 7-9)

Ex: 4 roots = −1, −1, 0, 2

$p(1) = 5 \rightarrow$ = coordinate (1, 5)

$x = -1$

$+1 \leftarrow$

$x + 1 = 0$

$(x+1)$

① generate equation

$$\underset{\overset{\uparrow}{5}}{y} = a\,\underset{\overset{\uparrow}{1}}{x}(\underset{\overset{\uparrow}{1}}{x}+1)^2(\underset{\overset{\uparrow}{1}}{x}-2)$$

② sub. the x, y coordinate + solve for 'a'

$$5 = a(1)\underbrace{(1+1)^2}_{2^2}\underbrace{(1-2)}_{-1} \qquad \text{Bedmas.}$$

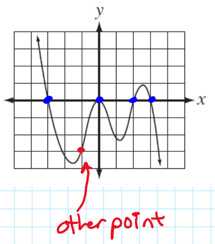
$$\frac{5}{-4} = \frac{-4a}{-4}$$

$$a = -\frac{5}{4}$$

③ rewrite final equation

$$\boxed{y = -\frac{5}{4}x(x+1)^2(x-2)}$$

Ex 6
p.125.



① x-int + multiplicity
= −3 M1, 0 M2, 2 M1, 3 M1

② other point = (−1, −3)

③ equation

$$y = ax^2(x+3)(x-2)(x-3)$$

$$-3 = a(-1)^2(-1+3)(-1-2)(-1-3)$$

$$-3 = a(1)(2)(-3)(-4)$$

$$\frac{-3}{24} = \frac{24a}{24}$$

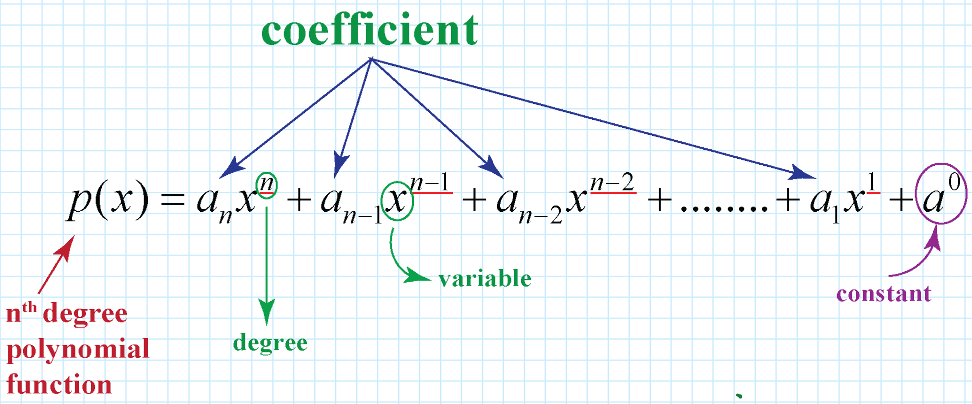
$$a = -\frac{1}{8}$$

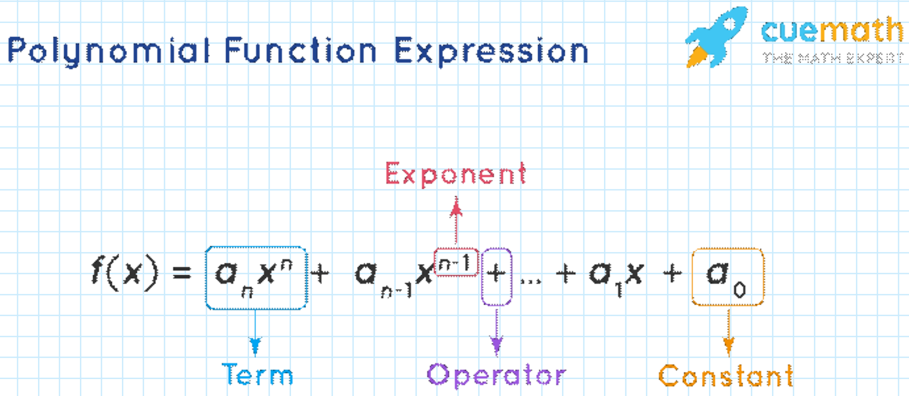
④ write final equation

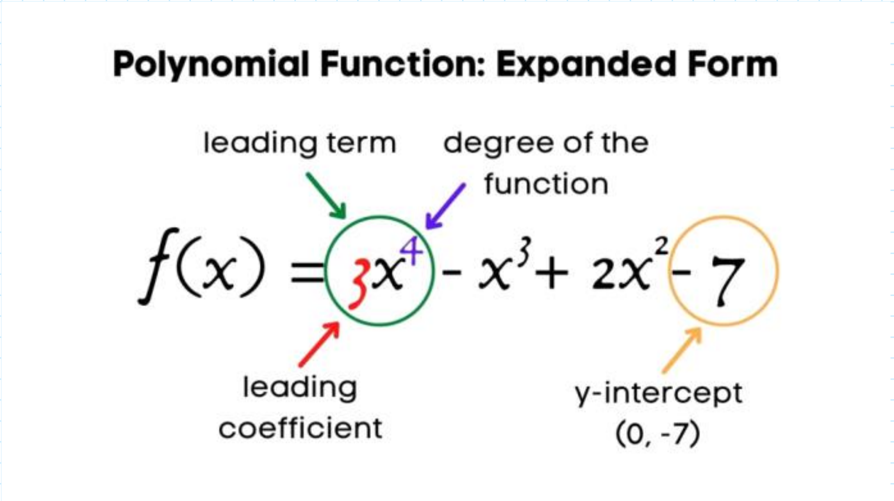
$$\boxed{y = -\frac{1}{8}x^2(x+3)(x-2)(x-3)}$$

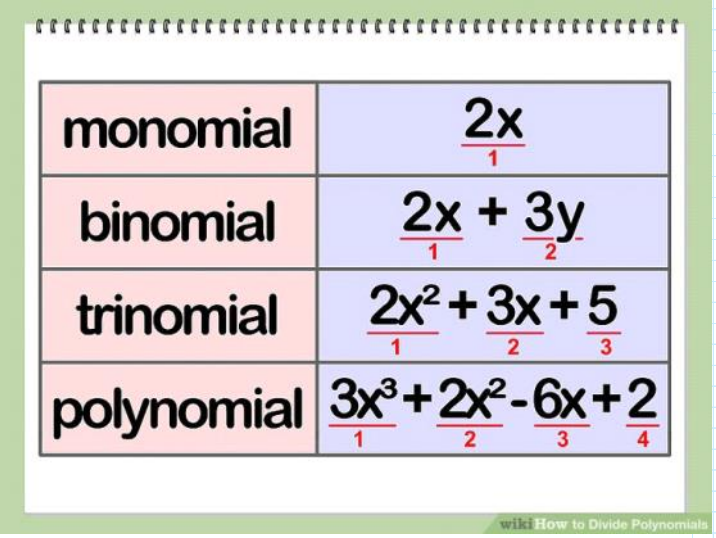
3.1 What's a Polynomial?

What is a polynomial function?



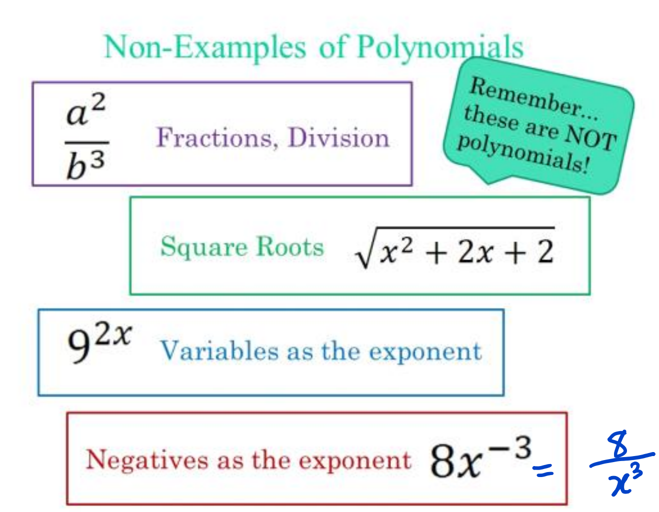






monomial	$\underset{1}{\underline{2x}}$
binomial	$\underset{1}{\underline{2x}} + \underset{2}{\underline{3y}}$
trinomial	$\underset{1}{\underline{2x^2}} + \underset{2}{\underline{3x}} + \underset{3}{\underline{5}}$
polynomial	$\underset{1}{\underline{3x^3}} + \underset{2}{\underline{2x^2}} - \underset{3}{\underline{6x}} + \underset{4}{\underline{2}}$

wikiHow to Divide Polynomials



Non-Examples of Polynomials

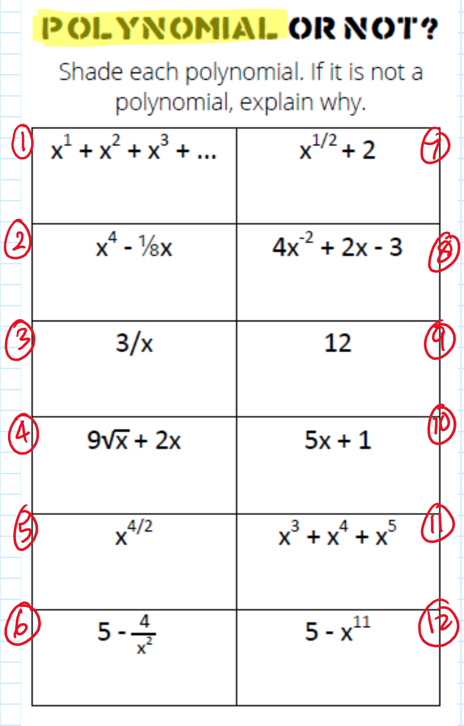
Remember... these are NOT polynomials!

$\frac{a^2}{b^3}$ Fractions, Division

Square Roots $\sqrt{x^2 + 2x + 2}$

9^{2x} Variables as the exponent

Negatives as the exponent $8x^{-3} = \frac{8}{x^3}$

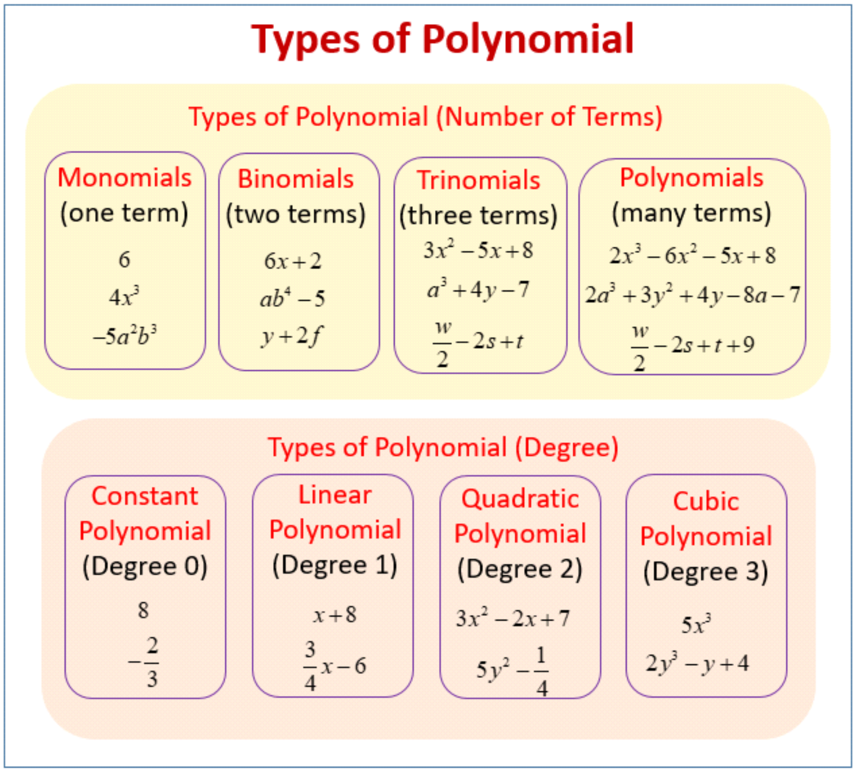


POLYNOMIAL OR NOT?

Shade each polynomial. If it is not a polynomial, explain why.

① $x^1 + x^2 + x^3 + \ldots$	$x^{1/2} + 2$ ⑦
② $x^4 - \frac{1}{8}x$	$4x^{-2} + 2x - 3$ ⑧
③ $3/x$	12 ⑨
④ $9\sqrt{x} + 2x$	$5x + 1$ ⑩
⑤ $x^{4/2}$	$x^3 + x^4 + x^5$ ⑪
⑥ $5 - \frac{4}{x^2}$	$5 - x^{11}$ ⑫

1, 2, 5, 9, 10, 11, 12



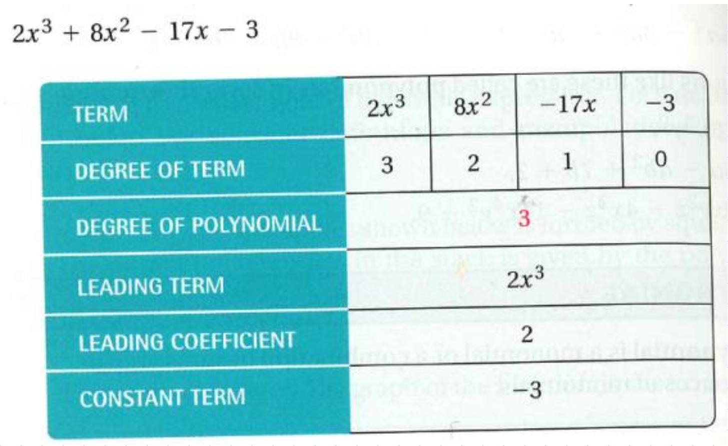
Types of Polynomial

Types of Polynomial (Number of Terms)

Monomials (one term)	Binomials (two terms)	Trinomials (three terms)	Polynomials (many terms)
6	$6x+2$	$3x^2-5x+8$	$2x^3-6x^2-5x+8$
$4x^3$	ab^4-5	a^3+4y-7	$2a^3+3y^2+4y-8a-7$
$-5a^2b^3$	$y+2f$	$\frac{w}{2}-2s+t$	$\frac{w}{2}-2s+t+9$

Types of Polynomial (Degree)

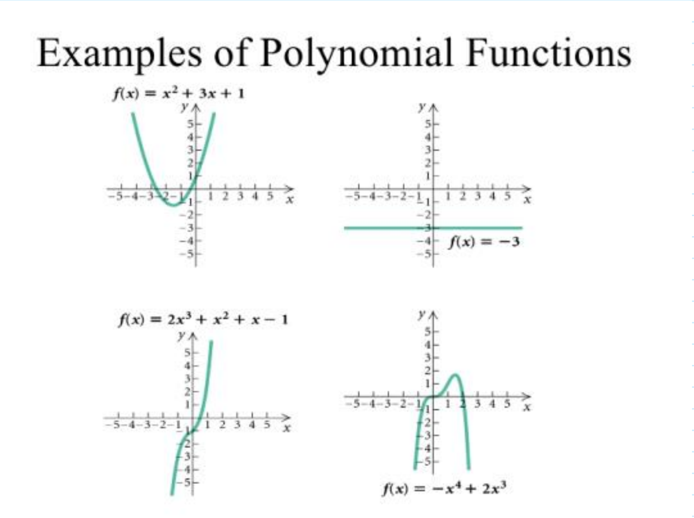
Constant Polynomial (Degree 0)	Linear Polynomial (Degree 1)	Quadratic Polynomial (Degree 2)	Cubic Polynomial (Degree 3)
8	$x+8$	$3x^2-2x+7$	$5x^3$
$-\frac{2}{3}$	$\frac{3}{4}x-6$	$5y^2-\frac{1}{4}$	$2y^3-y+4$

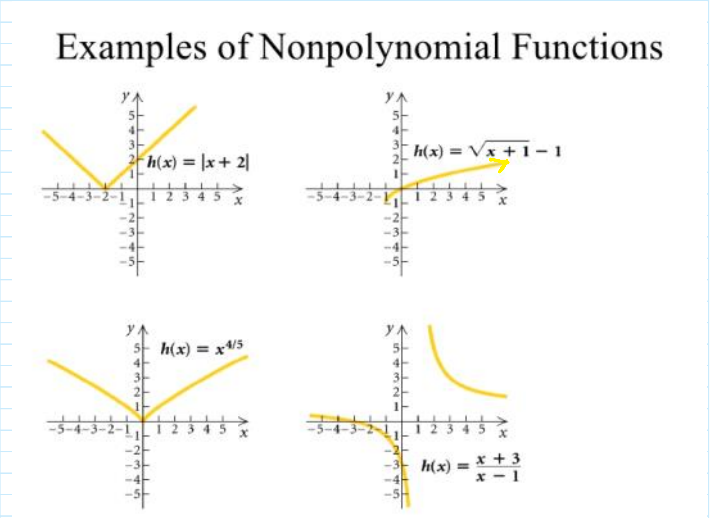


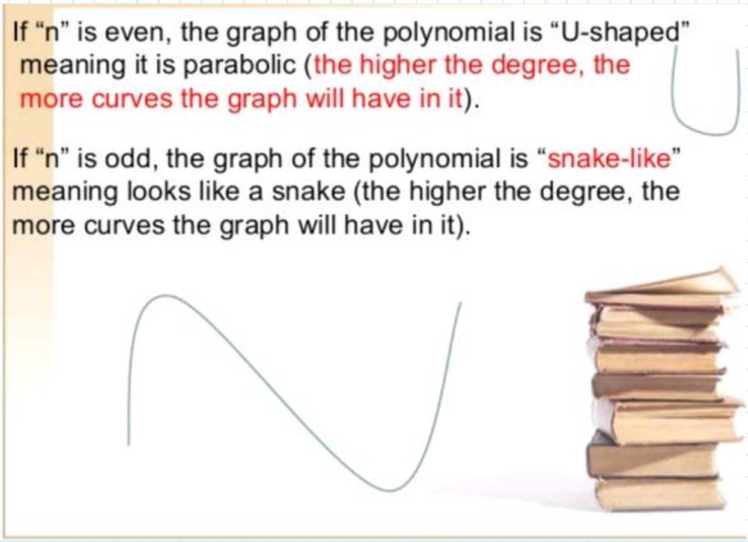
$2x^3+8x^2-17x-3$

TERM	$2x^3$	$8x^2$	$-17x$	-3
DEGREE OF TERM	3	2	1	0
DEGREE OF POLYNOMIAL	3			
LEADING TERM	$2x^3$			
LEADING COEFFICIENT	2			
CONSTANT TERM	-3			

3.2 Characteristics of Polynomial Functions

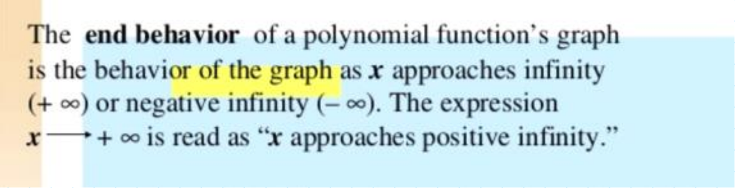

Examples of Polynomial Functions

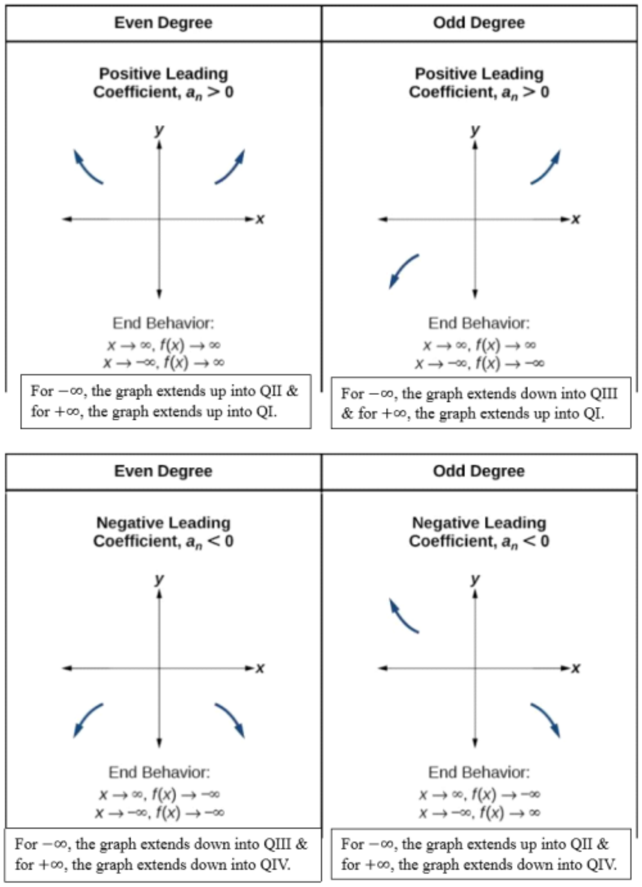

Examples of Nonpolynomial Functions


If "n" is even, the graph of the polynomial is "U-shaped" meaning it is parabolic (the higher the degree, the more curves the graph will have in it).

If "n" is odd, the graph of the polynomial is "snake-like" meaning looks like a snake (the higher the degree, the more curves the graph will have in it).

https://www.slideshare.net/morrobea/63-evaluatingandgraphingpolynomilafunctions

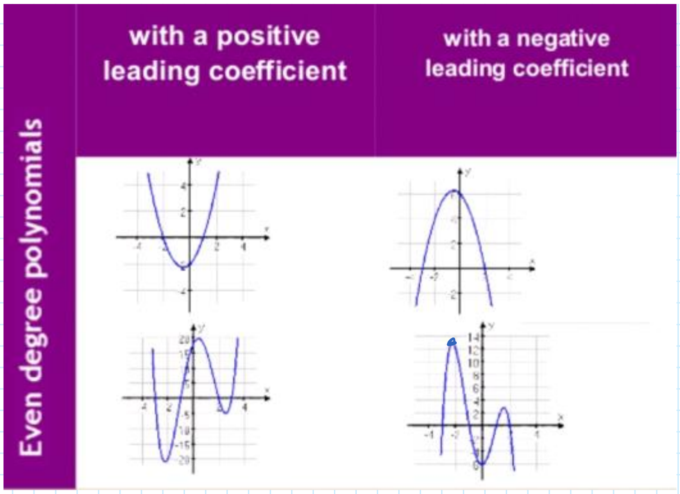

The **end behavior** of a polynomial function's graph is the behavior of the graph as x approaches infinity (+ ∞) or negative infinity (− ∞). The expression $x \longrightarrow + \infty$ is read as "x approaches positive infinity."

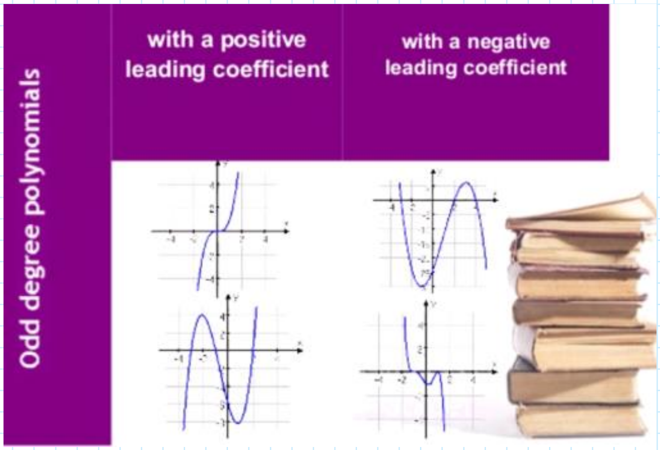


Even Degree	Odd Degree
Positive Leading Coefficient, $a_n > 0$ End Behavior: $x \to \infty, f(x) \to \infty$ $x \to -\infty, f(x) \to \infty$	**Positive Leading Coefficient, $a_n > 0$** End Behavior: $x \to \infty, f(x) \to \infty$ $x \to -\infty, f(x) \to -\infty$
For $-\infty$, the graph extends up into QII & for $+\infty$, the graph extends up into QI.	For $-\infty$, the graph extends down into QIII & for $+\infty$, the graph extends up into QI.

Even Degree	Odd Degree
Negative Leading Coefficient, $a_n < 0$ End Behavior: $x \to \infty, f(x) \to -\infty$ $x \to -\infty, f(x) \to -\infty$	**Negative Leading Coefficient, $a_n < 0$** End Behavior: $x \to \infty, f(x) \to -\infty$ $x \to -\infty, f(x) \to \infty$
For $-\infty$, the graph extends down into QIII & for $+\infty$, the graph extends down into QIV.	For $-\infty$, the graph extends up into QII & for $+\infty$, the graph extends down into QIV.

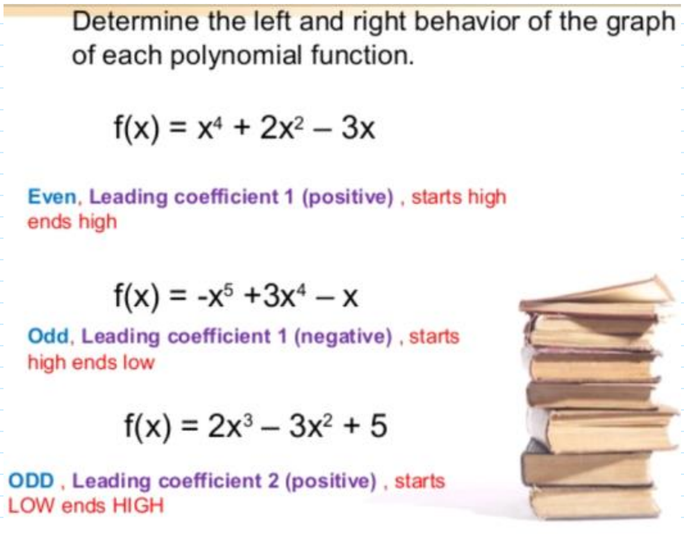
Degree is odd		Degree is even	
Leading coefficient is positive	Start low, End high	Leading coefficient is positive	Start high, End high
Leading coefficient is negative	Start high, End low	Leading coefficient is negative	Start low, End low





Degree and Leading coefficient tells us the behaviour of the graph.

$y = -x^4 + 2x^3 - x^2 + 3x + 20$	Negative even
$y = -10x^5 - 3x^3 + 2x - 4$	Negative odd
$y = (x+5)(x-3)(2x+4)$ $x(x)(2x) = 2x^3$	Positive odd
$y = x(x-4)^2(3x+1)$ $x(x)^2(3x) = 3x^4$	Positive even
$y = x^2(3-x)(x+4)^3$ $x^2(-x)(x)^3 = -x^6$	Negative even



Determine the left and right behavior of the graph of each polynomial function.

$f(x) = x^4 + 2x^2 – 3x$

Even, Leading coefficient 1 (positive) , starts high ends high

$f(x) = -x^5 +3x^4 – x$

Odd, Leading coefficient 1 (negative) , starts high ends low

$f(x) = 2x^3 – 3x^2 + 5$

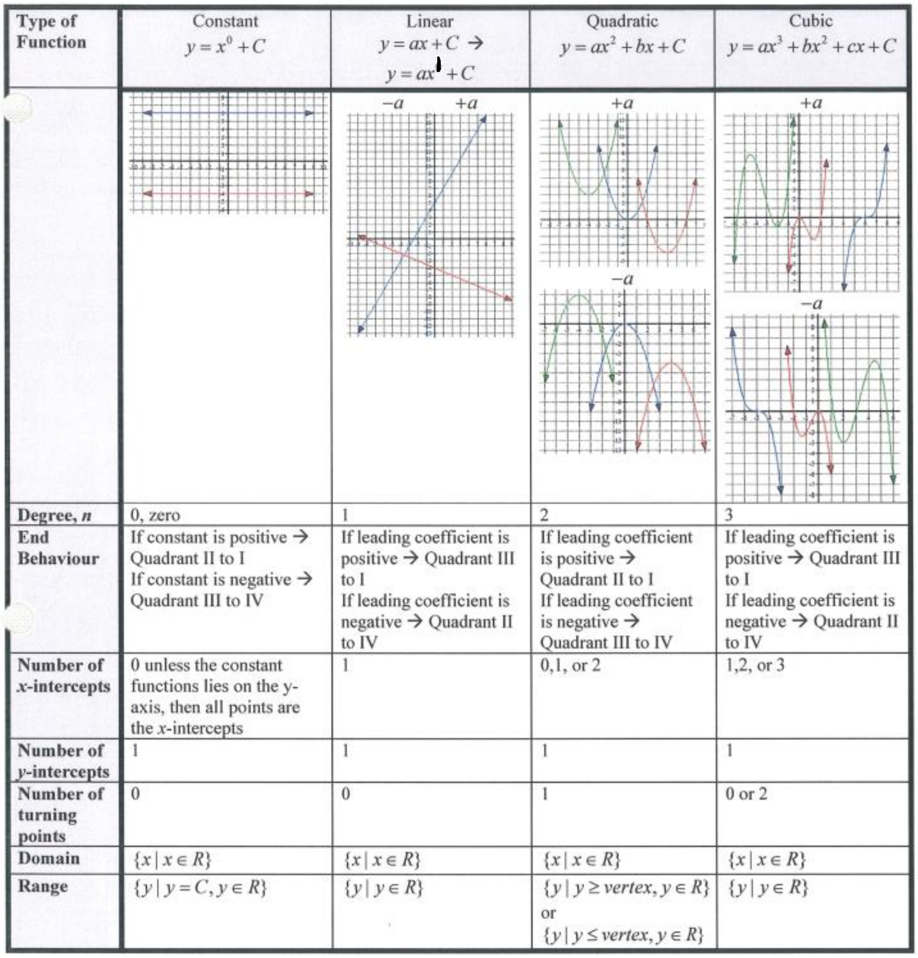
ODD , Leading coefficient 2 (positive) , starts LOW ends HIGH

Degree tells us the number of possible x-intercepts (roots or zeros)
Degree tells us the possible number of turns in the graph

$$\underset{x-intercept}{\underset{\uparrow}{(\boldsymbol{x}, 0)}} \qquad \underset{y-intercept}{\underset{\uparrow}{(0, \boldsymbol{y})}}$$

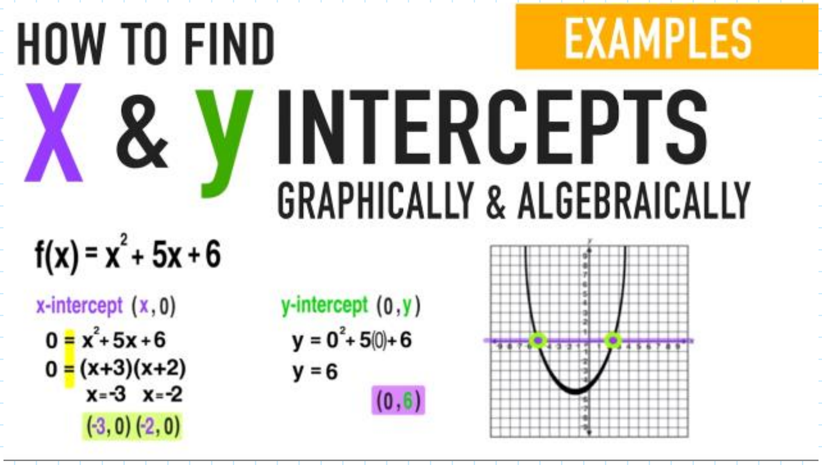
https://slideplayer.com/slide/9143093/

3.1 Characteristics of Polynomials Summary Sheet



Type of Function	Constant $y=x^0+C$	Linear $y=ax+C \rightarrow$ $y=ax^1+C$	Quadratic $y=ax^2+bx+C$	Cubic $y=ax^3+bx^2+cx+C$
		$-a$ $+a$	$+a$ $-a$	$+a$ $-a$
Degree, n	0, zero	1	2	3
End Behaviour	If constant is positive → Quadrant II to I If constant is negative → Quadrant III to IV	If leading coefficient is positive → Quadrant III to I If leading coefficient is negative → Quadrant II to IV	If leading coefficient is positive → Quadrant II to I If leading coefficient is negative → Quadrant III to IV	If leading coefficient is positive → Quadrant III to I If leading coefficient is negative → Quadrant II to IV
Number of x-intercepts	0 unless the constant functions lies on the y-axis, then all points are the x-intercepts	1	0,1, or 2	1,2, or 3
Number of y-intercepts	1	1	1	1
Number of turning points	0	0	1	0 or 2
Domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
Range	$\{y \mid y=C, y \in R\}$	$\{y \mid y \in R\}$	$\{y \mid y \geq vertex, y \in R\}$ or $\{y \mid y \leq vertex, y \in R\}$	$\{y \mid y \in R\}$

3.2 Graphs of Polynomial Functions



HOW TO FIND

EXAMPLES

X & y INTERCEPTS

GRAPHICALLY & ALGEBRAICALLY

$f(x) = x^2 + 5x + 6$

x-intercept $(x, 0)$

$0 = x^2 + 5x + 6$

$0 = (x+3)(x+2)$

$x = -3 \quad x = -2$

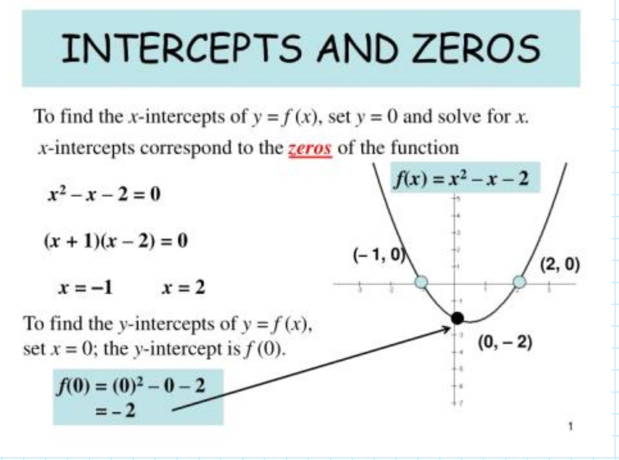
$(-3, 0)\ (-2, 0)$

y-intercept $(0, y)$

$y = 0^2 + 5(0) + 6$

$y = 6$

$(0, 6)$



INTERCEPTS AND ZEROS

To find the x-intercepts of $y = f(x)$, set $y = 0$ and solve for x.

x-intercepts correspond to the *zeros* of the function

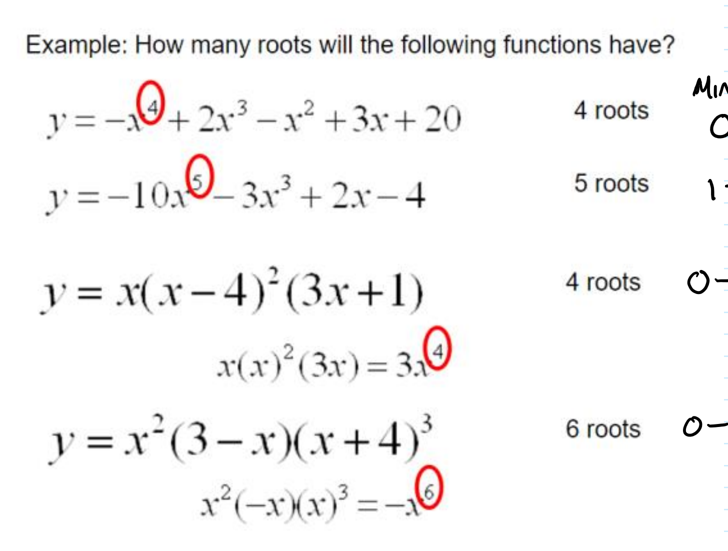
$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1 \qquad x = 2$

To find the y-intercepts of $y = f(x)$, set $x = 0$; the y-intercept is $f(0)$.

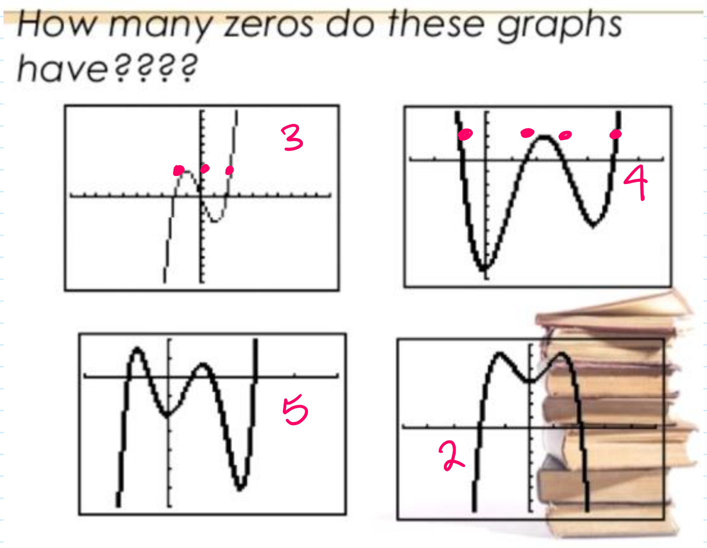
$f(0) = (0)^2 - 0 - 2$
$= -2$



Example: How many roots will the following functions have?

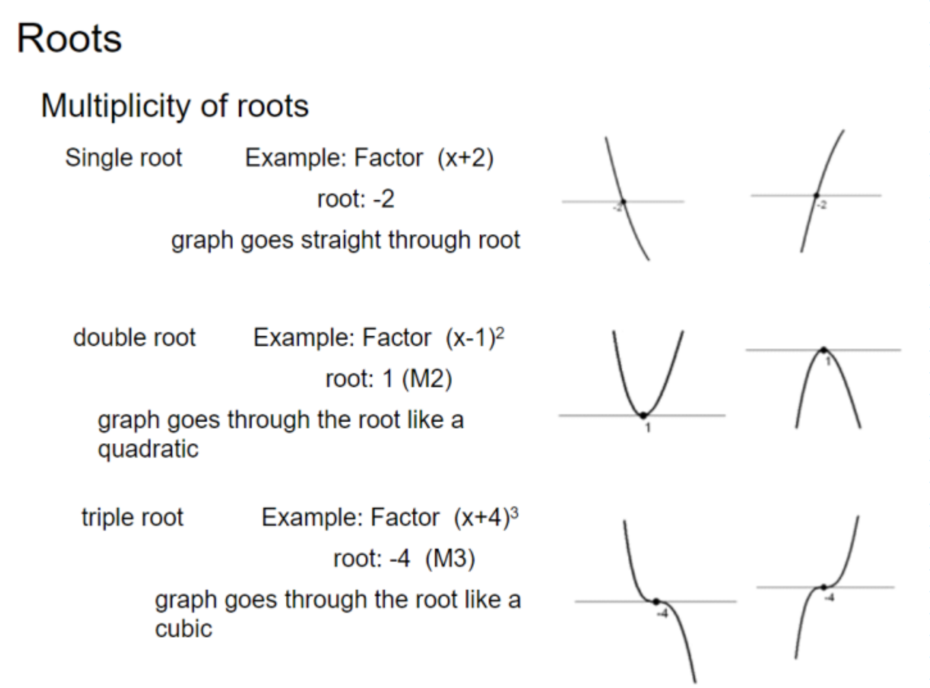
		MIN - MAX
$y = -x^{4} + 2x^3 - x^2 + 3x + 20$	4 roots	0-4
$y = -10x^{5} - 3x^3 + 2x - 4$	5 roots	1-5
$y = x(x-4)^2(3x+1)$ $x(x)^2(3x) = 3x^{4}$	4 roots	0-4
$y = x^2(3-x)(x+4)^3$ $x^2(-x)(x)^3 = -x^{6}$	6 roots	0-6

How many zeros do these graphs have????



3.4 Multiplicity

Multiplicity is the number of times an x-intercept occurs in a graph:



Roots

Multiplicity of roots

Single root Example: Factor (x+2)

root: -2

graph goes straight through root

double root Example: Factor $(x-1)^2$

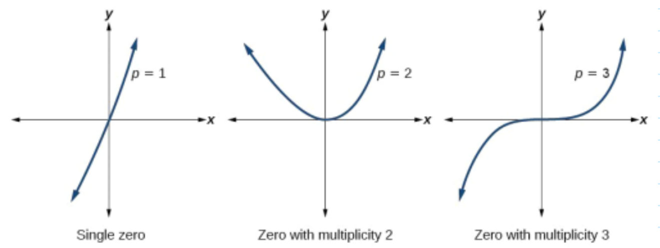
root: 1 (M2)

graph goes through the root like a quadratic

triple root Example: Factor $(x+4)^3$

root: -4 (M3)

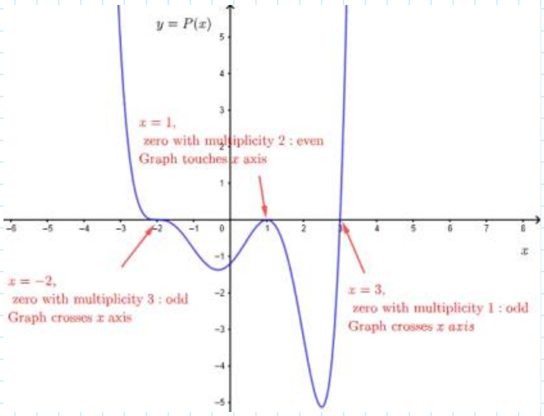
graph goes through the root like a cubic

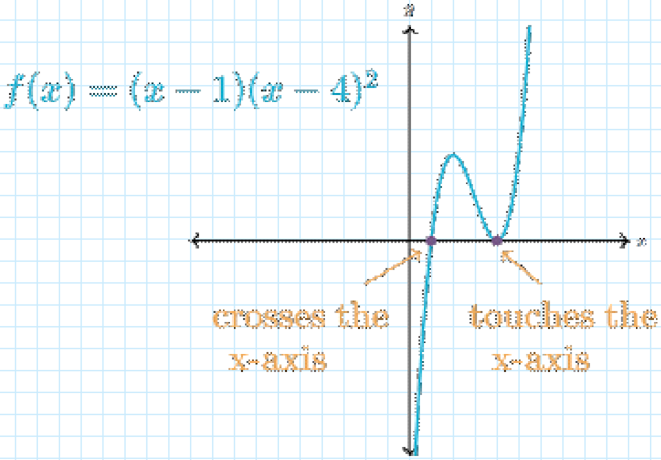


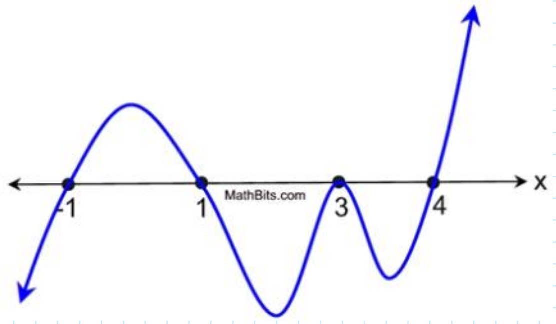
Single zero

Zero with multiplicity 2

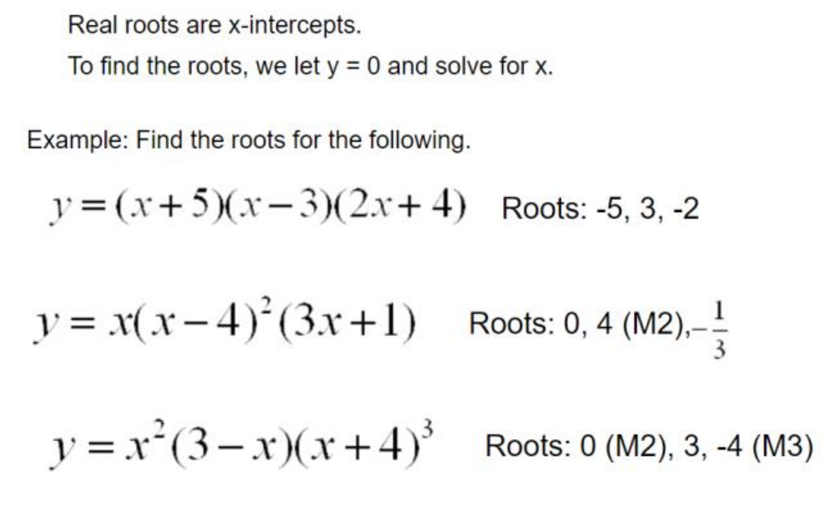
Zero with multiplicity 3







If you factor the polynomial, you can see the value of the x-intercepts (roots) and the corresponding multiplicity



Real roots are x-intercepts.

To find the roots, we let y = 0 and solve for x.

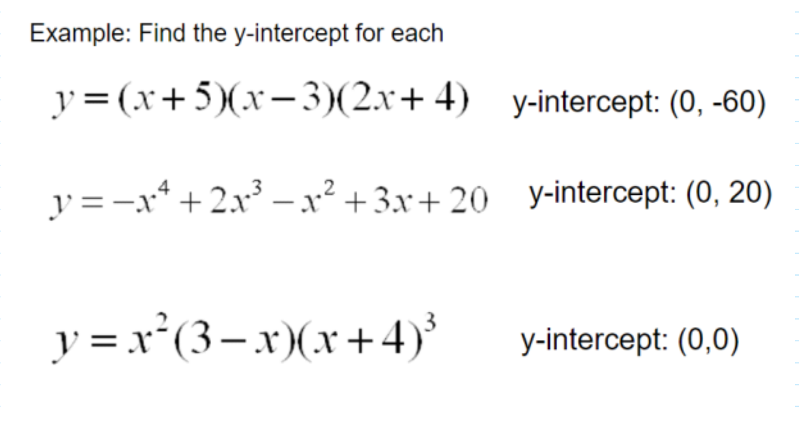
Example: Find the roots for the following.

$y=(x+5)(x-3)(2x+4)$ Roots: -5, 3, -2

$y=x(x-4)^2(3x+1)$ Roots: 0, 4 (M2), $-\frac{1}{3}$

$y=x^2(3-x)(x+4)^3$ Roots: 0 (M2), 3, -4 (M3)

To find the y-intercept, make x=0 and solve for y:



Example: Find the y-intercept for each

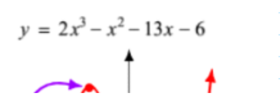
$y=(x+5)(x-3)(2x+4)$ y-intercept: (0, -60)

$y=-x^4+2x^3-x^2+3x+20$ y-intercept: (0, 20)

$y=x^2(3-x)(x+4)^3$ y-intercept: (0,0)

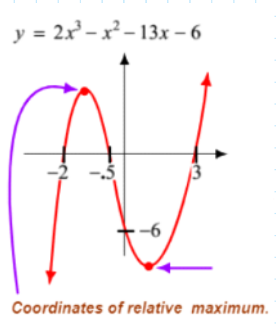
To graph a polynomial function:

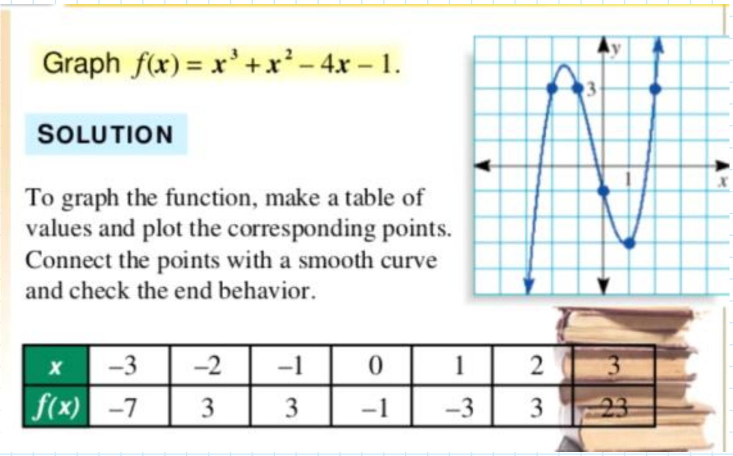
1. Determine the y-intercept by making x=0
2. Determine the x-intercepts by solving the polynomial when y=0
(or you can use your graphing calculator to find these



$y = 2x^3 - x^2 - 13x - 6$

2. Determine the x-intercepts by solving the polynomial when y=0
(or you can use your graphing calculator to find these characteristics)
3. Use the degree and leading coefficient to determine behaviour.
4. Use the table of values to estimate the relative and/or absolute Max's and Min's and get other points.
5. Draw the curve with the correct behaviour.



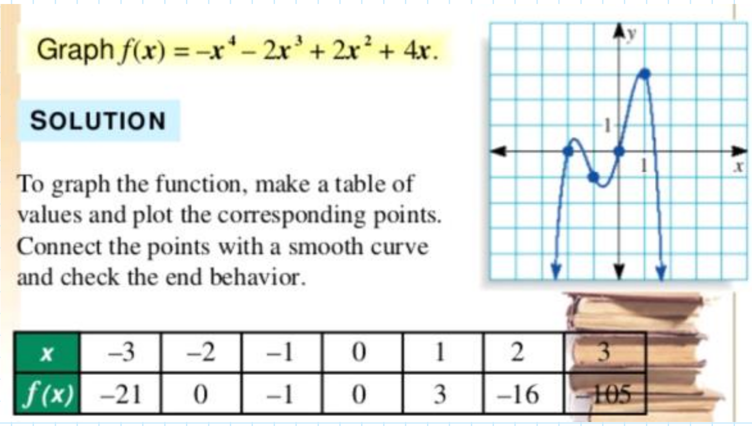


Graph $f(x) = x^3 + x^2 - 4x - 1$.

SOLUTION

To graph the function, make a table of values and plot the corresponding points. Connect the points with a smooth curve and check the end behavior.

x	–3	–2	–1	0	1	2	3
f(x)	–7	3	3	–1	–3	3	23



Graph $f(x) = -x^4 - 2x^3 + 2x^2 + 4x$.

SOLUTION

To graph the function, make a table of values and plot the corresponding points. Connect the points with a smooth curve and check the end behavior.

x	–3	–2	–1	0	1	2	3
f(x)	–21	0	–1	0	3	–16	–105

Now we are ready to graph.

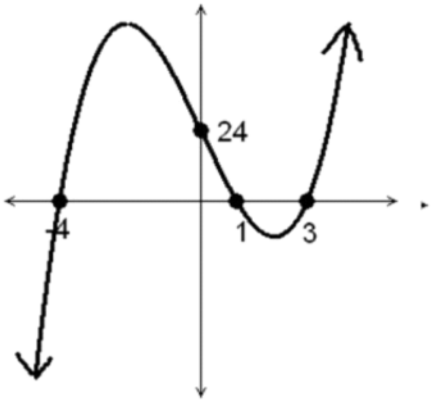
State the type, roots, y-intercept and graph.

$$y = 2(x-3)(x+4)(x-1)$$

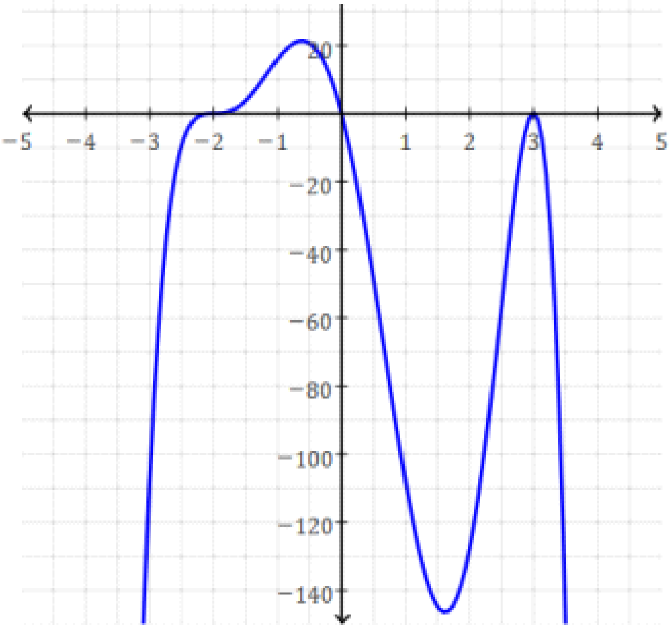
Type: positive odd

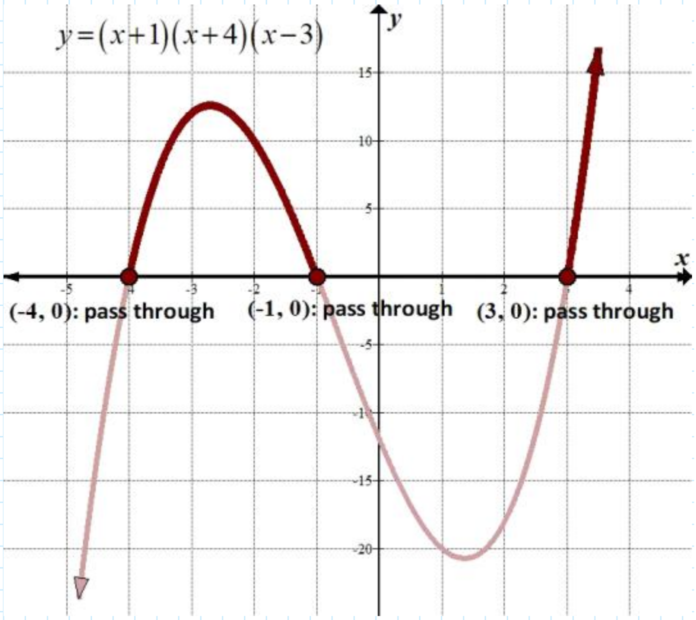
roots: 3, -4, 1

y-intercept: (0, 24)



$$y = -x^6 + 15x^4 + 10x^3 - 60x^2 - 72x$$
$$= -x(x+2)^3(x-3)^2$$





KEY Characteristics Practice

Determine the characteristics of the following polynomial functions.

1. $f(x) = x^3 + x^2 - x - 2$

1. Degree = 3
2. Leading Coefficient = 1
3. Positive/Negative = +
4. Behaviour =

↗

down into QIII

+ up into QI

↙

5. # of turning points = 0, 2
6. # of x-intercepts = 1 – 3
7. y-intercept = (0, -2)

2. $f(x) = x^4 - 4x^3 + 2x^2 + x + 4$

1. Degree = 4
2. Leading Coefficient = 1
3. Positive/Negative = +
4. Behaviour = up into QII +

up into QI

5. # of turning points = 1, 3
6. # of x-intercepts = 0 – 4
7. y-intercept = (0, 4)

3. $f(x) = x^5 - 4x^3 + 4x - 1$

1. Degree = 5
2. Leading Coefficient = 1
3. Positive/Negative = +
4. Behaviour = down into QIII

+ up into QI

5. # of turning points = 0, 2, 4
6. # of x-intercepts = 1 – 5
7. y-intercept = (0, -1)

4. $f(x) = -x^2 - 6x - 7$

1. Degree = 2
2. Leading Coefficient = -1
3. Positive/Negative = –
4. Behaviour = down into QIII

+ down into QIV

5. # of turning points = 1
6. # of x-intercepts = 0 – 2
7. y-intercept = (0, -7)

5. $f(x) = -x^3 + 10x^2 - 33x + 32$	6. $f(x) = -x^4 + 3x^3 - 5x - 2$
1. Degree = 3	1. Degree = 4
2. Leading Coefficient = -1	2. Leading Coefficient = -1
3. Positive/Negative = $-$	3. Positive/Negative = $-$
4. Behaviour = up into QII & down into QIV	4. Behaviour = down into QIII + down into QIV
5. # of turning points = 0, 2	5. # of turning points = 1, 3
6. # of x-intercepts = 1–3	6. # of x-intercepts = 0–4
7. y-intercept = (0, 32)	7. y-intercept = (0, −2)