

Thursday, May 16th

## Plan For Today:

**VICTORIA DAY ON MONDAY, MAY 20TH  
SCHOOL CLOSED**

1. Any Questions from last class?

▶ **Do Check-in Quiz (3.3-3.4)**

3. Start Chapter 3: Polynomial Functions

- ✓ 3.1: Characteristics of Polynomial Functions
- ✓ 3.2: Equations & Graphs of Polynomials Functions
- ✓ 3.3: The Remainder Theorem
- ✓ 3.4: Factoring Review & The Factor Theorem

✱ **3.5: Applications & Word Problems**

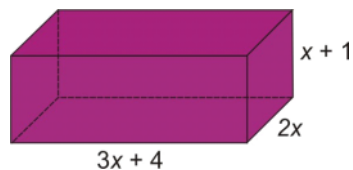
4. Work on practice questions from Workbook


5. Start Chapter 4: Rational Functions

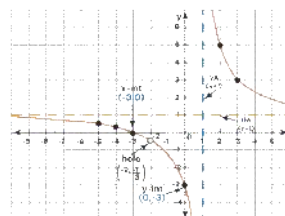
✱ **4.3: Rational Functions**

✱ 4.4: Graphing Rational Functions

6. Work on practice questions from Workbook



Graph of Rational Function  $f(x) = \frac{x^2 + 5x + 6}{x^2 + x - 2}$  



## Plan Going Forward:

1. Work on Ch3 practice from the workbook & Ch3 practice questions handout. Finish working on the Ch3 project.

✱ **CHAPTER 3 PROJECT DUE TUESDAY, MAY 21ST**

✱ **CHAPTER 3 TEST ON TUESDAY, MAY 21ST**

2. Start working on section 4.3 & 4.4 to prepare for next week we will start it after the Ch3 test on Tuesday and finish the majority of it on Wednesday. Next Thursday we will finish any remaining work in Ch4 and review for the U1 Exam but will also start Ch5 with exponential equations and logarithms.

✱ **UNIT 2 EXAM ON CH. 3 (ALL) & 4 (4.3-4.4 ONLY) ON MONDAY, MAY 27TH**

- 12 Multiple Choice & 20 marks on the Written
- ~1.5 hours - please prepare so you are not "learning" while doing the test
- Closed-book - no notes, formula sheet provided
- (rewrite opportunity is on last day of class Wednesday, June 19th)

3. We will only do the U2 test on the 27th and continue Ch5 on Tuesday, May 28th.

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at [anurita.weebly.com](http://anurita.weebly.com) after class. Anurita Dhiman = [adhiman@sd35.bc.ca](mailto:adhiman@sd35.bc.ca)

Thursday, May 16th In-Class Notes

$(x+1)$  b.  $x^4 + 4x^3 - 7x^2 - 34x - 24$   
 $\begin{array}{r|rrrrr} -1 & 1 & 4 & -7 & -34 & -24 \\ + & & & & & \\ \hline x & & & & & \end{array}$   
 $x^3 + 3x^2 - 10x - 24 \quad 0 \checkmark$

$(x+2)$   
 $\begin{array}{r|rrrr} -2 & 1 & 3 & -10 & -24 \\ + & & & & \\ \hline x & & & & \end{array}$   
 $x^2 + x - 12 \quad 0 \checkmark$   
 $(x-3)(x+4)$   
 $\frac{AK}{-12} = 1$   
 $\frac{1}{4}, \frac{-3}{-3} = 1$

$\rightarrow (x+1)(x+2)(x-3)(x+4)$

c.  $x^5 - 10x^4 + 22x^3 + 28x^2 - 55x - 50$

- ① possible factors  
 $-24 \rightarrow \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$
- ② Test factors to give  $R=0$   
 $(-1)^4 + 4(-1)^3 - 7(-1)^2 - 34(-1) - 24 = 0$   
 $\downarrow$   
 $x = -1 \quad x = -2 \checkmark \quad R=0$   
 $= (x+1) \quad (x+2)$
- ③ divide ....

May 16, 2024

Name: KEY

TOTAL = \_\_\_\_\_ / 7 marks

### Check-in Quiz Section 3.1-3.4: Polynomial Characteristics, Dividing & Factoring Polynomials

Complete the following questions **SHOWING ALL WORK** and steps where applicable.

1. Complete the following characteristics for the following function:

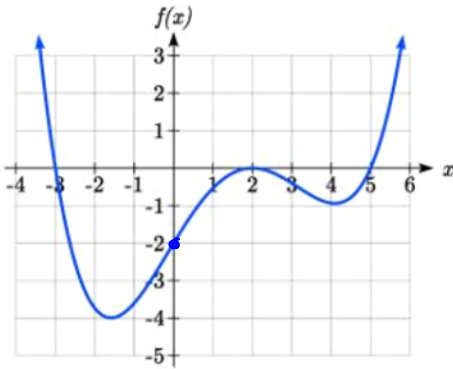
(1 mark)

$$f(x) = -x^4 + 2x^2 - 5x + 7$$

1. Degree = **4 (even)**
2. Leading Coefficient = **-1 (negative)**
3. Behaviour = **down + down = down into QII + QIV**
4. Possible # of turning points = **1 or 3**
5. Possible # of x-intercepts = **0 - 4**
6. Coordinate of y-intercept = **(0, 7)**

2. Answer the questions about the graph of the polynomial function shown:

(0.5 marks each = 2 marks)



a) What are all possible degrees this function can have?

**4, 6, 8, 10...**

b) How many x-intercepts does this function have?

**3**

c) What is the domain **and** range?

**$\{x \mid x \in \mathbb{R}\}$      $\{y \mid y \geq -4, y \in \mathbb{R}\}$**

d) What are the x-intercepts, including their multiplicity?

**$x = -3 (M1), 2 (M2), 5 (M1)$**

e) Using the y-intercept, determine the equation of this function.

**y-int = (0, -2)**

$$y = a(x+3)(x-2)^2(x-5)$$

$$-2 = a(0+3)(0-2)^2(0-5)$$

$$-2 = a(3)(4)(-5)$$

$$\frac{-2}{-60} = \frac{-60a}{-60}$$

$$a = \frac{1}{30}$$

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$$y = \frac{1}{30}(x+3)(x-2)^2(x-5)$$

3. If the remainder is 10 when the polynomial  $P(x) = 3x^3 + kx^2 - 5x + 7$  is divided by the binomial  $x - 2$ , determine  $k$ .

(1 mark)

$$\begin{aligned}
 & \downarrow x=2 \\
 & 3(2)^3 + k(2)^2 - 5(2) + 7 = 10 \\
 & 24 + 4k - 10 + 7 = 10 \\
 & 4k + 21 = 10 \\
 & 4k = \frac{-11}{4} \rightarrow k = -\frac{11}{4}
 \end{aligned}$$

4. Fully factor this polynomial  $P(x) = 3x^3 + 5x^2 - 4x - 4$

(3 marks)

- a. List all possible factors:  $\pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$

- b. Test factors and determine a binomial factor:

$$\begin{aligned}
 P(1) &= 3(1)^3 + 5(1)^2 - 4(1) - 4 \\
 &= 3 + 5 - 4 - 4 \\
 P(1) &= 0 \rightarrow (x - 1)
 \end{aligned}$$

- c. Do synthetic division with the binomial factor:

$$\begin{array}{r|rrrr}
 x-1 & 3x^3 & +5x^2 & -4x & -4 \\
 & 1 & 3 & 5 & -4 & -4 \\
 + & & \downarrow & \nearrow & \nearrow & \nearrow \\
 \hline
 x & & 3 & 8 & 4 & 0 \\
 & & & & & 3x^2 + 8x + 4
 \end{array}$$

- d. Fully factor and collect all factors for fully factored form of polynomial:

$$\begin{aligned}
 AC &= 12 \\
 & \swarrow \searrow \\
 & 2, 6 = 8 \\
 & \downarrow \downarrow \\
 & \frac{2}{3}, \frac{1}{2} \\
 & (3x+2)(x+2) \\
 \text{all factors} & \boxed{(x-1)(3x+2)(x+2)}
 \end{aligned}$$

15. Determine the x-intercept(s) and y-intercept algebraically (fully factor). Sketch the graph the equation below without the graphing calculator. SHOW WORK. (3 marks each)

a.  $y = -x^3 + 5x^2 - 7x + 3$   
 $\pm 1, \pm 3$

behaviour  
 (odd degree  $+ - LC$ )  
 up into  $QI$   
 + down into  $QIV$   
 y-int =  $(0, 3)$

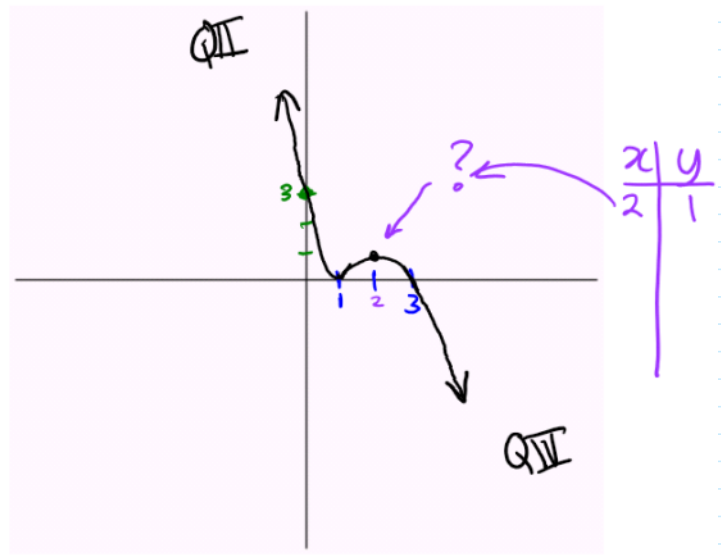
$P(3) = 0$   
 $P(1) = 0 \rightarrow (x-1)$   
 $P(-3) = 96$   
 $P(-1) = 16$

$$\begin{array}{r|rrrr} x-1 & -x^3 & +5x^2 & -7x & +3 \\ & 1 & -1 & 5 & -7 & 3 \\ + & & & -1 & 4 & -3 \\ \hline x & & -1 & 4 & -3 & 0 \\ & & & & & -x^2 + 4x - 3 \\ & & & & & -(x^2 - 4x + 3) \\ & & & & & -(x-3)(x-1) \end{array}$$

$y = -(x-3)(x-1)$

$\rightarrow$  x-int: 3, 1 (M2)

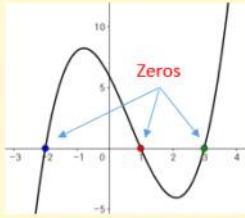
b.  $y = x^3 + 3x^2 - 14x + 40$



### 3.5 Solving Polynomial Equations & Word Problems

#### The Zeros or Roots of a Polynomial Function

**Graphically:** the real zeros or real roots of a polynomial function are the x-intercepts of the graph.

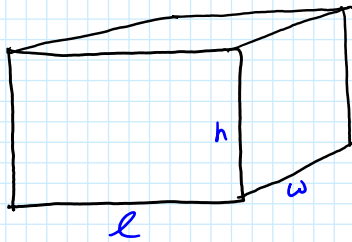


**Rational Zeros Theorem:** If a polynomial function has rational zeros, they will be a ratio of the factors of the constant to the factors of the leading coefficient.

Example:  $f(x) = 2x^3 - 9x^2 + 7x + 6$

$6: \pm 1, \pm 2, \pm 3, \pm 6$   
 $2: \pm 1, \pm 2$

### 3.5: Word Problems

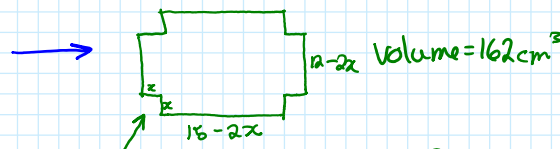
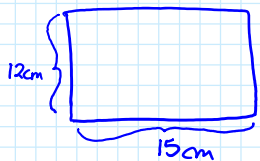


Volume =  $lwh$   
(units cubed)

In general we can determine the volume of a rectangular prism by multiplying the side in general.

$\hookrightarrow (x+a)(x+b)(x+c) = V$   
 $x^3 \dots = \text{Vol.}$

#1 p.148



remove  $x$  by  $x$  corners.

width =  $12 - 2x$   
 length =  $15 - 2x$   
 height =  $x$

$x(12-2x)(15-2x) = 162$

$$x(12-2x)(15-2x) = 162$$

$$x(180 - 24x - 30x + 4x^2) = 162$$

$$x(180 - 54x + 4x^2) = 162$$

$$180x - 54x^2 + 4x^3 = 162$$

$$4x^3 - 54x^2 + 180x - 162 = 0$$

$$\rightarrow 2(2x^3 - 27x^2 + 90x - 81) = 0$$

$P(3) = 2(3)^3 - 27(3)^2 + 90(3) - 81 = 0 \checkmark$

factors:  $\pm 1, \pm 3, \pm 9, \pm 27, \pm 81$   
 all factors = add  $2C = 2$   
 $\pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}, \pm \frac{27}{2}, \pm \frac{81}{2}$

$x-3$	2	-27	90	-81
3		6	-63	81
+	2	-21	27	0
x				

$2x^2 - 21x + 27$

$AC = 54$   
 $-3, -18 = -21$

$2x^2 - 3x - 18x + 27$

$x(2x-3) - 9(2x-3)$

$(x-9)(2x-3)$

$= 2(x-3)(x-9)(2x-3)$

$x=3$

$x=9$  reject b/c  $12-2(9)$   
 $-6 = \text{can't have negative dimensions}$

$x = \frac{3}{2}$  reject b/c  $x \geq 2$  from question

**3 cm must be removed**

new dimensions =  $9\text{cm} \times 6\text{cm} \times 3\text{cm}$

**Practice Word Problems**

**Your Turn**

- Three consecutive integers have a product of  $-210$ .
- Write a polynomial function to model this situation.
  - What are the three integers?



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**12.** The competition swimming pool at Saanich Commonwealth Place is in the shape of a rectangular prism and has a volume of  $2100 \text{ m}^3$ . The dimensions of the pool are  $x$  metres deep by  $25x$  metres long by  $10x + 1$  metres wide. What are the actual dimensions of the pool?

**13.** A boardwalk that is  $x$  feet wide is built around a rectangular pond. The pond is 30 ft wide and 40 ft long. The combined surface area of the pond and the boardwalk is  $2000 \text{ ft}^2$ . What is the width of the boardwalk?

**16.** Three consecutive odd integers have a product of  $-105$ . What are the three integers?

**14.** Determine the equation with least degree for each polynomial function. Sketch a graph of each.

- a)** a cubic function with zeros  $-3$  (multiplicity 2) and  $2$  and  $y$ -intercept  $-18$
- b)** a quintic function with zeros  $-1$  (multiplicity 3) and  $2$  (multiplicity 2) and  $y$ -intercept  $4$
- c)** a quartic function with a negative leading coefficient, zeros  $-2$  (multiplicity 2) and  $3$  (multiplicity 2), and a constant term of  $-6$