# Chapter 1 Practice Review Questions

## Arithmetic and Geometric Sequences & Series

#1–6

1. Determine the of the common difference and $20^{th}$ term of the following arithmetic sequence:

   -13, -11, -9, -7, -5, …

   $d = -11 - (-13)$
   $= -11 + 13$
   $\boxed{d = 2}$

   $a = -13$, $d = 2$, $n = 20$

   $t_{20} = -13 + (20-1)(2)$
   $= -13 + (19)(2)$
   $= -13 + 38$
   $\boxed{t_{20} = 25}$

2. Write an equation for the nth term of the following arithmetic sequence:

   12, 4, -4, 12, -20 …

   $a = 12$
   $d = 4 - 12$
   $d = -8$

   $t_n = 12 + (n-1)(-8)$
   $= 12 - 8n + 8$
   $\boxed{t_n = 20 - 8n}$

3. Kali deposited \$5 in the first week of a year and then deposited \$ 1.75 per week thereafter. ~~If~~ In the nth week, her weekly savings become \$ 20.75, find n

   $a = \$5$
   $d = \$1.75$
   $t_n = \$20.75$

   $5 + (n-1)(1.75) = 20.75$
   $-5 \qquad\qquad -5$
   $\frac{1.75(n-1)}{1.75} = \frac{15.75}{1.75}$
   $n - 1 = 9$
   $+1 \qquad +1$
   $\boxed{n = 10}$ = $10^{th}$ week

4. Determine the sum of the first 17 terms of the following arithmetic series:

   $-\frac{5}{9} - \frac{11}{18} - \frac{2}{3} + \ldots$

   $n=17$

   $a = -\frac{5}{9}$

   $d = -\frac{11}{18} - \left(-\frac{5}{9}\right)$
   $= -\frac{11}{18} + \frac{10}{18}$
   $d = -\frac{1}{18}$

   $S_{17} = \frac{17}{2}\left[2\left(-\frac{5}{9}\right) + (17-1)\left(-\frac{1}{18}\right)\right]$
   $= \frac{17}{2}\left[-\frac{10}{9} + \cancel{16}^{8}\left(-\frac{1}{\cancel{18}_{9}}\right)\right]$
   $= \frac{17}{2}\left[-\frac{10}{9} - \frac{8}{9}\right]$
   $= \frac{17}{2}\left(-\frac{18}{9}\right)$
   $= \frac{17}{\cancel{2}}\left(-\cancel{2}\right)$
   $\boxed{S_{17} = -17}$

5. The eleventh term of an arithmetic sequence is 30 and the sum of the first eleven terms is 55. What is the common difference?

$t_{11}=30 \rightarrow l=30$

$S_{11}=55 \rightarrow S_{11}=\frac{n}{2}(a+l)$

$55=\frac{11}{2}(a+30)$

$\frac{5\cancel{55}\times 2}{\cancel{11}}=a+30$

$10=a+30$

$-30$

$a=-20$

$t_{11}\rightarrow -20+(11-1)d=30$

$+20$

$10d=50$

$\boxed{d=5}$

6. How many terms of the arithmetic sequence 2, 8, 14, 20, ... are required to give a sum of 660?

$a=2 \quad d=6 \qquad S_n=660$

$\frac{n}{2}[2(2)+(n-1)(6)]=660$

$\frac{n}{2}(4+6n-6)=660$

$n(6n-2)=1320$

$6n^2-2n-1320=0$

$2(3n^2-n-660)=0$

AC $=1980$

$2(3n^2-45n+44n-660)=0$

$-45+44=-1$

$2(3n(n-15)+44(n-15))=0$

$2(3n+44)(n-15)=0$

$n\neq\frac{44}{3}$ reject

$\boxed{n=15}$

7. Determine the common ratio of the geometric sequence 8, 12, 18, 27, . . .

$r=\frac{12}{8}$

$\boxed{r=\frac{3}{2}}$

8. The general term of a geometric sequence is $t_n=8(-3)^{n-1}$. Determine the common ratio.

$\boxed{r=-3}$

or : find $a_1$ & $a_2$ then $r=\frac{a_2}{a_1}$

$a_1=8(-3)^{1-1}$

$=8(1)$

$a_1=8$

$a_2=8(-3)^{2-1}$

$=8(-3)$

$a_2=-24$

$r=\frac{-24}{8}$

$\boxed{r=-3}$

9. Calculate the $12^{th}$ term of the geometric sequence: 5, 15, 45, . . .

$n=12$ $\quad a=5 \quad r=3$

$t_{12}=5(3)^{11}$

$=5(177147)$

$\boxed{t_{12}=885735}$

10. Which term of the geometric sequence 5, 15, 45, . . . is 885 735?

$a=5 \quad r=3 \quad t_n=885735$

we know answer should be $n=12$

$\frac{5(3)^{n-1}}{5}=\frac{885735}{5}$

$3^{n-1}=177147$

common base

$\cancel{3}^{n-1}=\cancel{3}^{11}$

$n-1=11$ $+1$ $\rightarrow \boxed{n=12}$

11. Determine the number of terms in the geometric sequence:

$$\frac{1}{128},\frac{1}{32},\frac{1}{8},\ldots,2048$$

$t_n=2048$

$a=\frac{1}{128}$

$r=\frac{\frac{1}{32}}{\frac{1}{128}}$

$=\frac{1}{32}\times\frac{128}{1}$

$r=4$

$t_n$

$\frac{\frac{1}{128}(4)^{n-1}}{\frac{1}{128}}=2048\times\frac{128}{1}$

$4^{n-1}=262144$

common base

$\cancel{4}^{n-1}=\cancel{4}^{9}$

$n-1=9$ $+1$

$\boxed{n=10}$

12. The second term of a geometric series is -16 and the seventh term is 512. Determine the first term.

$t_2 = -16 \xrightarrow{\times r^5} t_7 = 512$

① $-16r^5 = 512$

$\frac{-16r^5}{-16} = \frac{512}{-16}$

$\sqrt[5]{r^5} = \sqrt[5]{-32}$

$r = -2$

② $t_2 \to a(-2)^{2-1} = -16$

$-2a = -16$

$\frac{-2a}{-2} = \frac{-16}{-2}$

$\boxed{a = 8}$

13. The 3rd term of a geometric sequence is 48 and the 6th term is $\frac{81}{4}$. Find the 1st term of the sequence.

$t_3 = 48 \xrightarrow{\times r^3} t_6 = \frac{81}{4}$

① $48r^3 = \frac{81}{4}$

$\frac{48r^3}{48} = \frac{\frac{81}{4}}{48}$

$r^3 = \frac{\cancel{81}^{27}}{4} \times \frac{1}{\cancel{48}_{16}}$

$\sqrt[3]{r^3} = \sqrt[3]{\frac{27}{64}}$

$r = \frac{3}{4}$

② $t_3 \to a\left(\frac{3}{4}\right)^{3-1} = 48$

$a\left(\frac{3}{4}\right)^2 = 48$

$\frac{9}{16}a = \cancel{48}^{16} \times \frac{16}{\cancel{9}_3}$

$\frac{\frac{9}{16}a}{\frac{9}{16}}$

$\boxed{a = \frac{256}{3}}$

14. A geometric sequence of positive terms has $t_1 = 320$ and $t_7 = 78\,125$. Find $t_4$.

$a = 320$

① $320(r)^6 = 78\,125$

$\frac{320(r)^6}{320} = \frac{78125}{320}$

$\sqrt[6]{r^6} = \sqrt[6]{\frac{15625}{64}}$

$r = \pm\frac{5}{2}$

② $t_4 = 320\left(\pm\frac{5}{2}\right)^3$

$= \cancel{320}^{40}\left(\pm\frac{125}{\cancel{8}_{1}}\right)$

$\boxed{t_4 = \pm 5000}$

15. For a geometric sequence, $t_7 = 5x + 2$ and $t_{10} = x - 23$. If the common ratio, r, is 2, find the value of $t_{10}$.

$\times r^3$

① $(5x+2)r^3 = x - 23$

$(5x+2)(2)^3 = x - 23$

$8(5x+2) = x - 23$

$40x + 16 = x - 23$

$-x \qquad -16$

$39x = -39$

$\frac{39x}{39} = \frac{-39}{39}$

$x = -1$

② $t_{10} = x - 23$

$= -1 - 23$

$\boxed{t_{10} = -24}$

16. If $x, 4, 8x$ are three consecutive terms in a geometric sequence, determine the values of $x$.

$$r = \frac{4}{x} = \frac{8x}{4}$$

$$\frac{4}{x} = \frac{8x}{4}$$

$$\frac{16}{8} = \frac{8x^2}{8}$$

$$\sqrt{2} = \sqrt{x^2}$$

$$\boxed{x = \pm\sqrt{2}}$$

17. If the sum of the first 5 terms of a geometric series is -328 and the common ratio is -4, determine the first term.

$S_5 = -328$ $\quad r = -4$ $\quad a = ?$

$$\frac{a\left(1 - \overbrace{(-4)^5}^{-1024}\right)}{\underbrace{1-(-4)}_{1+4}} = -328$$

$$\frac{a(1025)}{5} = -328$$

$$\frac{205a}{205} = \frac{-328}{205}$$

$$\boxed{a = \frac{8}{5}}$$

18. Determine the $\underbrace{\text{sum of the first 10}}_{S_{10}=?}$ terms of the geometric sequence -4, 6, -9, . . ., to the nearest tenth.

$a=-4 \quad r=-\frac{3}{2}$

$$S_{10}=\frac{-4\left(1-\left(-\frac{3}{2}\right)^{10}\right)}{1-\left(-\frac{3}{2}\right)}$$

$$=\frac{-4\left(1-\frac{59049}{1024}\right)}{\frac{5}{2}}$$

$$=-\cancel{4}\left(\frac{\overset{-11605}{-\cancel{58025}}}{\underset{128}{\cancel{1024}}}\right)\times\frac{\cancel{2}}{\cancel{5}}$$

$$\boxed{S_{10}=\frac{11605}{128}} \approx 90.66$$

19. A doctor prescribes medication to be taken for 7 days. The amount taken on the first day is 310 mg. On each successive day, the amount taken is one half the amount taken on the previous day. What is the total amount of medication taken? (Accurate to the nearest mg.)

$a=310\text{mg}$

$r=\frac{1}{2}$

$n=7$ days

$$S_7=\frac{310\left(1-\left(\frac{1}{2}\right)^7\right)}{1-\frac{1}{2}}$$

$$=\frac{310\left(1-\frac{1}{128}\right)}{\frac{1}{2}}$$

$$=\overset{155}{\cancel{310}}\left(\frac{127}{\underset{\underset{32}{\cancel{64}}}{\cancel{128}}}\right)\times\frac{\cancel{2}}{1}$$

$$\boxed{S_7=\frac{19685}{32}\text{mg}} \approx 615.16$$

$$=\boxed{615\text{ mg}}$$

20. Jim worked for a company for 8 years. His starting annual salary was $32 000. Each year his salary increased by 2% over the previous year's salary. What is the total amount of money Jim earned with this company?

$S_n$

$a = 32000$

$r = 1 + 0.02$

$r = 1.02$

$n = 8$

$$S_8 = \frac{32000(1 - 1.02^8)}{1 - 1.02}$$

$$= \frac{32000(1 - 1.02^8)}{-0.02}$$

$$= -1600000(1 - 1.02^8)$$

$$\boxed{S_8 = \$274655.01 \text{ earned}}$$

21. An aquarium originally containing 30 liters of water loses 6% of its water to evaporation every day. Determine a geometric sequence which shows the number of liters of water in the aquarium on 5 consecutive days.

$a = 30$

$r = 1 - 0.06$

$r = 0.94$

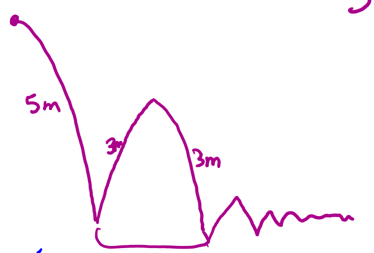
$n = 5$

$1^{st} \xrightarrow{\times 0.94} 2^{nd}$

$$\boxed{30L,\ 28.2L,\ 26.508L,\ 24.91752L,\ 23.4224688L}$$

22. A ball is dropped from a height of 5 m. After each bounce, it rises to 60% of its previous height. What is the total vertical distance the ball travels before it comes to rest?

$\underbrace{\text{total vertical distance}} \rightarrow S_\infty$




$5 \times 0.6 = 3m$

$6m$

$\left. \begin{array}{l} a = 6 \\ r = 0.6 \end{array} \right\}$

initial drop

$$S_\infty = \frac{6}{1-0.6} + 5$$

$$= \frac{6}{0.4} + 5$$

$$= 15 + 5$$

$$\boxed{S_\infty = 20\,m}$$

23. Determine the sum of the infinite geometric series: $3 - 1 + \frac{1}{3} - \frac{1}{9} + \ldots$

$a = 3 \quad r = -\frac{1}{3}$

$$S_\infty = \frac{3}{1-\left(-\frac{1}{3}\right)}$$

$$= \frac{3}{\frac{4}{3}}$$

$$= 3 \times \frac{3}{4}$$

$$\boxed{S_\infty = \frac{9}{4}}$$

24. For what values of x will the following infinite geometric series have a finite sum?

$$(x + 1) + (x + 1)^2 + (x + 1)^3 + \ldots$$

$r = x+1$ → restrictions on r for infinite series is $-1 < r < 1,\ r \neq 0$

$-1 < x+1 < 1,\quad x+1 \neq 0$

$-1 \qquad -1$

$\boxed{-2 < x < 0,\quad x \neq -1}$

25. For what values of x will the following infinite geometric series have a finite sum?

$$(x - 4) + (x - 4)^2 + (x - 4)^3 + \ldots$$

Same method as #24 above

$r = x-4$

$-1 < x-4 < 1,\quad x-4 \neq 0$

$+4 \qquad +4 \qquad +4$

$\boxed{3 < x < 5,\quad x \neq 4}$

26. Determine the sum of geometric series: $\underbrace{2}_{a=2}\underbrace{-1}_{r=-\frac{1}{2}}+\frac{1}{2}-\frac{1}{4}+...$

$$S_\infty = \frac{2}{1-\left(-\frac{1}{2}\right)}$$

$$= \frac{2}{\frac{3}{2}}$$

$$= 2 \times \frac{2}{3}$$

$$\boxed{S_\infty = \frac{4}{3}}$$

27. If the sum of an infinite geometric series is 90 and the common ratio is $-\frac{1}{5}$, determine the value of the first term.

$a = ?$

$S_\infty = 90$

$r = -\frac{1}{5}$

$$\frac{a}{1-\left(-\frac{1}{5}\right)} = 90$$

$$\frac{a}{\frac{6}{5}} = \overset{18}{\cancel{90}}\left(\frac{6}{\cancel{5}}\right)$$

$$\boxed{a = 108}$$

28. Evaluate the following:

$$\sum_{k=2}^{6} 4(3)^k$$

① $a_1 = 4(3)^2$ $(k=2)$
$= 4(9)$
$a_1 = 36$

② $a_2 = 4(3)^3$ $(k=3)$
$= 4(27)$
$a_2 = 108$

③ $r = \frac{108}{36}$
$r = 3$

④ $n = 6 - 2 + 1$
$n = 5$

⑤ $S_5 = \frac{36(1-3^5)}{1-3}$
$= \frac{\cancel{36}^{18}(1-243)}{-\cancel{2}}$
$= -18(-242)$
$\boxed{S_5 = 4356}$

29. Evaluate the following:

$$\sum_{k=1}^{\infty} 50\left(\frac{1}{4}\right)^k$$

① $a_1 = 50\left(\frac{1}{4}\right)^1$
$= \cancel{50}^{25}\left(\frac{1}{\cancel{4}_2}\right)$
$a_1 = \frac{25}{2}$

② $a_2 = 50\left(\frac{1}{4}\right)^2$
$= \cancel{50}^{25}\left(\frac{1}{\cancel{16}_8}\right)$
$= \frac{25}{8}$

③ $r = \frac{\frac{25}{8}}{\frac{25}{2}}$
$= \frac{\cancel{25}}{\cancel{8}_4} \times \frac{\cancel{2}^1}{\cancel{25}}$
$r = \frac{1}{4}$

④ $n = \infty$
$S_\infty = \frac{\frac{25}{2}}{1-\frac{1}{4}}$
$= \frac{\frac{25}{2}}{\frac{3}{4}}$
$= \frac{25}{\cancel{2}} \times \frac{\cancel{4}^2}{3}$
$\boxed{S_\infty = \frac{50}{3}}$

30. Write an expression to represent the sum of the series given by

$$\sum_{k=0}^{15} 16(2)^{k+1}$$

① $a_1 = 16(2)^{0+1}$

$= 16(2)$

$a_1 = 32$

② $r = 2$

③ $n = 15 - 0 + 1$

$n = 16$ terms

$$S_{15} = \frac{32(1-2^{15})}{1-2}$$

$$= \frac{32(1-2^{15})}{-1}$$

$$\boxed{S_{15} = -32(1-2^{15})} \longrightarrow S_{15} = -32(-32766)$$

$$\boxed{S_{15} = 1048512}$$