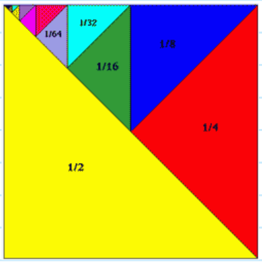
Tuesday, Jan. 16th

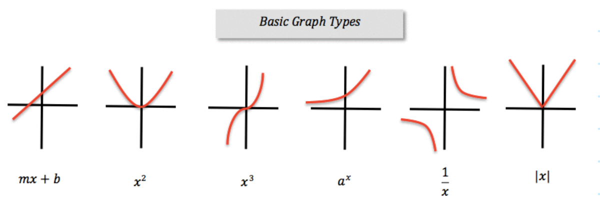
Plan For Today:

WELCOME BACK TO PRE-CALCULUS 12 Winter 2024

1. Any general questions about course?
2. Any questions about material from last class (1.3-1.4)?
   - Do 1.3-1.4 Geometric Sequences & Series Check-in Quiz
3. Go over Arithmetic & Geometric Sequences & Series
   - ✓ 1.1 Arithmetic Sequences
   - ✓ 1.2 Arithmetic Series
   - ✓ 1.3 Geometric Sequences
   - ✓ 1.4 Geometric Series
   - ✓ Sigma Notation
   - **1.5 Infinite Geometric Series**



4. Work on practice questions and project.
5. Review basic graphing for Chapter 2
   - **2.0 Graphing Review**
   - 2.1 Horizontal and Vertical Translations
   - 2.2 Reflections and Stretches
   - 2.3 Combining Transformations
   - 2.4 Inverse of a Relation



6. Work on Graphing Review Handout.

Plan Going Forward:

1. Finish going through practice question in Chapter 1 and finish the Chapter 1 Practice Questions Handout to prepare for the test next class. The KEY for this handout will be posted on my website after class.
2. Finish working on the Ch1 Geometric Sequences and Series Project.

- **CH1 PROJECT DUE THURSDAY, JAN. 18TH**
- **CH1 TEST ON THURSDAY, JAN. 18TH**

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.

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1.5 Infinte Series.

divergent series → when terms of series increase

ex: 5, −25, 125, −625, 3125, ...

☆ $r > 1$ or $r < -1$ → $\boxed{|r| > 1}$

convergent series → when terms of series decrease & get closer to zero

ex: $\frac{2}{3}, -\frac{4}{15}, \frac{8}{75}, -\frac{16}{375}, \ldots$

☆ $-1 < r < 1$ → $|r| < 1$

b/c the terms get smaller, the terms do not <u>add</u> a significant amount to total ∴ you can determine sum without knowing the number of terms.

infinte sum formula $S_\infty = \frac{a}{1-r}$ if $|r| < 1$ or $-1 < r < 1$

∞ ↑ infinity

Examples -

p.35 #2e) $\frac{27}{2} - 9 + 6 - 4 + \ldots$

Step ①: $a = \frac{27}{2}$

$r = \frac{-9}{\frac{27}{2}}$

$= \cancel{-9}^{1} \times \frac{2}{\cancel{27}_{3}}$

$r = -\frac{2}{3}$

Step ② check terms are geometric
- multiply by r

$-9 \times \frac{-2}{3} = 6$ ✓

Step ③ $S_\infty = \frac{a}{1-r}$

$r = -\frac{2}{3}$

Step 3 $S_\infty = \frac{a}{1-r}$

$$S_\infty = \frac{\frac{27}{2}}{1-\left(-\frac{2}{3}\right)}$$

$$= \frac{\frac{27}{2}}{\frac{3}{3}+\frac{2}{3}}$$

$$= \frac{\frac{27}{2}}{\frac{5}{3}}$$

$$= \frac{27}{2} \times \frac{3}{5}$$

$$\boxed{S_\infty = \frac{81}{10}}$$

p.36 #3 $\sum_{i=1}^{\infty} (-1)^i \left(\frac{2}{3}\right)^{i+1}$

$t_1 = a$ | t_2 | t_3

$(-1)^1\left(\frac{2}{3}\right)^{1+1=2}$ | $(-1)^2\left(\frac{2}{3}\right)^{2+1}$ | $(-1)^3\left(\frac{2}{3}\right)^{3+1=4}$

$(-1)\left(\frac{4}{9}\right)$ | $(1)\left(\frac{8}{27}\right)$ | $(-1)\left(\frac{16}{81}\right)$

$a = -\frac{4}{9}$ | $t_2 = \frac{8}{27}$ | $t_3 = -\frac{16}{81}$

$\times -\frac{2}{3}$ ✓

$$r = \frac{\frac{8}{27}}{-\frac{4}{9}}$$

$$= \frac{\cancel{8}^{2}}{\cancel{27}_{3}} \times -\frac{\cancel{9}^{1}}{\cancel{4}_{1}}$$

$r = -\frac{2}{3} \leftarrow |r| < 1$ ✓ $S_\infty = \frac{a}{1-r}$

$$S_\infty = \frac{-\frac{4}{9}}{1-\left(-\frac{2}{3}\right)}$$

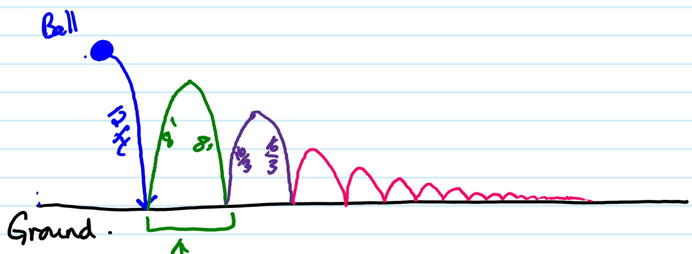
$$= \frac{-\frac{4}{9}}{\frac{3}{3}+\frac{2}{3}}$$

$$\overline{\quad\quad}$$
$$\frac{3}{3}+\frac{2}{3}$$

$$= -\frac{4}{\cancel{9}_3} \times \frac{\cancel{3}^1}{5}$$

$$\boxed{S_\infty = -\frac{4}{15}}$$

Ex: 4 p.34.



first term

$a = 16$ ft

$r = \frac{2}{3}$

rebounds $\frac{2}{3}$

$\cancel{12}^4 \times \frac{2}{\cancel{3}} = 8$

$8 \times \frac{2}{3} = \frac{16}{3}$

(≈ 5.3)

total starting 2nd bounce (up/down)

initial drop.

total vertical distance

$$S_\infty = \frac{16}{1-\frac{2}{3}} + 12$$

$$= \frac{16}{\frac{3}{3}-\frac{2}{3}} + 12$$

$$= 16 \times \frac{3}{1} + 12$$

$$= 48 + 12.$$

$$\boxed{S_\infty = 60 \text{ feet}}$$

1.5 Infinite Geometric Series

Sum to infinity of a geometric series

$$S_\infty = \frac{a_1}{1-r}$$

exists for $-1 < r < 1$

a_1 is the first term

r is the ratio

$r = a_n \div a_{n-1}$

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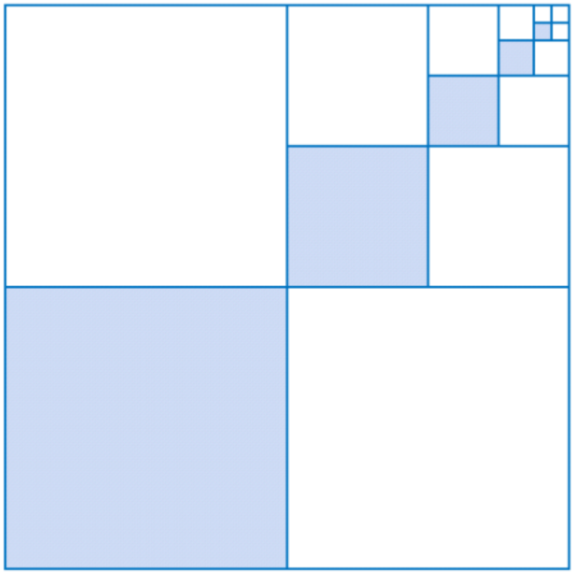
2. Find the value of the infinite geometric series $4+2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$

$$a_1 = 4, \quad r = \frac{1}{2}$$

$$S_\infty = \frac{a_1}{1-r}$$

$$S_\infty = \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8$$

Apply the Sum of an Infinite Geometric Series

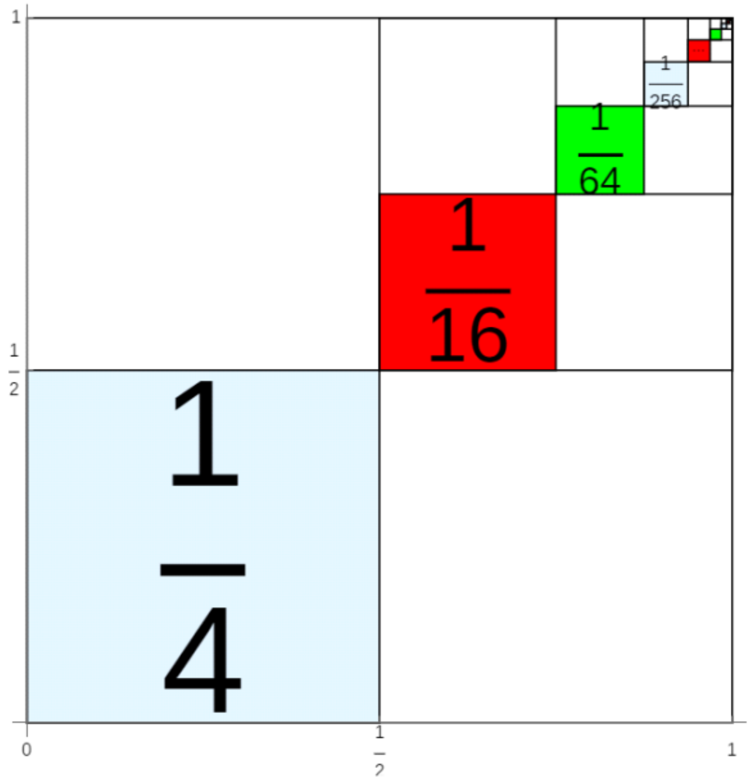


Assume that each shaded square represents $\frac{1}{4}$ of the area of the larger square bordering two of its adjacent sides and that the shading continues indefinitely in the indicated manner.

a) Write the series of terms that would represent this situation.

b) How much of the total area of the largest square is shaded?

Solution



Consider the infinite geometric series $4 - \frac{4}{5} + \frac{4}{25} - \ldots$.

$r = -\frac{1}{5}$

a) Explain why the series has a sum to infinity.

b) Determine the sum to infinity.

The sum to infinity is $\frac{10}{3}$.

Which infinite geometric series has a sum? What is the sum?

a) $4 - 6 + 9 - 13.5 + \ldots$

b) $6 + 2 + \frac{2}{3} + \frac{2}{9} + \ldots$

Solution

a) $4 - 6 + 9 - 13.5 + \ldots$

For this series, $r = -1.5$; since $|r| > 1$, the series has no sum.

b) $6 + 2 + \frac{2}{3} + \frac{2}{9} + \ldots$

For this series, $r = \frac{1}{3}$; since $|r| < 1$, the series has a sum. Use the formula.

$$\begin{aligned} S &= \frac{a}{1-r} \\ &= \frac{6}{1 - \frac{1}{3}} \\ &= \frac{6}{\frac{2}{3}} \\ &= 9 \end{aligned}$$

The sum of the series is 9.

Sigma Notation

Sigma Notation

There is a special notation that is used to represent a series. For example, the geometric series $3 + 6 + 12 + 24 + 48 + 96$ has 6 terms, with first term 3 and common ratio 2. The general term is $t_n = 3(2)^{n-1}$.

Each term in the series can be expressed in this form.

$t_1 = 3(2)^{1-1}$ $\quad t_2 = 3(2)^{2-1}$ $\quad t_3 = 3(2)^{3-1}$

$t_4 = 3(2)^{4-1}$ $\quad t_5 = 3(2)^{5-1}$ $\quad t_6 = 3(2)^{6-1}$

The series is the sum of all these terms, and is represented as shown.

The sum of … $\longrightarrow$ $\displaystyle\sum_{k=1}^{6} 3(2)^{k-1}$ $\longleftarrow$ … all numbers of the form $3(2)^{k-1}$ …

… for integral values of k from 1 to 6.

The symbol Σ is the capital Greek letter sigma, which corresponds to S, the first letter of the word "sum." When Σ is used as shown above, it is called *sigma notation*. In sigma notation, k is frequently used as the variable under the Σ sign and in the expression following it. Any letter can be used, as long as it is not used elsewhere.

Example 1:

Finite geometric sequence: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots, \frac{1}{32768}$

Related finite geometric series: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots + \frac{1}{32768}$

Written in sigma notation: $\sum_{k=1}^{15} \frac{1}{2^k}$

Example 2:

Infinite geometric sequence: $2, 6, 18, 54, \ldots$

Related infinite geometric series: $2 + 6 + 18 + 54 + \ldots$

Written in sigma notation: $\sum_{n=1}^{\infty} \left(2 \cdot 3^{n-1}\right)$

1) Find the sum of the infinite series:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$$

As first term equals a_1, $a_1 = \frac{1}{2}$

Using $a_n = r a_{n-1}$ and first two terms, $r = \frac{1}{2}$

$$\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{k-1} = \frac{a_1}{1-r} = \frac{(1/2)}{1-(1/2)} = 1$$

2) Find the sum of $\sum_{k=1}^{\infty} 4\left(-\frac{1}{3}\right)^{k-1}$

$$a_1 = 4$$

$$r = -\frac{1}{3}$$

$$\therefore$$

$$\sum_{k=1}^{\infty} 4\left(-\frac{1}{3}\right)^{k-1} = \frac{a_1}{1-r} = \frac{4}{1-(-1/3)} = \frac{4}{(4/3)} = \cancel{4} \cdot \frac{3}{\cancel{4}} = 3$$