Thursday, May 16th

Plan For Today:

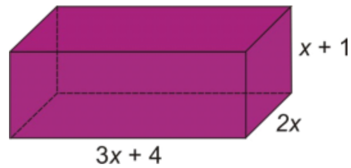
VICTORIA DAY ON MONDAY, MAY 20TH
SCHOOL CLOSED

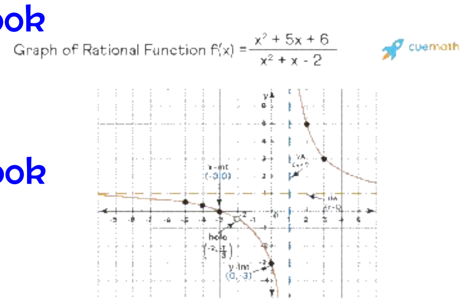
1. Any Questions from last class?
   - ► Do Check-in Quiz (3.3-3.4)

3. Start Chapter 3: Polynomial Functions
   - ✓ 3.1: Characteristics of Polynomial Functions
   - ✓ 3.2: Equations & Graphs of Polynomials Functions
   - ✓ 3.3: The Remainder Theorem
   - ✓ 3.4: Factoring Review & The Factor Theorem
   - ✸ **3.5: Applications & Word Problems**

4. Work on practice questions from Workbook
5. Start Chapter 4: Rational Functions
   - ✸ **4.3: Rational Functions**
   - ✸ 4.4: Graphing Rational Functions

6. Work on practice questions from Workbook





Plan Going Forward:

1. Work on Ch3 practice from the workbook & Ch3 practice questions handout. Finish working on the Ch3 project.

   - ✸ **CHAPTER 3 PROJECT DUE TUESDAY, MAY 21ST**
   - ✸ **CHAPTER 3 TEST ON TUESDAY, MAY 21ST**

2. Start working on section 4.3 & 4.4 to prepare for next week we will start it after the Ch3 test on Tuesday and finish the majority of it on Wednesday. Next Thursday we will finish any remaining work in Ch4 and review for the U1 Exam but will also start Ch5 with exponential equations and logarithms.

   - ✸ **UNIT 2 EXAM ON CH. 3 (ALL) &4 (4.3-4.4 ONLY) ON MONDAY, MAY 27TH**
     - 12 Multiple Choice & 20 marks on the Written
     - ~1.5 hours - please prepare so you are not "learning" while doing the test
     - Closed-book - no notes, formula sheet provided
     - (rewrite opportunity is on last day of class Wednesday, June 19th)

3. We will only do the U2 test on the 27th and continue Ch5 on Tuesday, May 28th.

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class. Anurita Dhiman = adhiman@sd35.bc.ca

4.3: Rational Function Transformations

Graphing the basic rational function with transformations

8-4 Rational Functions

The rational function $f(x) = \frac{1}{x}$ can be transformed by using methods similar to those used to transform other types of functions.

$|a|$ → vertical stretch or compression factor
$a < 0$ → reflection across the x-axis

k → vertical translation
down for $k < 0$; up for $k > 0$

$$f(x) = \frac{a}{x-h} + k$$

h → horizontal translation
left for $h < 0$; right for $h > 0$

Holt Algebra 2

Core Concept

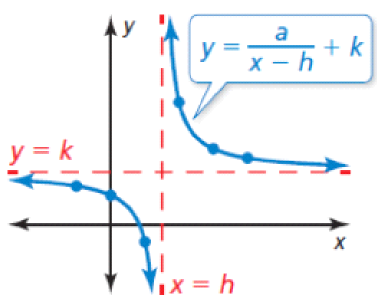
Graphing Translations of Simple Rational Functions

To graph a rational function of the form $y = \frac{a}{x-h} + k$, follow these steps:

Step 1 Draw the asymptotes $x = h$ and $y = k$.

Step 2 Plot points to the left and to the right of the vertical asymptote.

Step 3 Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



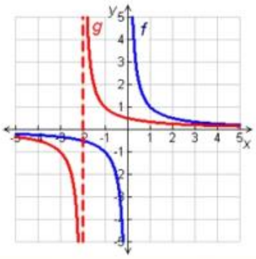
Rational Functions

Example 1: Transforming Rational Functions

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

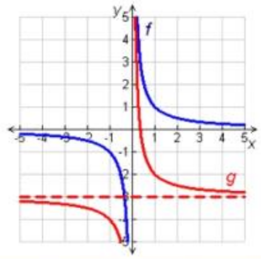
A. $g(x) = \frac{1}{x + 2}$

Because $h = -2$, translate f 2 units left.



B. $g(x) = \frac{1}{x} - 3$

Because $k = -3$, translate f 3 units down.



Rational Functions and Transformations

$$f(x) = \frac{a}{x - h} + k$$

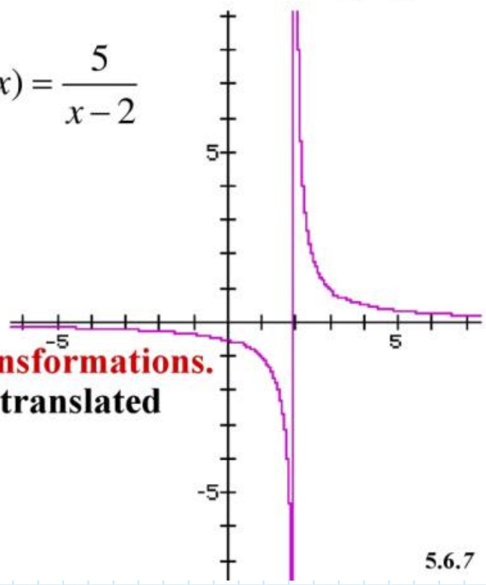
Graph $f(x) = \frac{5}{x-2}$.

Compare the graph of $f(x) = \frac{5}{x-2}$ **with** $f(x) = \frac{1}{x}$.

Vertical stretch by a factor of 5, horizontal translation 2 units right.

Sketch the graph using transformations.

The horizontal asymptote is translated 2 units right as well.



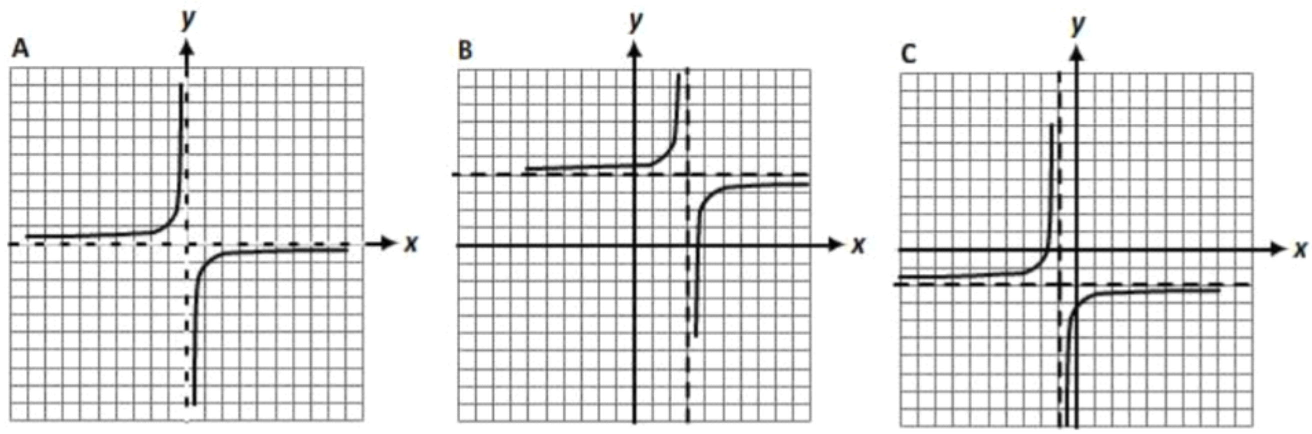
Domain: $x \neq 2, x \in R$.

Range: $y \neq 0, y \in R$

5.6.7

1. Match the rational equation with the correct graph. Explain your rationale for how you matched the equation with the graph.

a) $y = -\dfrac{1}{x}$ b) $y = -\dfrac{1}{x-3} + 4$ c) $y = -\dfrac{1}{x+1} - 2$



4.3: Analysing Rational Function Graphs

Graph Rational Functions with Holes

If the degree of the numerator < degree of the denominator then horizontal asymptote is at y = 0.

The vertical asymptotes will occur where the denominator equals zero.

If there is a common factor in the numerator and denominator then the graph of a rational function will have a hole when a value of x causes both the numerator and the denominator to equal 0. We can set the common factor to zero and solve for x to find the hole.

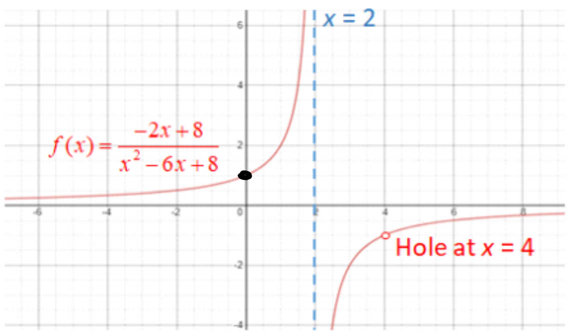
Example:

$$f(x) = \frac{-2x+8}{x^2-6x+8}$$

$$= \frac{-2(x-4)}{(x-4)(x-2)}$$

hole at $x = 4$

vertical asymptote at $x = 2$



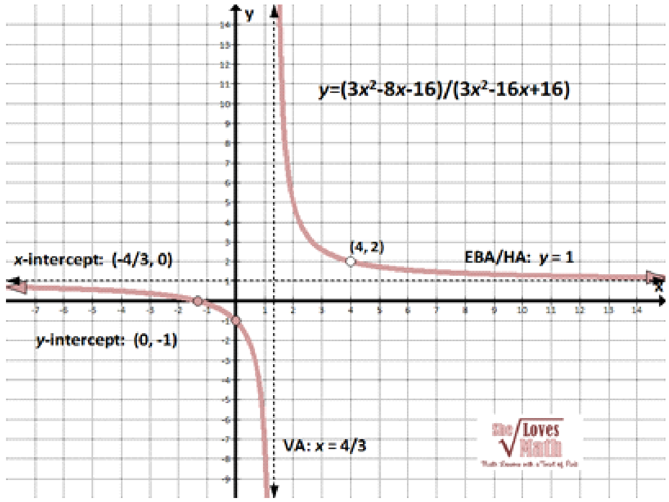
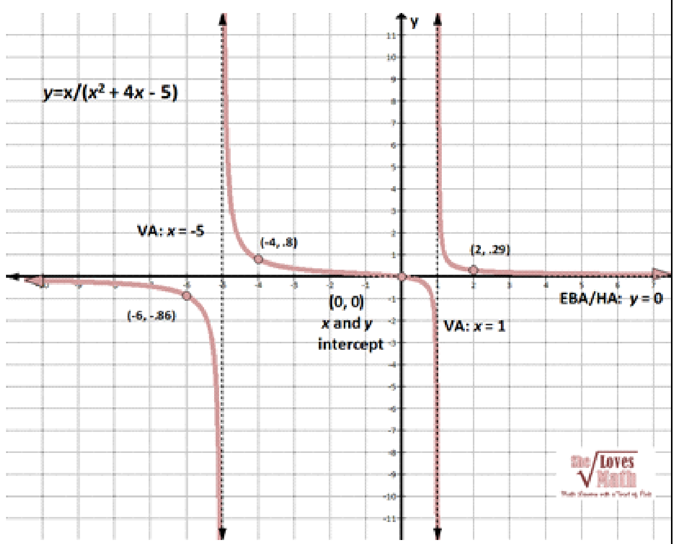
Sketching the Graph of a Rational Function by Hand

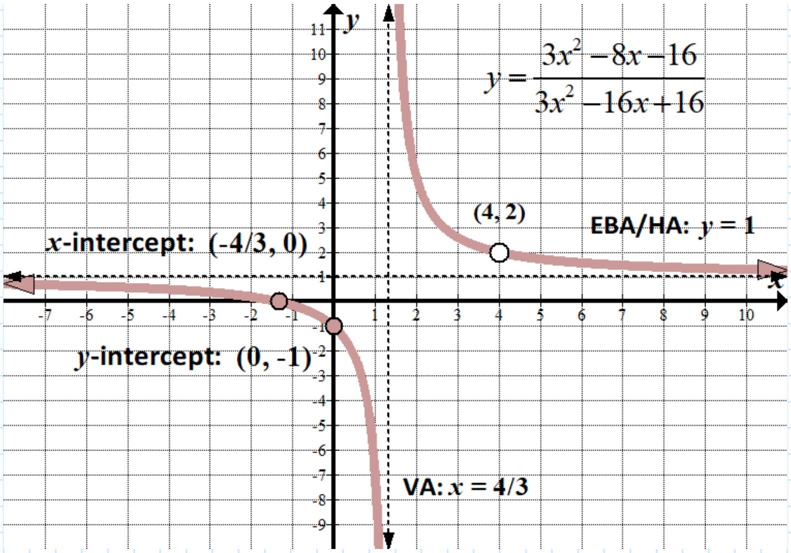
Guidelines for Graphing Rational Functions

1. Write the rational expression in **simplest form**, by factoring the numerator and denominator and dividing out common factors.
2. Find the coordinates of any **"holes"** in the graph.
3. Find and plot the **y-intercept**, if any, by evaluating **f(0).**
4. Find and plot the **x-intercept(s)**, if any, by **finding the zeros of the numerator**.
5. Find the **vertical asymptote(s)**, if any, by **finding the zeros of the denominator**. Sketch these using dashed lines.
6. Find the **horizontal asymptote**, if any, by **comparing the degrees of the numerator and denominator**. Sketch these using dashed lines.
7. Find the **oblique asymptote**, if any, by **dividing the numerator by the denominator using long division**.
8. Plot **5-10 additional points**, including points close to each x-intercept and vertical asymptote.
9. Use **smooth curves** to complete the graph.

Rules for Graphing Rationals	**Examples**
To get the **end behavior asymptote (EBA)**, you want to compare the degree in the numerator to the degree in the denominator. There can be **at most 1 EBA** and most of the time, these are horizontal. ➢ If the degree (largest exponent) on the **bottom is greater** than the degree on the top, the EBA (which is also a **horizontal asymptote** or **HA**) is $\mathbf{y=0}$.	$y=\frac{x+2}{x^2-4}$ Notice that even though we can take out a removable discontinuity $(x+2)$, the bottom still has a higher degree than the top, so the HA/EBA is $\mathbf{y=0}$.
➢ If the degree on the **top is greater** than the degree on the bottom, there is no EBA/HA. **However, if the degree on the top is one more than the degree on the bottom, than there is a slant (oblique) EBA asymptote, which is discussed below.**	$y=\frac{x^3+2}{x-4}$ No HA/EBA. Vertical asymptote is still $\mathbf{x=4}$.
➢ If the degree is the **same on the top and the bottom**, than **divide coefficients of the variables with the highest degree on the top and bottom**; this is the HA/EBA. You can determine this asymptote even without factoring.	$y=\frac{2x^3+2}{3x^3-4}$ Since the degree on the top and bottom are both 3, the HA/EBA is $\mathbf{y=\frac{2}{3}}$.
➢ If the degree on the top is **one more** than the degree on the bottom, then the function has a **slant** or **oblique EBA** in the form $y=mx+b$. We have to use **long division** to find this equation. We can just ignore or "throw away" the remainder and just use the linear equation. Weird, huh?	$y=\frac{2x^2+x+1}{x-4}$ $\begin{array}{r} 2x\ +9 \\ x-4\overline{)2x^2+x+1} \\ \underline{2x^2-8x} \\ 9x+1 \\ \underline{9x-36} \\ \not{37} \end{array}$ EBA: $\mathbf{y=2x+9}$
➢ (more Advanced) Find the point where any horizontal asymptotes **cross the function** by **setting the function to the horizontal asymptote**, and solving for "***x***". You already have the "***y***" (from the HA equation).	**Q:** Where does $y=\frac{-x^2+x}{x^2+x-12}$ **intersect** its EBA? **A:** Note that the EBA is $y=\frac{-1}{1}=-1$. Now set $\frac{-x^2+x}{x^2+x-12}=-1$ and cross multiply: $-x^2+x=-1\left(x^2+x-12\right)$; $\mathbf{x=6}$. So the point where the function intersects the EBA is **(6, −1)**.

Rules for Graphing Rationals	Examples
First factor both the numerator and denominator, and **cross out any factors** in both the numerator and denominator. ➢ If any of these factors contain variables, these are **removable discontinuities**, or "**holes**" and will be little circles on the graphs. The idea is that if you cross out a polynomial, you can't forget that it was in the denominator and can't "legally" be set to 0. (We will see graph later.) The **domain** of a rational function is all real numbers, except those that make the denominator equal zero, as we saw earlier. (Note that if after you cross out factors, you still have that same factor on the bottom, the "hole" will turn into a **vertical asymptote**; follow the rules below).	$y=\frac{x^2-5x+6}{x-3}=\frac{\cancel{(x-3)}(x-2)}{\cancel{(x-3)}}=x-2$ This function reduces to the line $y=x-2$ with a **removable discontinuity** (a little circle on the graph) where $x=2$ and $y=(2)-2=0$ (plug 2 in for ***y*** in original or reduced fraction). So the hole is at (2, 0). Domain is $(-\infty,3)\cup(3,\infty)$, since a 3 would make the denominator = 0. It's like we have to "skip over" the 3 with interval notation.
To get **vertical asymptotes or VAs**: ➢ After determining if there are any holes in the graph, factor (if necessary) what's left in the **denominator** and set the factors to 0. For any value of ***x*** where these factors could be 0, this creates a vertical asymptote at "***x* =** " for these values. Note: There could a multiple number of vertical asymptotes, or no vertical asymptotes. Don't forget to include the factors with "***x***" alone (***x* = 0** is the vertical asymptote).	$y=\frac{x^2-5x+6}{x(x^2-9)}=\frac{\cancel{(x-3)}(x-2)}{x\cancel{(x-3)}(x+3)}=\frac{x-2}{x(x+3)}$ Vertical asymptotes occur when $(x-0)=0$ or $(x+3)=0$, or $x=0$ or $x=-3$. Domain is $(-\infty,-3)\cup(-3,0)\cup(0,3)\cup(3,\infty)$, since anything that could make the denominator 0 (even a hole) can't be included. So we have to "skip over" –3, 0, and 3.

Steps		Graph
$y=\frac{3x^2-8x-16}{3x^2-16x+16}$ Factor: $y=\frac{(3x+4)\cancel{(x-4)}}{(3x-4)\cancel{(x-4)}}$ **Removable Discontinuity or Hole:** ***x*** = 4, plug in 4 for ***x*** to get ***y*** = 2. RD is (4, 2). **VA:** Set denominator to 0 after removing the hole: $3x-4=0 \quad x=\frac{4}{3}$.	**EBA/HA:** Since the degree is the same on the top and bottom (both are 2), we take the coefficients and divide them: $y=\frac{3}{3}=1$. ***x*-intercept (root):** Set ***y*** (or top) to 0: $3x+4=0; \quad x=-\frac{4}{3} \quad \left(-\frac{4}{3},0\right)$ ***y*-Intercept:** Set ***x*** to 0: $y=\frac{3(0)+4}{3(0)-4}=-1 \quad (0,-1)$ **Domain:** Can't be any value of ***x*** that makes the bottom zero: $\left(-\infty,\frac{4}{3}\right)\cup\left(\frac{4}{3},4\right)\cup(4,\infty)$	
$y=\frac{x}{x^2+4x-5}$ Factor: $y=\frac{x}{(x+5)(x-1)}$ **VA:** Set denominator to 0 after factoring; we have 2 of them: $x=-5, \quad x=1$. **EBA/HA:** Since the degree on the top (1) is less than the degree on the bottom (2), the EBA or VA is ***y*** = 0.	***x*-intercept (root):** Set ***y*** (or top) to 0: $x=0. \quad (0,0)$ ***y*-Intercept:** Set ***x*** to 0: $y=\frac{(0)}{(0)^2+4(0)-5}=0: \ (0,0)$ **"T-chart"**: Try some points around the vertical asymptotes: $x=-6, \quad y=-.86$ $x=-4, \quad y=.8$ $x=2, \quad y=.29$ **Domain:** Can't be any value of ***x*** that makes the bottom zero: $(-\infty,-5)\cup(-5,1)\cup(1,\infty)$.	



Rules for Asymptotes

Exponent of highest degree term in denominator larger than highest degree term in numerator then Horizontal Asymptote is

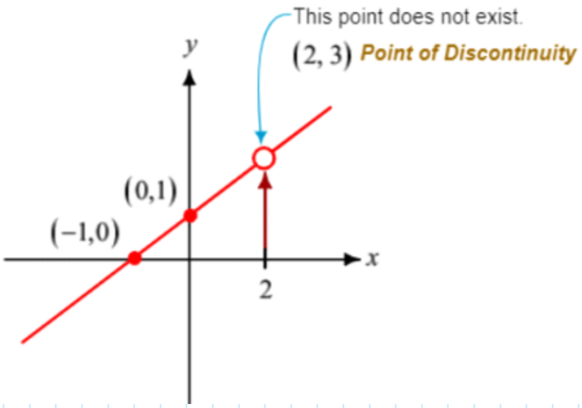
$y = 0$

Exponent values of highest degree of terms in numerator and denominator the same then Horizontal Asymptote is

$y =$ ratio of their coefficients

2.2 Point of Discontinuity

$$f(x) = \frac{x^2 - x - 2}{x - 2}$$

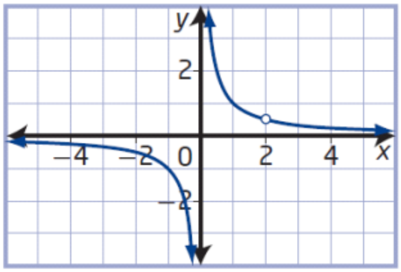


5. Which graph matches each rational function? Explain your choices.

a) $A(x) = \frac{x^2 + 2x}{x^2 + 4}$ **b)** $B(x) = \frac{x - 2}{x^2 - 2x}$

c) $C(x) = \frac{x + 2}{x^2 - 4}$ **d)** $D(x) = \frac{2x}{x^2 + 2x}$

A



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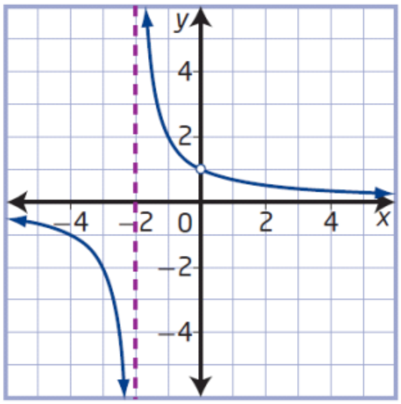

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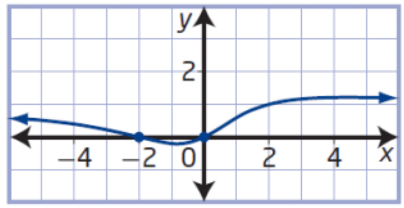

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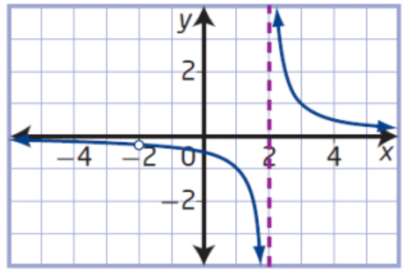

8. Write the equation of a possible rational function with each set of characteristics.

a) vertical asymptotes at $x = \pm 5$ and x-intercepts of -10 and 4

b) a vertical asymptote at $x = -4$, a point of discontinuity at $\left(-\frac{11}{2}, 9\right)$, and an x-intercept of 8

c) a point of discontinuity at $\left(-2, \frac{1}{5}\right)$, a vertical asymptote at $x = 3$, and an x-intercept of -1

d) vertical asymptotes at $x = 3$ and $x = \frac{6}{7}$, and x-intercepts of $-\frac{1}{4}$ and 0