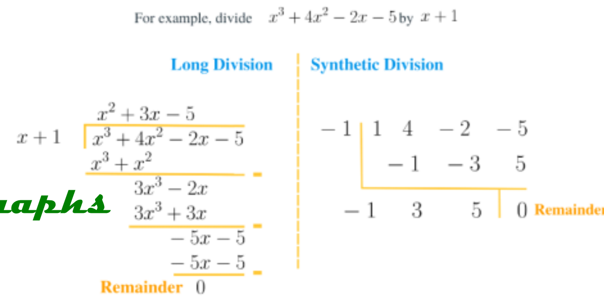
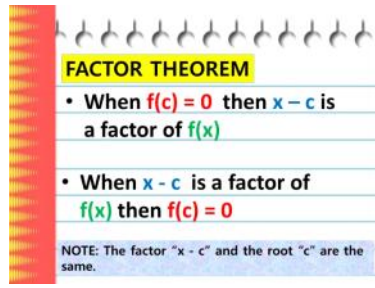
Wednesday, May 15th

Plan For Today:



1. Go over U1 Exam - any questions?
2. Any Questions from last class?
    - ✧ *Polynomial characteristics of graphs*
3. Start Chapter 3: Polynomial Functions
    - ✓ 3.1: Characteristics of Polynomial Functions
    - ✓ 3.2: Equations & Graphs of Polynomials Functions
    - ✸ **3.3: The Remainder Theorem**
    - ✸ **3.4: Factoring Review & The Factor Theorem**
    - ✸ **3.5: Applications & Word Problems**
4. Work on practice questions from Workbook



Plan Going Forward:

1. Work on 3.3-3.5 practice from the workbook for tomorrow.

    - ✸ **CHECK-IN QUIZ ON THURSDAY, MAY 16TH**

2. We will finish/review Chapter 3 Polynomials tomorrow and possibly start an intro to Ch4 on Thursday if time.

    - ✸ **CHAPTER 3 PROJECT DUE TUESDAY, MAY 21ST**
    - ✸ **CHAPTER 3 TEST ON TUESDAY, MAY 21ST**

VICTORIA DAY ON MONDAY, MAY 20TH - SCHOOL CLOSED

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class. Anurita Dhiman = adhiman@sd35.bc.ca

MC Characteristics of Graphs Practice & KEY

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

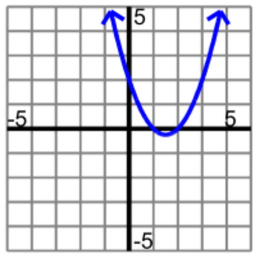
Identify Polynomial Functions

Identify which graph represents the given polynomial function.

1) $y = -x^2 - 2x + 2$

____

A)



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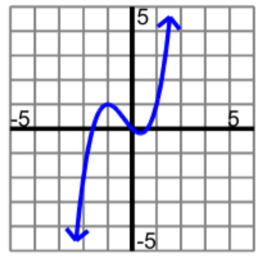

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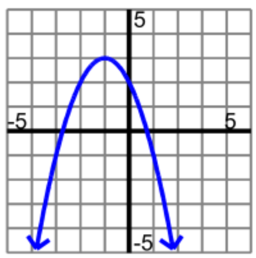

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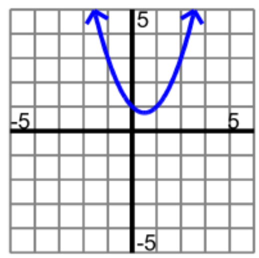

2) $y = 3x^2 + 2x + 2$

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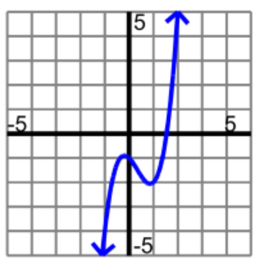
A)



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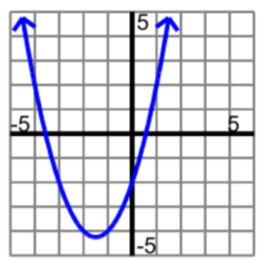

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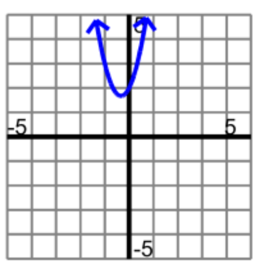

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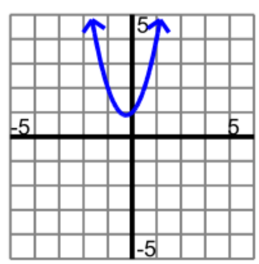


Name : ______________________ Score : __________

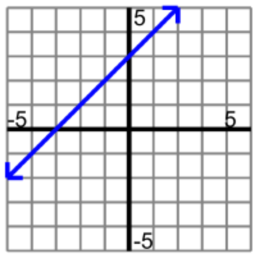
Teacher : ______________________ Date : __________

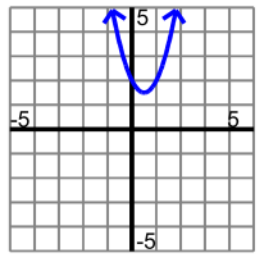
Identify Polynomial Functions

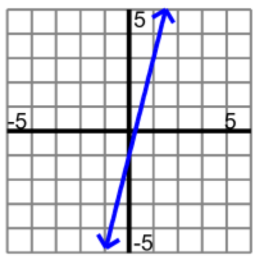
Identify which graph represents the given polynomial function.

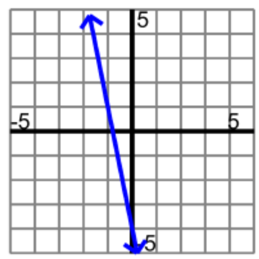
3) $y = -5x - 4$

____

A) 

B) 

C) 

D) 

4) $y = -x^3 - 2x^2 + x + 1$

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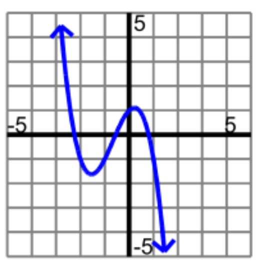
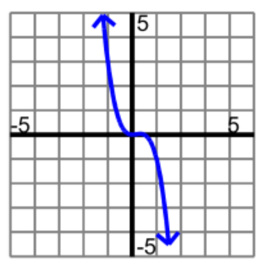
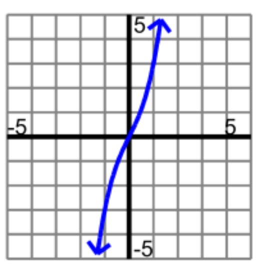
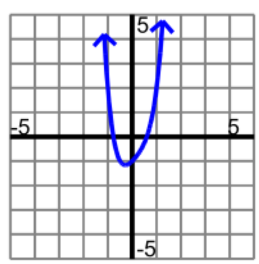
A) 

B) 

C) 

D) 

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

Identify Polynomial Functions

Identify which graph represents the given polynomial function.

5) $y = x^3 - 2x^2 - 2$

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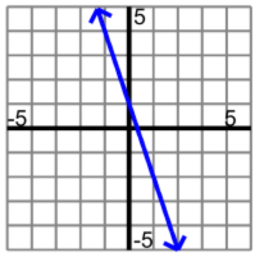
A)



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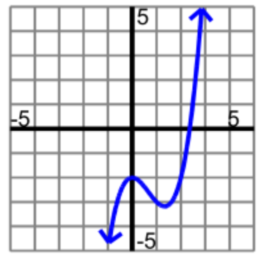

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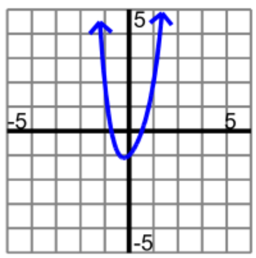

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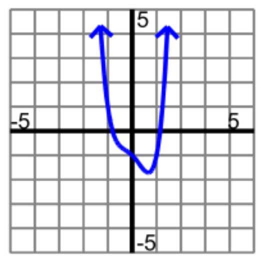

6) $y = 5x + 2$

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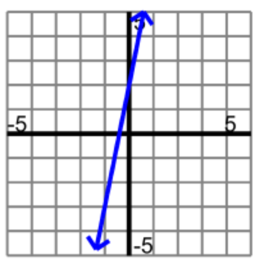
A)



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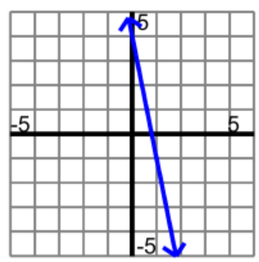

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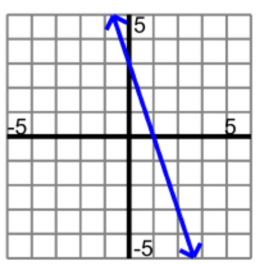

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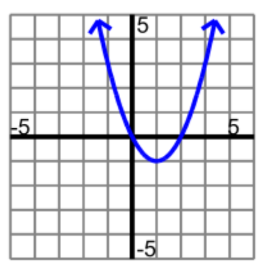


Name : ______________________________ Score : ____________

Teacher : ____________________________ Date : ____________

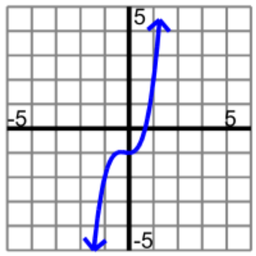
Identify Polynomial Functions

Identify which graph represents the given polynomial function.

7) $y = -x^3 - x^2 + x + 2$

____

A)



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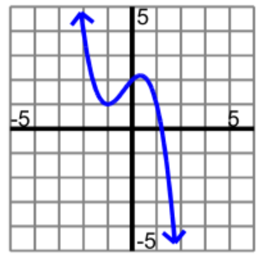

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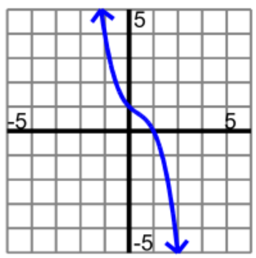

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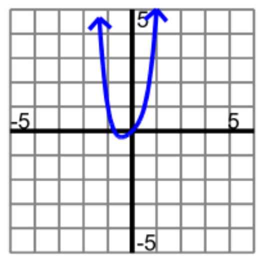

8) $y = x^3 + 2x - 1$

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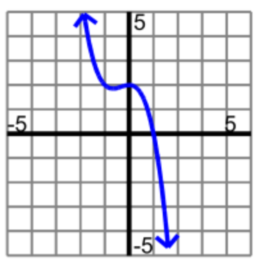
A)



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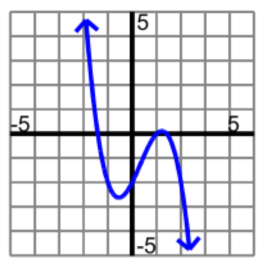

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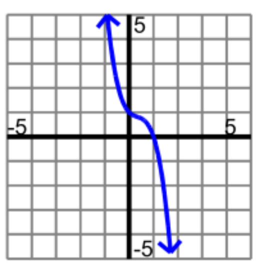

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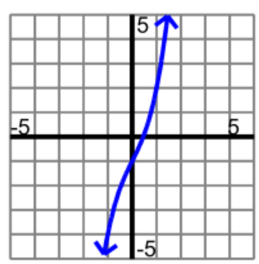

Name : ____________________ Score : __________

Teacher : ____________________ Date : __________

Identify Polynomial Functions

Identify which graph represents the given polynomial function.

9) $y = -x^4 + x^3 - x^2 - 2x + 1$

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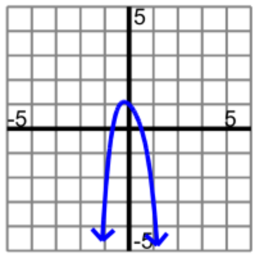
A)



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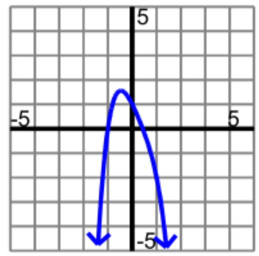

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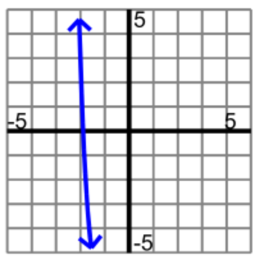

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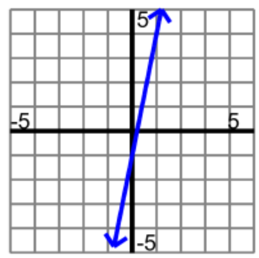
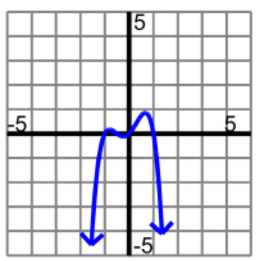


10) $y = x^4 - 2x^3 + 2x^2 + 1$

____

A)



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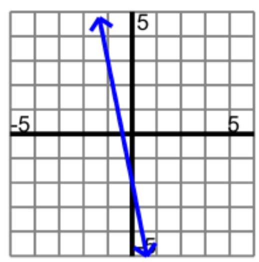

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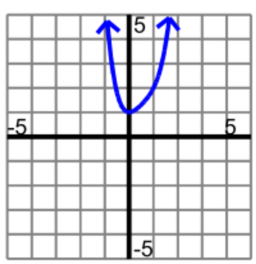

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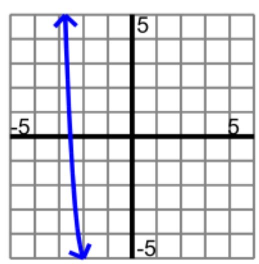


Name : ____________________ Score : __________

Teacher : ____________________ Date : __________

KEY

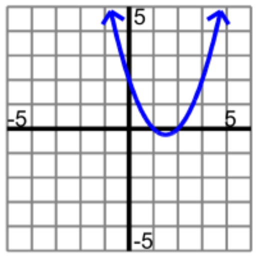
Identify Polynomial Functions

Identify which graph represents the given polynomial function.

1) $y = -x^2 - 2x + 2$

C

A)



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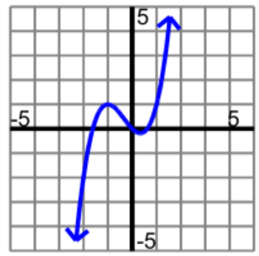

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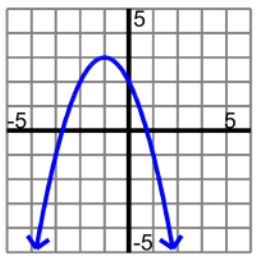

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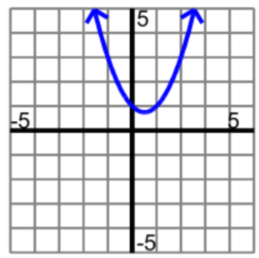
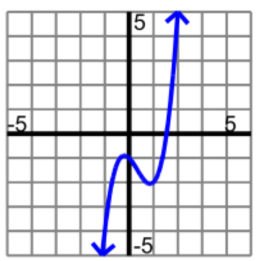


2) $y = 3x^2 + 2x + 2$

C

A)



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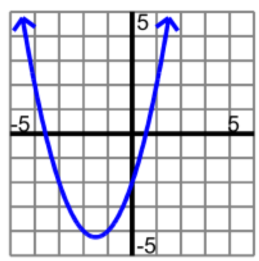

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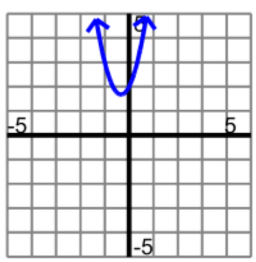

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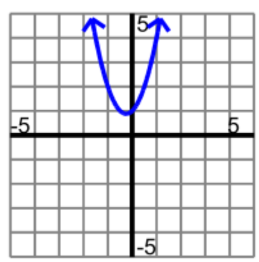


Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

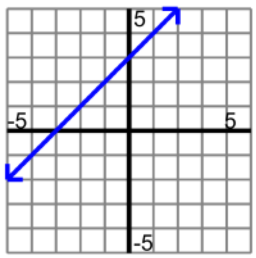
Identify Polynomial Functions

Identify which graph represents the given polynomial function.

3) $y = -5x - 4$

D______

A)



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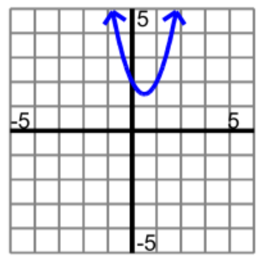

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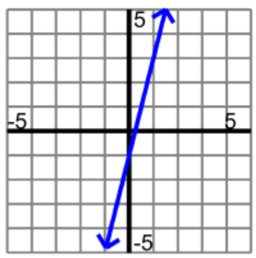

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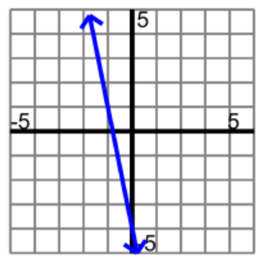
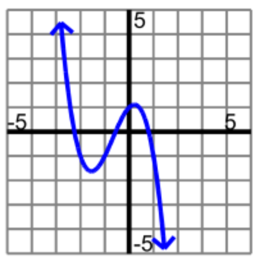


4) $y = -x^3 - 2x^2 + x + 1$

A______

A)



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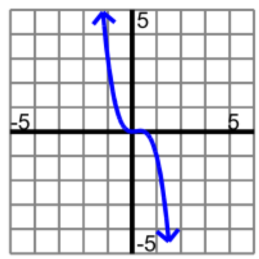

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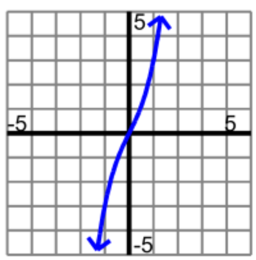

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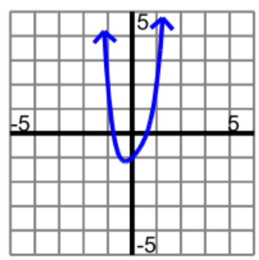


Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

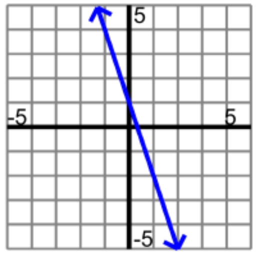
Identify Polynomial Functions

Identify which graph represents the given polynomial function.

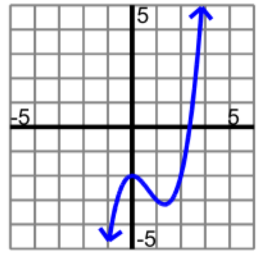
5) $y = x^3 - 2x^2 - 2$

B

A)



B)



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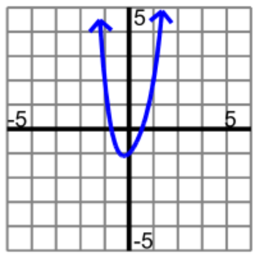

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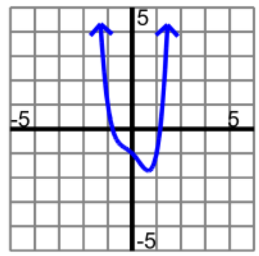
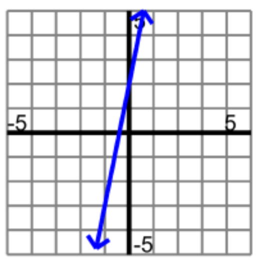


6) $y = 5x + 2$

A

A)



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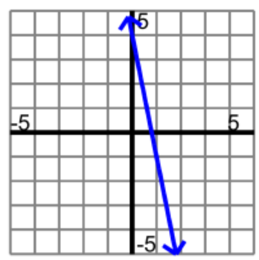

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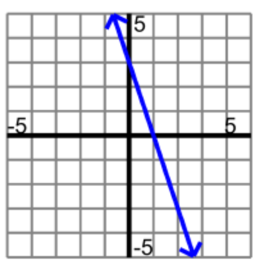

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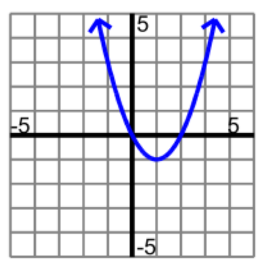


Name : ____________________ Score : __________

Teacher : ____________________ Date : __________

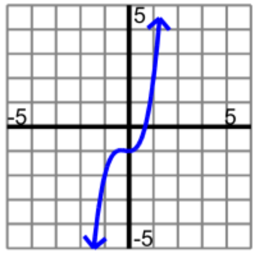
Identify Polynomial Functions

Identify which graph represents the given polynomial function.

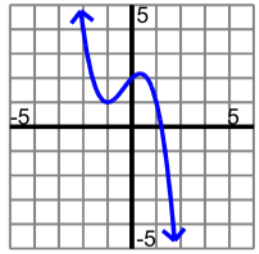
7) $y = -x^3 - x^2 + x + 2$

B______

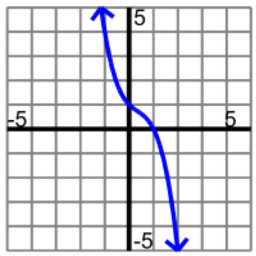
A)



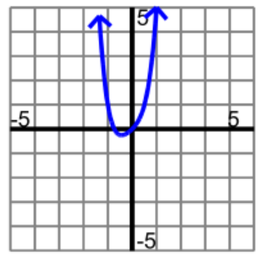
B)



C)



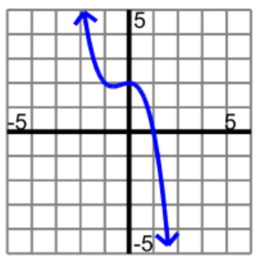
D)



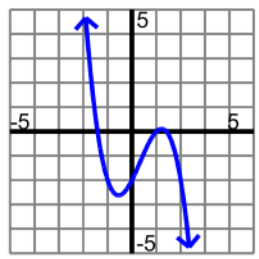
8) $y = x^3 + 2x - 1$

D______

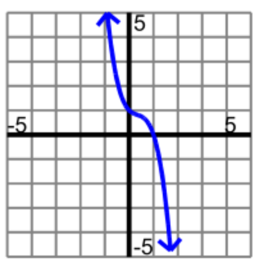
A)



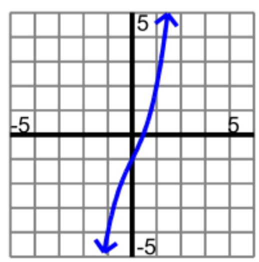
B)



C)



D)



Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

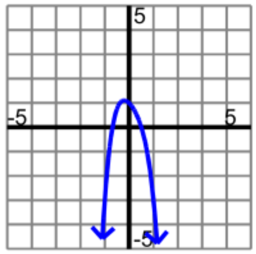
Identify Polynomial Functions

Identify which graph represents the given polynomial function.

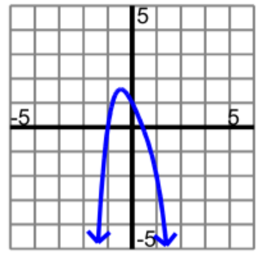
9) $y = -x^4 + x^3 - x^2 - 2x + 1$

B______

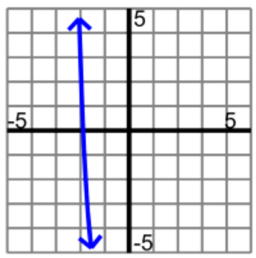
A)



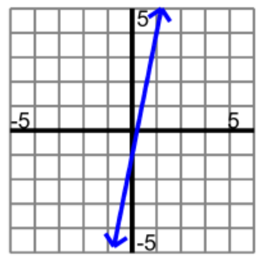
B)



C)



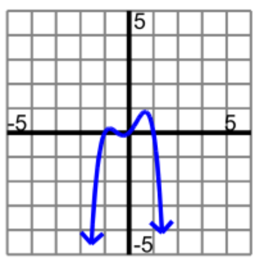
D)



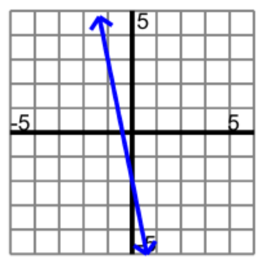
10) $y = x^4 - 2x^3 + 2x^2 + 1$

C______

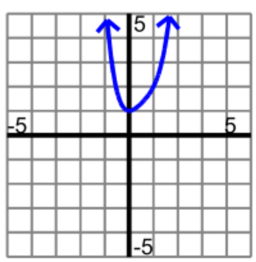
A)



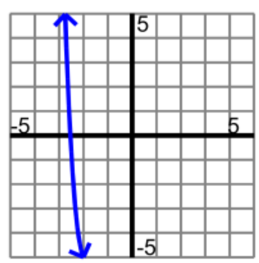
B)



C)



D)



Review

p. 129 #7 abc

b) $\underbrace{0,0}_{M2}, \underbrace{1,1,1}_{M3}, 2$ $\qquad$ $\underbrace{P(-1)=12}_{x=-1 \quad y=12}$

$P(x) = a x^2 (x-1)^3 (x-2)$ $\qquad$ $x=1$, $x-1=0$, $(x-1)$ $\qquad$ $x=2$, $x-2=0$, $(x-2)$

$12 = a(-1)^2(-1-1)^3(-1-2)$

$12 = a(-1)^2(-2)^3(-3)$

$12 = a(1)(-8)(-3)$

$\frac{12}{24} = \frac{24a}{24}$

$a = \frac{1}{2}$ $\rightarrow$ $\boxed{P(x) = \frac{1}{2}x^2(x-1)^3(x-2)}$

3.3 Dividing Polynomials by Binomials. $= \frac{P(x)}{x-a}$

Ex 3 p. 133 is great for notes!

Ex 4 p. 134 $\quad P(x) = 4x^5 - 30x^3 - 50x + 2 \div x+3$

add in missing degrees with zero coefficient

$4x^5 + 0x^4 - 30x^3 + 0x^2 - 50x + 2$

Synthetic Division:

① write the x-value from binomial

write all coefficients for all degrees in order from highest to lowest.

$$\begin{array}{c|cccccc} -3 & 4 & 0 & -30 & 0 & -50 & 2 \\ + & & -12 & 36 & -18 & 54 & -12 \\ \hline \times & 4 & -12 & 6 & -18 & 4 & -10 \end{array}$$

ADD

Bring down digit first

Remainder $= -10$

② any numbers below the line multiply by x-value from binomial

any numbers above the line ADD.

these coefficients form the quotient.

☆ quotient begins at x^{n-1}

$4x^4 - 12x^3 + 6x^2 - 18x + 4 - \frac{10}{x+3}$

OR $(x+3)(4x^4 - 12x^3 + 6x^2 - 18x + 4) - 10$

Ex 6 $\quad 6x^4 - 7x^3 + 4x^2 - 11x + 9 \div 2x-1 = 0$

$x = \frac{1}{2}$

Section 3.3: Practice Dividing Polynomials

Do long division AND synthetic division for each of the following and write the division statement at the end.

1. $(x^3 + 7x^2 + 14x + 3) \div (x + 2)$

Long Division	**Synthetic Division**
$$\begin{array}{r} x^2 + 5x + 4 \\ x+2 \overline{) x^3 + 7x^2 + 14x + 3} \\ -\underline{\ x^3 + 2x^2} \quad\quad\quad\quad \\ 5x^2 + 14x \quad\quad \\ -\underline{\ 5x^2 + 10x} \quad\quad \\ 4x + 3 \\ -\underline{\ 4x + 8} \\ -5 \end{array}$$	$$\begin{array}{c\|cccc} x+2 & x^3 & +7x^2 & +14x & +3 \\ -2 & 1 & 7 & 14 & 3 \\ + & & -2 & -10 & -8 \\ \hline x & 1 & 5 & 4 & -5 \end{array}$$ $\underbrace{x^2 + 5x + 4}$ $- \frac{5}{x+2}$ OR: $(x+2)(x^2+5x+4) - 5$

Division Statement:

$$\frac{x^3 + 7x^2 + 14x + 3}{x + 3} = x^2 + 5x + 4 - \frac{5}{x + 2}$$

2. $(10x^3+37x^2+37x+6)\div(x+2) = x^2$

Synthetic Division

$x+2 \quad 10x^3+37x^2+37x+6$

-2	10	37	37	6
+		-20	-34	-6
x	10	17	3	0

no remainder

$\boxed{10x^2+17x+3}$ quotient.

SHORT-CUT

AC = 30

+15, +2 17 = B

$+15$ / 10, $+2$ / 10 = 17

$\frac{3}{2}$ $\frac{1}{5}$ ← ± constant ← coeff. with x

$(2x+3)(5x+1)$

Notice you can factor the quadratic further:

$\underbrace{10x^2+15x}_{GCF=5x} / \underbrace{+2x+3}_{GCF=1}$

$5x(2x+3)+1(2x+3) \rightarrow \boxed{(2x+3)(5x+1)}$

In this case this polynomial can be fully factored to the following:

$(x+2)(2x+3)(5x+1) = 10x^3+37x^2+37x+6$

original polynomial.

$$\frac{(\cancel{x+2})(2x+3)(5x+1)}{(\cancel{x+2})} = \frac{10x^3+37x^2+37x+6}{(x+2)}$$

$$(2x+3)(5x+1) = \frac{10x^3+37x^2+37x+6}{x+2}$$

quotient original question

Remainder Theorem

$P(a) = R$ for $\frac{P(x)}{x-a}$

1. use 'a' value from binomial + substitute into 'x' of polynomial
2. evaluate to determine remainder.

p.143 #1 Ex: $P(x) = x^4+3x^3-7x+2$

$(x-a) = (x+2)$

find the remainder for $\frac{P(x)}{x-a} = \frac{x^4+3x^3-7x+2}{x+2}$

1. $x+2 \rightarrow x = -2$
2. $P(-2) = (-2)^4 + 3(-2)^3 - 7(-2) + 2$ (-8)
3. evaluate $= 16 - 24 + 14 + 2$

$\boxed{\text{Remainder} = 8}$

#2 p.143 2c) $2x^4 + kx^2 - 3x + 5 \div x-2$

$R = 3$

solve for 'k'

1. $x-2 \rightarrow x = 2$

solve for 'k'

1. $x-2 \rightarrow x=2$
2. $P(2) \rightarrow 2(2)^4 + k(2)^2 - 3(2) + 5 = 3$
3. solve for 'k' $\quad 32 + 4k - 6 + 5 = 3$

$$4k + 31 = 3 \quad -31$$

$$\frac{4k}{4} = \frac{-28}{4}$$

$$\boxed{k = -7}$$

Factor Theorem

For $\frac{P(x)}{x-a} \rightarrow P(a) = 0$ (remainder is zero) when $x-a$ is a factor of $P(x)$

Ex: $P(x) = 2x^3 - 7x^2 + 2x + 3 \div x-1$

= is $x-1$ a factor of $P(x)$?

solution: ① $x-1 \rightarrow x=1$

② $P(1) = 0$? = <u>IS</u> a factor

$\neq 0$ = <u>NOT</u> a factor.

$$2(1)^3 - 7(1)^2 + 2(1) + 3 =$$

$$2 - 7 + 2 + 3 = 0 \checkmark$$

YES : $(x-1)$ <u>IS</u> a factor of $P(x)$ b/c the remainder is zero.

IF the binomial <u>IS</u> a factor, do synthetic division + fully factor the polynomial.

Note: if the degree of $P(x)$ is $n>3$, find extra factors. ∴ use synthetic division until quotient is x^2 for trinomial factoring

$x-1$	$2x^3$	$-7x^2$	$+2x$	$+3$
1		2	-5	-3
+ / ×	2	-5	-3	0

$$2x^2 - 5x - 3 \quad \text{factor}$$

$AC = -6$

$-6, +1 \rightarrow -5$

$\frac{-6}{2}, \frac{+1}{2} \rightarrow -\frac{3}{1}, +\frac{1}{2}$

$$(x-3)(2x+1)$$

$2x^2 - 5x - 3$

$2x^2 - 6x + 1x - 3$

$2x(x-3) + (x-3)$

$(x-3)(2x+1)$

Final answer is all binomial factors written together

$$\boxed{(x-1)(x-3)(2x+1)}$$

$x = 1 \quad x = 3 \quad x = -\frac{1}{2}$

if question says solve.

together

$x=1$ $\quad x=3$ $\quad x=-\frac{1}{2}$ $\quad$ solve.

Rational Root Theorem

① use the constant term to determine all possible factors of the polynomial $P(x)$

$$ex: \; x^3+3x^2+4x\underbrace{-12}_{}$$

factors of $-12 = \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

② Test factors in polynomial to find the factor which gives a remainder of zero.

$$\text{test } x=1 \longrightarrow P(1) = (1)^3+3(1)^2+4(1)-12$$
$$= 1+3+4-12$$
$$= -4$$

$\therefore\; x-1$ is <u>NOT</u> a factor.

③ use a factor to divide & fully factor the polynomial

$$Ex \quad P(x) = 2x^4+5x^3-11x^2-20x+12$$

① possible factors $\longrightarrow$ 12 $\rightarrow \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

② test factors to get $R=0$ $\quad P(-3)=0 \quad 2(-3)^4+5(-3)^3-11(-3)^2-20(-3)+12=0$

$x=-3$ ✓

③ do division with this factor

$x+3$ $\quad 2x^4+5x^3-11x^2-20x+12$

-3	2	5	-11	-20	12
+		-6	3	24	-12
x	2	-1	-8	4	0 ✓

$$2x^3-x^2-8x+4$$

$P(2)=0$

or $P(-2)=0$

$x-2$

2	2	-1	-8	4
+		4	6	-4
x	2	3	-2	0 ✓

$2x^2 + 3x - 2$

$AC = -4$

$+4, -1 = 3$

$\frac{+4}{2}, \frac{-1}{2}$

$\frac{+2}{1} \to x+2$

$\to 2x-1$

$(2x-1)(x+2)$

Final Solution
- collect all factors

$\boxed{(x+3)(x-2)(2x-1)(x+2)}$

Solutions: $x = -3, 2, \frac{1}{2}, -2$

or in order

$\boxed{x = -3, -2, \frac{1}{2}, 2}$

Dividing Polynomials

Dividing Polynomials

Long Division

$$
\begin{array}{r|l}
 & 2x^2 + x - 5 \\ \hline
x-3 & 2x^3 - 5x^2 - 8x + 15 \\
 & \underline{2x^3 - 6x^2} \\
 & \quad x^2 - 8x \\
 & \quad \underline{x^2 - 3x} \\
 & \qquad -5x + 15 \\
 & \qquad \underline{-5x + 15} \\
\text{Remainder} & \qquad\qquad 0
\end{array}
$$

Synthetic Division

$x-3 \quad 2x^3 - 5x^2 - 8x + 15$

$$
\begin{array}{c|cccc}
3 & 2 & -5 & -8 & 15 \\
 & & 6 & 3 & -15 \\ \hline
\times & 2 & 1 & -5 & 0
\end{array}
$$

Remainder

$\boxed{2x^2 + x - 5}$

Long Division

Review long division of polynomials.

$$(x^3 - 7x^2 + 11x + 9) \div (x - 4)$$

$$
\begin{array}{r|l}
 & x^2 - 3x - 1 \\ \hline
x-4 & x^3 - 7x^2 + 11x + 9 \\
 & \underline{-x^3 + 4x^2} \\
 & \quad -3x^2 + 11x \\
 & \quad \underline{+3x^2 - 12x} \\
 & \qquad -x + 9 \\
 & \qquad \underline{+x - 4} \\
 & \qquad\quad 5
\end{array}
$$

quotient: $x^2 - 3x - 1$; divisor: $x - 4$; dividend: $x^3 - 7x^2 + 11x + 9$; add the opposite; constant 5 → remainder

Division Statement

$$x^3 - 7x^2 + 11x + 9 = (x-4)(x^2 - 3x - 1) + 5$$

Synthetic Division

divide $2x^3 + 3x^2 - 4x + 15$ by $x + 3$

$$
\begin{array}{c|cccc}
 & 2x^3 & 3x^2 & -4x & 15 \\
+3 & 2 & 3 & -4 & 15 \\
- & & & & \\ \hline
\times & & & &
\end{array}
$$

$$
\begin{array}{c|cccc}
+3 & 2 & 3 & -4 & 15 \\
- & & 6 & -9 & 15 \\ \hline
\end{array}
$$

$P(x) = x^3 - 10x + 6$ is divided by $x + 4$

$$
\begin{array}{c|cccc}
+4 & 1 & 0 & -10 & 6 \\
- & & 4 & -16 & 24 \\ \hline
\times & 1 & -4 & 6 & -18
\end{array}
$$

remainder

$= \boxed{x^2 - 4x + 6 - \frac{18}{x+4}}$

+3	2	3	−4	15
−		6	−9	15
×	2	−3	5	0

remainder

$$(2x^3 + 3x^2 - 4x + 15) \div (x + 3) = 2x^2 - 3x + 5$$

Restriction: $x + 3 \neq 0$ or $x \neq -3$

Remainder Theorem

If a polynomial f(x) is divided by (x – a), the remainder is f(a).
f(x) = (x – a) Q(x) + f(a)

Factor Theorem

A polynomial f(x) has a factor (x – a) if and only if f(a) = 0.

3.4 Factoring Review

Common Factor First: GCF = Greatest Common Factor

Factor Out the GCF

The first step to factoring is to factor out the greatest common factor (GCF) from each term.

Example:

$$12y^3 - 15y^2 + 6y$$

3y 3y 3y

$$2\times 2\times \boxed{3\times y}\times y\times y \qquad 5\times \boxed{3\times y}\times y \qquad 2\times \boxed{3\times y}$$

$$\boxed{3y}(4y^2 - 5y + 2)$$

Factoring Techniques

Factor out the GCF

$$2yx^2 - 8xy - 24y$$
$$= 2y(x^2 - 4x - 12)$$

Special Cases

Difference of Two Squares

$$x^2 - 9$$
$$= x^2 - 3^2$$
$$= (x+3)(x-3)$$

Perfect Square Trinomial

$$x^2 - 10x + 25$$
$$= (x-5)^2$$

Grouping

$$4x^2 - 4x - 15$$
$$= 4x^2 - 10x + 6x - 15$$
$$= 2x(2x-5) + 3(2x-5)$$
$$= (2x+3)(2x-5)$$

Difference of Squares

$$a^2 - b^2 = (a+b)(a-b)$$

Examples:

$$9x^2 - 16$$
$$= (3x)^2 - 4^2$$
$$= (3x+4)(3x-4)$$

$$4x^2 - 81y^2$$
$$= (2x)^2 - (9y)^2$$
$$= (2x+9y)(2x-9y)$$

Trinomial Factoring Where the Leading Coefficient 'a' = 1

$$ax^2 + bx + c$$

To factor $x^2 + bx + c$:

1. First arrange in descending order.
2. Use a trial-and-error procedure that looks for factors of c whose sum is b.
   - If c is positive, then the signs of the factors are the same as the sign of b.
   - If c is negative, then one factor is positive and the other is negative. (If the sum of the two factors is the opposite of b, changing the signs of each factor will give the desired factors whose sum is b.)
3. Check your result by multiplying.

Thus the factorization is

$$(x+3)(x+5), \quad \text{or} \quad (x+5)(x+3)$$

by the commutative law of multiplication. In general,

$$(x+p)(x+q) = x^2 + (p+q)x + pq.$$

To factor, we can use this equation in reverse.

Factoring Trinomials with a =1

	$x^2 + bx + c$	$x^2 + 2x - 8$
Find the two numbers that will make these equations true.	$\square \times \square = c$ $\square + \square = b$	$\boxed{4} \times \boxed{-2} = -8$ $\boxed{4} + \boxed{-2} = 2$
Put the two numbers in the expression.	$(x+\square)(x+\square)$	$(x+\boxed{4})(x+\boxed{-2})$

$$x^2 - 5x - 24$$

$$(x \quad)(x \quad)$$

$-24 \cdot 1$

$-12 \cdot 2$

$-8 \cdot 3$

$-6 \cdot 4$

$$x^2 - 5x - 24$$

$$(x-8)(x+3)$$

Factor Trinomial with Negative Leading Coefficient

When the leading coefficient of a polynomial is negative, we can factor out a common factor with a negative coefficient.

Examples:

$$\begin{aligned}&-5x^2+x+4\\&=-(5x^2-x-4)\\&=-(5x^2-5x+4x-4)\\&=-\left(5x(x-1)+4(x-1)\right)\\&=-(5x+4)(x-1)\end{aligned}$$

$$\begin{aligned}&-3x^3+12x^2+15x\\&=-3x(x^2-4x-5)\\&=-3x(x^2-5x+x-5)\\&=-3x\left(x(x-5)+(x-5)\right)\\&=-3x(x+1)(x-5)\end{aligned}$$

Perfect Square Trinomials (PST)

$$a^2+2ab+b^2=(a+b)(a+b)=(a+b)^2$$

$$a^2-2ab+b^2=(a-b)(a-b)=(a-b)^2$$

Examples:

$$\begin{aligned}&x^2+12x+36\\&=x^2+(2)(6)x+6^2\\&=(x+6)^2\end{aligned}$$

$$\begin{aligned}&9x^2-12x+4\\&=(3x)^2-(3)(2)x+2^2\\&=(3x-2)^2\end{aligned}$$

There are many methods for factoring a trinomial where $a \neq 1$

1. The FOIL method is the same as a 'Guess and Check' Method

THE FOIL METHOD

To factor trinomials of the type $ax^2 + bx + c$, $a \neq 1$, using the **FOIL method**:

1. Factor out the largest common factor.
2. Find two First terms whose product is ax^2:

$$(\square x + \quad)(\square x + \quad) = ax^2 + bx + c.$$

FOIL

3. Find two Last terms whose product is c:

$$(\quad x + \square)(\quad x + \square) = ax^2 + bx + c.$$

FOIL

4. Repeat steps (2) and (3) until a combination is found for which the sum of the Outside and Inside products is bx:

$$(\square x + \square)(\square x + \square) = ax^2 + bx + c.$$

I

O

FOIL

5. Always check by multiplying.

TIPS FOR FACTORING $ax^2 + bx + c$, $a \neq 1$, USING THE FOIL METHOD

1. If the largest common factor has been factored out of the original trinomial, then no binomial factor can have a common factor (other than 1 or -1).
2. a) If the signs of all the terms are positive, then the signs of all the terms of the binomial factors are positive.
   b) If a and c are positive and b is negative, then the signs of the factors of c are negative.
   c) If a is positive and c is negative, then the factors of c will have opposite signs.
3. Be systematic about your trials. Keep track of those you have tried and those you have not.
4. Changing the signs of the factors of c will change the sign of the middle term.

1. The AC Method here is the Decomposition method; also knows as the Factor by Grouping method.

THE *ac*-METHOD

To factor $ax^2 + bx + c, a \neq 1$, using the ac-method:

1. Factor out the largest common factor.
2. Multiply the leading coefficient a and the constant c.
3. Try to factor the product ac so that the sum of the factors is b. That is, find integers p and q such that $pq = ac$ and $p + q = b$.
4. Split the middle term. That is, write it as a sum using the factors found in step (3).
5. Factor by grouping.
6. Always check by multiplying.

Factoring Polynomials:

Type 2:

Quadratic Trinomials with a Leading coefficient $\neq 1$

a) Factoring by Decomposition

1. Multiply a and c
2. Look for two numbers that multiply to that product and add to b
3. Break down the middle term into two terms using those two numbers
4. Find the common factor for the first pair and factor it out & then find the common factor for the second pair and factor it out.
5. From the two new terms, place the common factor in one bracket and the factored out factors in the other bracket.

$$a \times c = -20$$

The 2 nos. are -20 & 1

$$5x^2 - 19x - 4$$

$$= 5x^2 - 20x + 1x - 4$$

$$= 5x(x-4) + 1(x-4)$$

$$= (x-4)(5x+1)$$

Factor by Grouping

Example:

$$6x^2 + 15x - 21$$

$$= 3\left(2x^2 + 5x - 7\right)$$ 1. Factor out Greatest Common Factor

$$2 \times -7 = -14 \qquad +5$$

Find two numbers when multiplied get -14 and when added get +5

$$+7 \times -2 = -14$$

$$+7 + -2 = +5$$

Rewrite $5x$ as $-2x + 7x$

$$= 3\left(2x^2 - 2x + 7x - 7\right)$$

2. Split the middle term into two terms.

$$= 3\left[2x(x-1) + 7(x-1)\right]$$

3. Rewrite the pairs of terms and take out the common factor.

$$= 3(x-1)(2x+7)$$

Solving General Trinomials - the Decomposition Method

$3x^2 - 10x + 8$

The product is $3 \times 8 = 24$.
The sum is -10.

Factors of 24	
1	24
2	12
3	8
4	6

$3x^2 - 6x - 4x + 8$

Rewrite the middle term of the polynomial using -6 and -4.
(-6x - 4x is just another way of expressing -10x.)

$3x(x - 2) - 4(x - 2)$

Factor by grouping.

$(x - 2)(3x - 4)$

3. This is the Short-cut Method I like to use.

Decomposition	**Short-cut Factoring**
$3x^2-19x-14$ **A = 3, C = -14 → AC = -42**	$3x^2-19x-14$ **A = 3, C = -14 → AC = -42**
Two numbers that multiply to -42 but add to -19	**Two numbers that multiply to -42 but add to -19**
These numbers are: -21 and +2	**These numbers are: -21 and +2**
Replace the middle (b) term with these two factors written with an *x*	**Place the first term of the trinomial without the squared in the first spot of each factored bracket**
$3x^2-21x+2x-14$	$(3x\quad)(3x\quad)$
Split it in half and factor each half	**Place the two factors (-21 & +2) in the brackets to form the binomials**
$3x^2-21x \mid +2x-14$ $3x(x-7)+2(x-7)$	$(3x-21)(3x+2)$
Place each term in front of the brackets in its own bracket and write the other common binomial in one bracket.	**Reduce the terms in the binomials like you would fractions**
$(3x+2)(x-7)$	$\left({}_1\cancel{3}x-\cancel{21}_7\right)(3x+2)$ $=(x-7)(3x+2)$
Done!	**Done!**

Factoring Review & Practice

Review of Algebra and Factoring

Common Factoring

Determine the greatest common factor by checking what the largest term divisible by all terms is (numbers and variables).

Ex. $2x^2 - 6x \rightarrow \boxed{2x(x-3)}$

Complete the following for practice:

a) $3x^3 - 9x^2$

b) $-8x^3 + 2x^2 - 22x$

Binomial Factoring with a Difference of Squares

When the 2 terms of the binomial are perfect squares and there is a subtraction between them, you can use this method for factoring. Form must be $\left(a^2 - b^2\right)$.

Ex. $x^2 - 9 \rightarrow \boxed{(x+3)(x-3)}$ $\quad\quad 4x^2 - 25y^2 \rightarrow \boxed{(2x+5y)(2x-5y)}$

Here, you put the square root of x and the square root of 9 in each bracket with different signs between them: this is the difference of squares factoring.

NOTE: $x^2 + 9$ is a sum of squares and cannot be factored.

Complete the following:

a) $a^2 - 16$ b) $144 - 9y^2$ c) $36x^2 - 49$

Trinomial Factoring

A trinomial is in the form: ax^2+bx+c. There are different methods for trinomial factoring; including decomposition, guess and check, short-cut factoring, box method. I will show you decomposition and short-cut factoring (I usually do short-cut factoring in class).

When a trinomial has a leading coefficient of 1, the method is simple:

x^2-4x-5 → **C = -5.**

Find two numbers that multiply to -5 but add to -4. Here the two numbers or factors are -5 and +1. Place these two factors in the brackets with x and you're done.

$x^2-4x-5=\boxed{(x-5)(x+1)}$

When the leading coefficient is not 1, use one of the following methods.

<table>
<tr>
<th>Decomposition</th>
<th>Short-cut Factoring</th>
<th>Another Short-cut Factoring</th>
<th>Box Method</th>
</tr>
<tr>
<td>$3x^2-19x-14$
A = 3, C = -14 → AC = -42

Two numbers that multiply to -42 but add to -19

These numbers are: -21 and +2

Replace the middle (b) term with these two factors written with an *x*

$3x^2-21x+2x-14$

Split it in half and factor each half

$3x^2-21x \mid +2x-14$
$3x(x-7)+2(x-7)$

Place each term in front of the brackets in its own bracket and write the other common binomial in one bracket.

$(3x+2)(x-7)$

Done!</td>
<td>$3x^2-19x-14$

A = 3, C = -14 → AC = -42

Two numbers that multiply to -42 but add to -19

These numbers are: -21 and +2

Place the first term of the trinomial without the squared in the first spot of each factored bracket

$(3x \quad)(3x \quad)$

Place the two factors (-21 & +2) in the brackets to form the binomials

$(3x-21)(3x+2)$

Reduce the terms in the binomials like you would fractions

$({}_1\cancel{3}x-\cancel{21}_7)(3x+2)$
$=(x-7)(3x+2)$

Done!</td>
<td>$3x^2-19x-14$

A = 3, C = -14 → AC = -42

Two numbers that multiply to -42 but add to -19

These numbers are: -21 and +2

Rewrite the trinomial with the AC as the last term

$x^2-19x-42$

Place the two factors (-21 & +2) in the brackets to form the binomials

$(x-21)(x+2)$

Divide and reduce the constant in each bracket by the A from original trinomial

$\left(x-\frac{21}{3}\right)\left(x+\frac{2}{3}\right)=(x-7)\left(x+\frac{2}{3}\right)$

If there is still a denominator, write that number in front of the *x* in the same brackets to get final factored form.

$(x-7)\left(x+\frac{2}{3}\right)=(x-7)(3x+2)$</td>
<td>$3x^2-19x-14$

A = 3, C = -14 → AC = -42

Two numbers that multiply to -42 but add to -19
These numbers are: -21 and +2

Place the first and last term in the box in the first and last spot

$\begin{array}{|c|c|}\hline 3x^2 & \\ \hline & -14 \\ \hline\end{array}$

Place the two factors with *x* in the second and third box

$\begin{array}{|c|c|}\hline 3x^2 & -21x \\ \hline 2x & -14 \\ \hline\end{array}$

Common factor each row and each column and collect the factors in two brackets for final factored form.

$\begin{array}{|c|c|c}\hline 3x^2 & -21x & 3x \\ \hline 2x & -14 & 2 \\ \hline x & -7 & \end{array}$

$(3x+2)(x-7)$</td>
</tr>
</table>

Complete the following using a method of your choice:

a) $6x^2 - 5x - 4$

b) $2x^2 + 11x + 5$

c) $2x^2 + x - 1$

d) $2x^2 - 3x - 2$

To solve, you make each binomial bracket equal zero and solve for *x*.

Solving the example from above:

$$(3x+2)(x-7)$$

$$3x + 2 = 0 \qquad x - 7 = 0$$

$$3x = -2 \qquad \boxed{x = 7}$$

$$\boxed{x = -\frac{2}{3}}$$

Kuta Software - Infinite Algebra 1

Name________________________________

Factoring Trinomials (a = 1)

Date________________ Period____

Factor each completely.

1) $b^2 + 8b + 7$

2) $n^2 - 11n + 10$

3) $m^2 + m - 90$

4) $n^2 + 4n - 12$

5) $n^2 - 10n + 9$

6) $b^2 + 16b + 64$

7) $m^2 + 2m - 24$

8) $x^2 - 4x + 24$

9) $k^2 - 13k + 40$

10) $a^2 + 11a + 18$

11) $n^2 - n - 56$

12) $n^2 - 5n + 6$

13) $b^2 - 6b + 8$

14) $n^2 + 6n + 8$

15) $2n^2 + 6n - 108$

16) $5n^2 + 10n + 20$

17) $2k^2 + 22k + 60$

18) $a^2 - a - 90$

19) $p^2 + 11p + 10$

20) $5v^2 - 30v + 40$

21) $2p^2 + 2p - 4$

22) $4v^2 - 4v - 8$

23) $x^2 - 15x + 50$

24) $v^2 - 7v + 10$

25) $p^2 + 3p - 18$

26) $6v^2 + 66v + 60$

Kuta Software - Infinite Algebra 1

Name____________________

Factoring Trinomials (a = 1)

Date________________ Period____

Factor each completely.

1) $b^2 + 8b + 7$

$(b + 7)(b + 1)$

2) $n^2 - 11n + 10$

$(n - 10)(n - 1)$

3) $m^2 + m - 90$

$(m - 9)(m + 10)$

4) $n^2 + 4n - 12$

$(n - 2)(n + 6)$

5) $n^2 - 10n + 9$

$(n - 1)(n - 9)$

6) $b^2 + 16b + 64$

$(b + 8)^2$

7) $m^2 + 2m - 24$

$(m + 6)(m - 4)$

8) $x^2 - 4x + 24$

Not factorable

9) $k^2 - 13k + 40$

$(k - 5)(k - 8)$

10) $a^2 + 11a + 18$

$(a + 2)(a + 9)$

11) $n^2 - n - 56$

$(n + 7)(n - 8)$

12) $n^2 - 5n + 6$

$(n - 2)(n - 3)$

13) $b^2 - 6b + 8$

$(b-4)(b-2)$

14) $n^2 + 6n + 8$

$(n+2)(n+4)$

15) $2n^2 + 6n - 108$

$2(n+9)(n-6)$

16) $5n^2 + 10n + 20$

$5(n^2 + 2n + 4)$

17) $2k^2 + 22k + 60$

$2(k+5)(k+6)$

18) $a^2 - a - 90$

$(a-10)(a+9)$

19) $p^2 + 11p + 10$

$(p+10)(p+1)$

20) $5v^2 - 30v + 40$

$5(v-2)(v-4)$

21) $2p^2 + 2p - 4$

$2(p-1)(p+2)$

22) $4v^2 - 4v - 8$

$4(v+1)(v-2)$

23) $x^2 - 15x + 50$

$(x-10)(x-5)$

24) $v^2 - 7v + 10$

$(v-5)(v-2)$

25) $p^2 + 3p - 18$

$(p-3)(p+6)$

26) $6v^2 + 66v + 60$

$6(v+10)(v+1)$

Kuta Software - Infinite Algebra 1 Name____________________________________

Factoring Trinomials (a > 1)

Date________________ Period____

Factor each completely.

1) $3p^2 - 2p - 5$

2) $2n^2 + 3n - 9$

3) $3n^2 - 8n + 4$

4) $5n^2 + 19n + 12$

5) $2v^2 + 11v + 5$

6) $2n^2 + 5n + 2$

7) $7a^2 + 53a + 28$

8) $9k^2 + 66k + 21$

9) $15n^2 - 27n - 6$

10) $5x^2 - 18x + 9$

11) $4n^2 - 15n - 25$

12) $4x^2 - 35x + 49$

13) $4n^2 - 17n + 4$

14) $6x^2 + 7x - 49$

15) $6x^2 + 37x + 6$

16) $-6a^2 - 25a - 25$

17) $6n^2 + 5n - 6$

18) $16b^2 + 60b - 100$

Kuta Software - Infinite Algebra 1

Name________________________________

Factoring Trinomials (a > 1)

Date________________ Period____

Factor each completely.

1) $3p^2 - 2p - 5$

$(3p - 5)(p + 1)$

2) $2n^2 + 3n - 9$

$(2n - 3)(n + 3)$

3) $3n^2 - 8n + 4$

$(3n - 2)(n - 2)$

4) $5n^2 + 19n + 12$

$(5n + 4)(n + 3)$

5) $2v^2 + 11v + 5$

$(2v + 1)(v + 5)$

6) $2n^2 + 5n + 2$

$(2n + 1)(n + 2)$

7) $7a^2 + 53a + 28$

$(7a + 4)(a + 7)$

8) $9k^2 + 66k + 21$

$3(3k + 1)(k + 7)$

9) $15n^2 - 27n - 6$

$3(5n + 1)(n - 2)$

10) $5x^2 - 18x + 9$

$(5x - 3)(x - 3)$

11) $4n^2 - 15n - 25$

$(n - 5)(4n + 5)$

12) $4x^2 - 35x + 49$

$(x - 7)(4x - 7)$

13) $4n^2 - 17n + 4$

$(n - 4)(4n - 1)$

14) $6x^2 + 7x - 49$

$(3x - 7)(2x + 7)$

15) $6x^2 + 37x + 6$

$(x + 6)(6x + 1)$

16) $-6a^2 - 25a - 25$

$-(2a + 5)(3a + 5)$

17) $6n^2 + 5n - 6$

$(2n + 3)(3n - 2)$

18) $16b^2 + 60b - 100$

$4(b + 5)(4b - 5)$

3.4 Factor Theorem: Factoring Polynomials (n>2)

The factor theorem is when the remainder is zero

Remainder Theorem

If a polynomial f(x) is divided by (x – a), the remainder is f(a).
f(x) = (x – a) Q(x) + f(a)

Factor Theorem

A polynomial f(x) has a factor (x – a) if and only if f(a) = 0.

Remainder Theorem vs Factor Theorem

Remainder Theorem	Factor Theorem
The remainder theorem states that the remainder when p(x) is divided by (x - a) is p(a).	The factor theorem states that (x - a) is a factor of p(x) if and only if f(a) = 0.
It is used to find the remainder.	It is used to decide whether a linear polynomial is a factor of the given polynomial or not.

The remainder and factor theorems together are used to solve/factorize polynomials.

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Determining Potential Integral Zeros

List the potential integral zeros of $f(x)$.

$f(x) = x^3 + 2x^2 - 11x + 20$

Factors of the constant term: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

Potential Zeros: $\pm\{1, 2, 3, 4, 5, 10, 20\}$

Actual Zeros: P(-5) = 0

Use the Factor Theorem: $f(a) = 0$

One Factor of the Polynomial is $(x + 5)$

The graph of the function will have an x-intercept at -5.

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Potential Integral Zeros

List the possible integral zeros of $f(x)$.

$f(x) = x^3 + 9x^2 + 23x + 15$

Factors of the constant term: $\pm 1, \pm 3, \pm 5, \pm 15$

Potential zeros: $\pm\{1, 3, 5, 15\}$

Actual Zeros: -5, -3, -1

Use the Factor Theorem: $f(a) = 0$

Factors of the Polynomial: $(x + 5)\ (x + 3)\ (x + 1)$

The graph of the function will have x-intercepts at -5, -3 and -1.

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When a polynomial cannot be factored using grouping, you can use the factor theorem to find a factor, then use synthetic division until you get to a quadratic which you can finish with quadratic factoring.

Applying the Factor Theorem

Factor $P(x) = x^3 - 9x^2 + 23x - 15$.

Potential Zeros $\{\pm1, \pm3, \pm5, \pm15\}$

Try P(1)
$P(1) = (1)^3 - 9(1)^2 + 23(1) - 15$
$= 0$

Since P(1) = 0, then $(x - 1)$ is a factor of the polynomial.

$$\begin{array}{r|rrrr} -1 & 1 & -9 & 23 & -15 \\ & & -1 & 8 & -15 \\ \hline & 1 & -8 & 15 & \boxed{0} \end{array}$$

$(x - 1)(x^2 - 8x + 15)$
$(x - 1)(x - 3)(x - 5)$

$\boxed{\text{Therefore, } P(x) = (x - 1)(x - 3)(x - 5).}$

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Applying the Factor Theorem

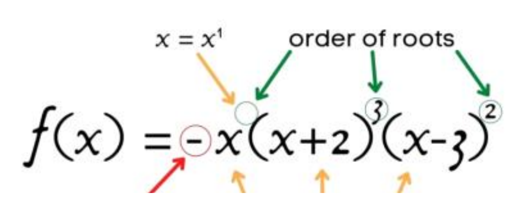
Factor $P(x) = x^4 - 4x^3 + 5x^2 + 2x - 8$.

Potential Zeros $\{\pm1, \pm2, \pm4, \pm8\}$

Try P(-1)
$P(-1) = (-1)^4 - 4(-1)^3 + 5(-1)^2 + 2(-1) - 8$
$= 1 + 4 + 5 - 2 - 8$

Since P(-1) = 0, then $(x + 1)$ is a factor

Polynomial Function: Factored Form



Try P(-1)

P(-1) = $(-1)^4 - 4(-1)^3 + 5(-1)^2 + 2(-1) - 8$
$= 1 + 4 + 5 - 2 - 8$
$= 0$

Since P(-1) = 0, then $(x + 1)$ is a factor.

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & 5 & 2 & -8 \\ & & 1 & -5 & 10 & -8 \\ \hline -2 & 1 & -5 & 10 & -8 & \boxed{0} \\ & & -2 & 6 & -8 & \\ \hline & 1 & -3 & 4 & \boxed{0} & \end{array}$$

$(x + 1)(x^3 - 5x^2 + 10x - 8)$

Try P(2) into original

P(2) = $(2)^4 - 4(2)^3 + 5(2)^2 + 2(2) - 8$
$= 16 - 32 + 20 + 4 - 8$
$= 0$

Since P(2) = 0, then $(x - 2)$ is a factor.

Therefore, $P(x) = (x + 1)(x - 2)(x^2 - 3x + 4)$.

$$f(x) = -x(x+2)^3(x-3)^2$$

sign of leading coefficient

x-intercepts (roots) (-2,0), (0,0), (3,0)

Practice Word Problems

Your Turn

Three consecutive integers have a product of -210.

a) Write a polynomial function to model this situation.

b) What are the three integers?

12. The competition swimming pool at Saanich Commonwealth Place is in the shape of a rectangular prism and has a volume of 2100 m³. The dimensions of the pool are x metres deep by $25x$ metres long by $10x + 1$ metres wide. What are the actual dimensions of the pool?

13. A boardwalk that is x feet wide is built around a rectangular pond. The pond is 30 ft wide and 40 ft long. The combined surface area of the pond and the boardwalk is 2000 ft². What is the width of the boardwalk?

16. Three consecutive odd integers have a product of -105. What are the three integers?

14. Determine the equation with least degree for each polynomial function. Sketch a graph of each.

a) a cubic function with zeros -3 (multiplicity 2) and 2 and y-intercept -18

b) a quintic function with zeros -1 (multiplicity 3) and 2 (multiplicity 2) and y-intercept 4

c) a quartic function with a negative leading coefficient, zeros -2 (multiplicity 2) and 3 (multiplicity 2), and a constant term of -6