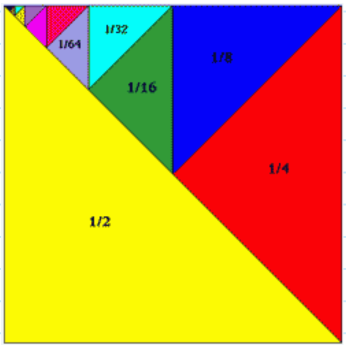
Thursday, Jan. 11th

Plan For Today:

WELCOME BACK TO PRE-CALCULUS 12 Winter 2024

1. Any general questions about course?
2. Any questions about material from last class (1.1-1.2)?
    - Do 1.1-1.2 Arithmetic Sequences & Series Check-in Quiz
3. Go over Arithmetic & Geometric Sequences & Series
    - ✓ 1.1 Arithmetic Sequences
    - ✓ 1.2 Arithmetic Series
    - **1.3 Geometric Sequences**
    - **1.4 Geometric Series**
    - **1.5 Infinite Geometric Series**
    - **Sigma Notation**
4. Work on practice questions from Workbook and work on project.



Plan Going Forward:

1. Finish going through practice question from 1.3-1.4 in workbook and start working on Ch1 Geometric Project.
2. We will finish Geometric Sequences & Series on Tuesday.

- **1.3-1.4 CHECK-IN QUIZ TUESDAY, JAN. 16TH**
- **UNIT 1 PROJECT DUE THURSDAY, JAN. 18TH**
- **UNIT 1 TEST ON THURSDAY, JAN. 18TH**

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.

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1.3 Geometric Sequences.

$a, ar, ar^2, ar^3 \ldots\ldots ar^{n-1}$

* a geometric sequence has a common ratio which multiplies to get terms

(arithmetic the common difference is added/subtracted)

ex: $2, 10, 50, 250, \ldots$ geometric

$2 \xrightarrow{\times 5} 10 \xrightarrow{\times 5} 50 \xrightarrow{\times 5} 250$

$2, 10, 18, 26, \ldots$ arithmetic

$2 \xrightarrow{+8} 10 \xrightarrow{+8} 18 \xrightarrow{+8} 26$

Term formula determines a specific term of the sequence:

$$t_n = ar^{n-1}$$

Ex #4a p.23

$a_{11} = ?$ $a_1 = \frac{1}{128}$ $r = 2$

$n = 11$

$$t_{11} = \frac{1}{128}(2)^{11-1}$$

$$= \frac{1}{128}(2)^{10}$$

$$= \frac{1}{\cancel{128}_{1}}(\cancel{1024}^{8})$$

$$\boxed{t_{11} = 8}$$

Ex 3 p. 21

$t_4 = 125 \qquad t_9 = \frac{125}{32} \qquad t_{13} = ?$

$$\underbrace{t_4 \xrightarrow{\times r} t_5 \xrightarrow{\times r} t_6 \xrightarrow{\times r} t_7 \xrightarrow{\times r} t_8 \xrightarrow{\times r} t_9}_{\times r^5},\ \underbrace{t_{10},\ t_{11},\ t_{12},\ t_{13}}_{r^4}$$

$$\frac{125 \times r^5}{125} = \frac{125}{32} \div 125$$

$\div$ changes to $\times$ by reciprocal

$$r^5 = \frac{\cancel{125}}{32} \times \frac{1}{\cancel{125}}$$

$$\sqrt[5]{r^5} = \sqrt[5]{\frac{1}{32}}$$

$$\sqrt{x^2} = \sqrt{9}$$

$$x = \pm 3$$

$$r = \frac{1}{2}$$

Method 1
solve for a
(in Book)

$$ar^3 = 125$$

$$a\left(\frac{1}{2}\right)^3 = 125$$

$$\cancel{8} \times \frac{a}{\cancel{8}} = 125 \times 8$$

$$a = 1000$$

$$t_{13} = 1000\left(\frac{1}{2}\right)^{13-1}$$

$$= 1000\left(\frac{1}{2}\right)^{12}$$

Method 2.
- use t_9 + $\times r^4$ to get t_{13}

$$t_9 = \frac{125}{32}$$

$$t_{13} = \frac{125}{32} r^4$$

$$= \frac{125}{32}\left(\frac{1}{2}\right)^4$$

$$= \frac{125}{32}\left(\frac{1}{16}\right)$$

$$\boxed{t_{13} = \frac{125}{512}}$$

$$t_{13} = 1000\left(\frac{1}{4096}\right)$$

$$= \frac{1000}{4096} \begin{matrix} \div 8 \\ \div 8 \end{matrix}$$

$$\boxed{t_{13} = \frac{125}{512}.}$$

1.4 Geometric Series

↳ sum of terms

$$a + ar + ar^2 + ar^3 + \ldots\ldots ar^{n-1}$$

- to determine the sum of terms use this formula.

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{or} \quad S_n = \frac{a - rl}{1-r}$$

- to represent the sum of a geometric series, write the series in condensed form with sigma notation

$$\sum_{k=1}^{n} ar^{k-1}$$

Ex: p.28

#1 e) $S_5 = ?$ $\quad t_3 = 3 \quad r = \frac{1}{2}$

① find 'a'

use term formula

$t_n = ar^{n-1}$

$$3 = a\left(\tfrac{1}{2}\right)^{3-1}$$

$$3 = a\left(\tfrac{1}{2}\right)^{2}$$

$$4 \times 3 = a\left(\tfrac{1}{\cancel{4}}\right) \times \cancel{4}$$

$$\frac{3}{\frac{1}{4}}$$

$$a = 12$$

$$3 \times 4$$

$4 \times 3 = a\left(\cancel{4}\right)$

$a = 12$

$\frac{3}{4}$

$3 \times \frac{4}{1}$

$= 12$

② find S_5

$$S_5 = \frac{12\left(1 - \left(\frac{1}{4}\right)^5\right)}{1 - \frac{1}{4}}$$

recall common denominator.

$$= \frac{12\left(1 - \frac{1}{1024}\right)}{\frac{3}{4}}$$

$\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$

$\frac{1024}{1024} - \frac{1}{1024} = \frac{1023}{1024}$

$$= \overset{1}{\cancel{\overset{4}{\cancel{12}}}}\left(\frac{1023}{\underset{\underset{64}{\cancel{256}}}{\cancel{1024}}}\right) \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}}$$

$$\boxed{S_5 = \frac{1023}{64}}$$

$$S_5 = \frac{12\left(1 - \left(\frac{1}{2}\right)^5\right)}{1 - \frac{1}{2}}$$

$\frac{2}{2} - \frac{1}{2} = \frac{1}{2}$

$$= \frac{12\left(1 - \frac{1}{32}\right)}{\frac{1}{2}}$$

$\frac{32}{32} - \frac{1}{32} = \frac{31}{32}$

$$= \frac{12\left(\frac{31}{32}\right)}{\frac{1}{2}}$$

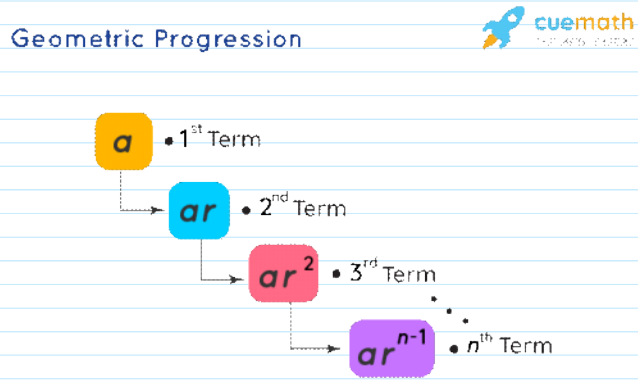
multiply by reciprocal.

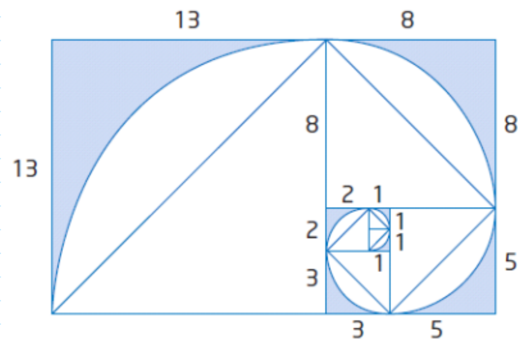
$$= \overset{3}{\cancel{12}}\left(\frac{31}{\underset{\cancel{16}\,4}{\cancel{32}}}\right) \times \frac{\overset{1}{\cancel{2}}}{1}$$

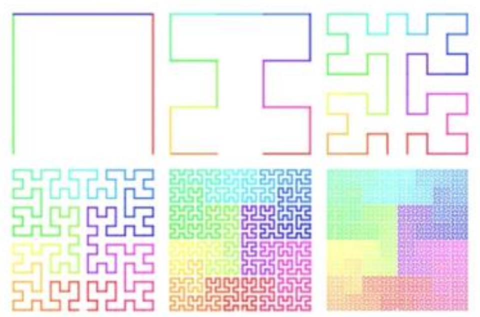
$$\boxed{S_5 = \frac{93}{4}}$$

(23.25)

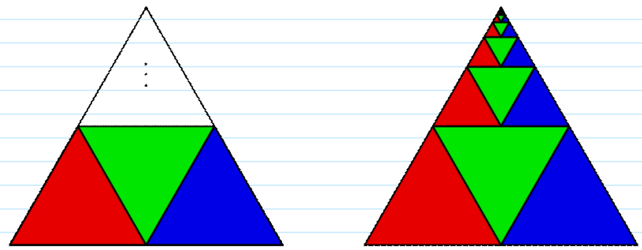
1.3 Geometric Sequences







Fractal are self-similar patterns that repeat at all levels of scale.



Example: Area of the green triangle is reducing by 1/4 for each fractal level.

Geometric Sequence

A geometric sequence has a common ratio.

The formula for the n^{th} term is

$$a_n = ar^{n-1}$$

where a_n = n^{th} term of the sequence

a = first term of the sequence

r = common ratio

Geometric Sequences

- Ratio of consecutive terms is constant.
  - Called the "***common ratio***."
- Examples:
- 1, 3, 9, 27, 81, … ratio= 3
- 64, -32, 16, -8, 4, … ratio = -1/2
- a, ar, ar^2, ar^3, ar^4, … ratio = r

WRITING A RULE FOR A GEOMETRIC SEQUENCE

$$a_n = a_1 r^{n-1}$$

3, 15, 75, 375, 1,875, …

$$a_1 = 3$$

$$r = \frac{15}{3} = 5$$

determine the 9th term

$$a_9 = 3(5)^{(9-1)} \rightarrow a_9 = 3(5)^{(8)}$$

$$a_9 = 1{,}171{,}875$$

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- Consider the geometric sequence:

3, 6, 12, 24, 48, …

This sequence has $t_1 = 3$ and common ratio $r = 2$. Thus:

$$t_1 = 3$$
$$t_2 = 3 \bullet 2$$
$$t_3 = 3 \bullet 2 \bullet 2 = 3 \bullet 2^2$$
$$t_4 = 3 \bullet 2 \bullet 2 \bullet 2 = 3 \bullet 2^3$$
$$t_n = 3 \bullet 2^{n-1}$$

Dempsey's Double

The Ian Dempsey Breakfast Show on Today FM runs a daily competition called *Dempsey's Double.* Every morning one lucky listener will have the chance to try their hand at winning thousands of euro:

"We ask you ten questions starting with €5 for the first correct answer. For every answer you get right after that, we double your cash."

What's the top prize in this competition?

Geometric Series – Ex. 3

Most lottery games in the USA allow winners of the jackpot prize to choose between two forms of the prize: an annual-payments option or a cash-value option.

In the case of the New York Lotto, there are 26 annual payments in the annual-payments option, with the first payment immediately, and the last payment in 25 years time.

The payments increase by 4% each year. The amount advertised as the jackpot prize is the total amount of these 26 payments. The cash-value option pays a smaller amount than this.

1.4 Geometric Series

Geometric Sequence and Geometric Series

A geometric sequence is a sequence of numbers in which the ratio between consecutive terms is constant.

The formula for the n^th term of a geometric sequence is

$$a_n = ar^{n-1}$$

where a is the first term and r is the ratio

A geometric series results from adding the terms of a geometric sequence.

The formula for the sum of a finite geometric series is

$$S_n = \sum_{i=1}^{n} ar^{i-1} = \frac{a\left(1-r^n\right)}{1-r}, r \neq 1$$

where n is the number of terms, a is the first term and r is the ratio

The formula for the sum of an infinite geometric series is

$$S_n = \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}, |r| < 1$$

where a is the first term and r is the ratio

If $|r| \geq 1$, then the infinite series does not have a sum

Geometric Series: ***An indicated sum of terms in a geometric sequence.***

Example:

Geometric **Sequence**	VS	Geometric **Series**
3, 6, 12, 24, 48		**3 + 6 + 12 + 24 + 48**

Example #2: Find the sum of the following series.

$7 + 14 + 28 + \ldots$ for 12 terms

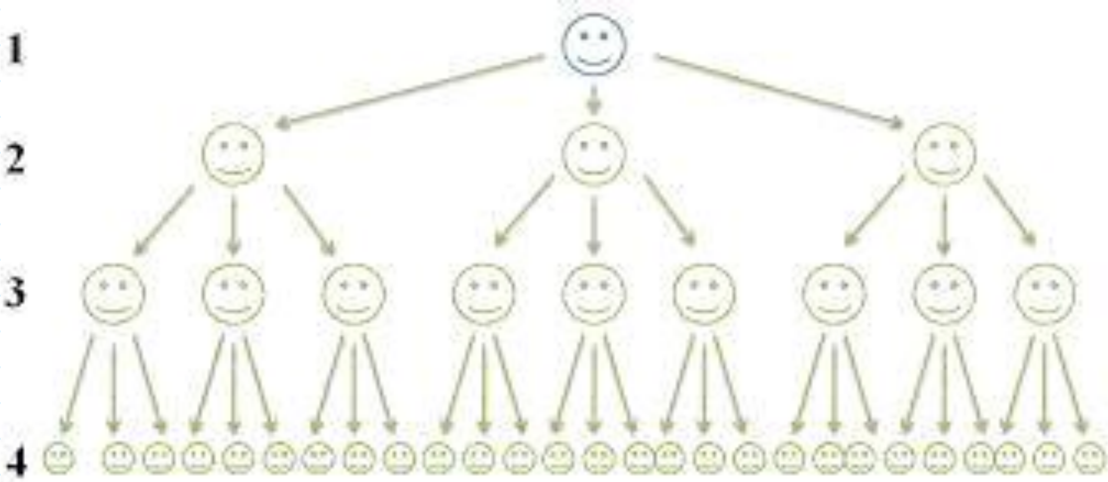
Step #1: Identify the variables.

$\frac{14}{7} = 2$ and $\frac{28}{14} = 2$, so $r = 2$.

$a = 7,\ r = 2,\ n = 12$

Step #2: Substitute and evaluate.

$$S_n = \frac{a(1-r^n)}{(1-r)}$$
$$S_n = \frac{7(1-2^{12})}{1-2}$$
$$S_n = \frac{7(-4095)}{-1}$$
$$S_n = 28665$$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