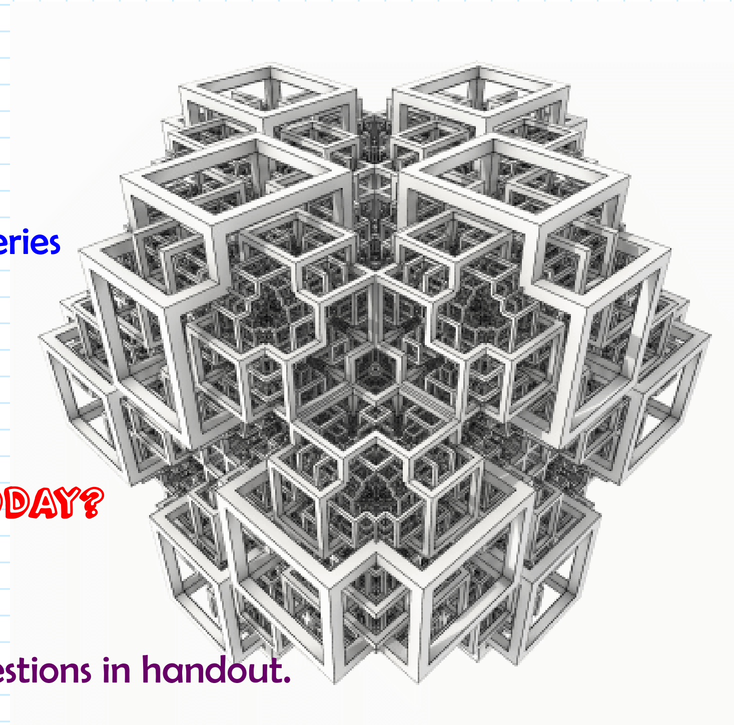
Tuesday, Dec. 12th

Plan For Today:



1. Any questions from 10.2 Sum Formulae?
   - Do 10.3-10.4 Check-in Quiz
2. Finish Topic 10: Geometric Sequences & Series
   - ✓ 10.1 Geometric Sequences
   - ✓ 10.2 Geometric Series
   - ✓ 10.3 Infinite Geometric Series
   - **Word Problems**
   - **10.4 Sigma Notation**
3. Work on Practice Questions
   - **HAND-IN TOPIC10 PROJECT TODAY?**

Plan Going Forward:

1. Finish going through Topic 10 practice questions in handout.

- **UNIT 4 EXAM ON CH9 & T10 ON THURSDAY, DEC. 14TH**
  - 15 MC & 20 Marks on Written
  - Rewrite next Tuesday during class (last class) on Tuesday, Dec. 19th
  - I'll email you when I post marks by Saturday

2. We will not be doing anything new on Thursday after the test so you may leave when finished OR stay to continue reviewing for the rewrite or finishing any missing projects.

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.
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Tuesday, Dec. 12th In-Class Notes

WORD PROBLEMS AND OTHER QUESTIONS

In 1990 the population of Canada was approximately 26.6 million. The population projection for 2025 is approximately 38.4 million. If this projection were based on a geometric sequence, what would be the annual growth rate? Given that this is a geometric sequence what assumptions would you have to make?

$a = \underset{1990}{26.6}$ $\quad 2025 - 38.4$

$n = 2025 - 1990 + 1$

$n = 36$

$t_1 = 26.6 \quad t_{36} = 38.4$

$$\frac{38.4}{26.6} = \frac{26.6(r)^{36-1}}{26.6}$$

$$\sqrt[35]{\frac{38.4}{26.6}} = \sqrt[35]{r^{35}}$$

$$r = \sqrt[35]{\frac{38.4}{26.6}}$$

$r = 1.01$ ratio $\rightarrow$ rate : $1.01 - 1$

$\boxed{\text{rate : } 0.01 \quad 1\%}$

$\sqrt{x^2} = \sqrt{9}$

$\sqrt[4]{x^4} = \sqrt[4]{16}$

If $x + 2$, $2x + 1$, and $4x - 3$ are three consecutive terms of a geometric sequence, determine the value of the common ratio and the three given terms.

1st, 2nd, 3rd

$$r \Rightarrow \frac{2x+1}{x+2} = \frac{4x-3}{2x+1}$$

LCD $(x+2)(2x+1)$

$(2x+1)(2x+1) = (4x-3)(x+2)$

FOIL FOIL

$\cancel{4x^2} + 4x + 1 = \cancel{4x^2} + 5x - 6$

$4x + 1 = 5x - 6$

$-x = -7$

$x = 7$

① Factor ② quad formula

1st : $7 + 2 = \boxed{9}$

2nd : $2(7) + 1 = \boxed{15}$

3rd : $4(7) - 3 = \boxed{25}$

$r = \frac{15}{9} \rightarrow \boxed{r = \frac{5}{3}}$

$\frac{25}{15} = \frac{5}{3}$ ✓

The Russian nesting doll or Matryoshka had its beginnings in 1890. The dolls are made so that the smallest doll fits inside a larger one, which fits inside a larger one, and so on, until all the dolls are hidden inside the largest doll. In a set of 50 dolls, the tallest doll is 60 cm and the smallest is 1 cm. If the decrease in doll size approximates a geometric sequence, determine the common ratio. Express your answer to three decimal places.

$n=50$

$t_1 \Rightarrow a=60$

$t_{50}=1$

$\frac{1}{60} = \frac{60(r)^{50-1}}{60}$

$\sqrt[49]{\frac{1}{60}} = \sqrt[49]{r^{49}}$

$r = \sqrt[49]{\frac{1}{60}}$

$\boxed{r = 0.920}$

Bread and bread products have been part of our diet for centuries. To help bread rise, yeast is added to the dough. Yeast is a living unicellular micro-organism about one hundredth of a millimetre in size. Yeast multiplies by a biochemical process called budding. After mitosis and cell division, one cell results in two cells with exactly the same characteristics.

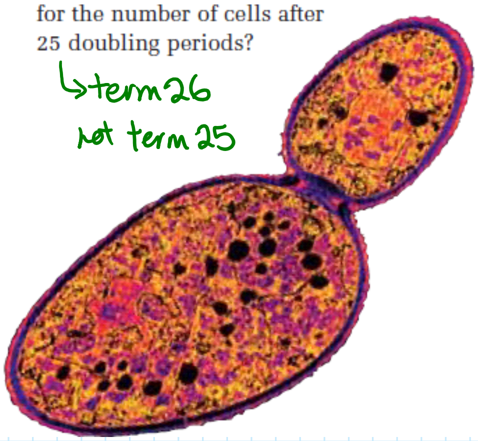
a) Write a sequence for the first six terms that describes the cell growth of yeast, beginning with a single cell.

b) Write the general term for the growth of yeast.

c) How many cells would there be after 25 doublings?

d) What assumptions would you make for the number of cells after 25 doubling periods?

↳ term 26
not term 25



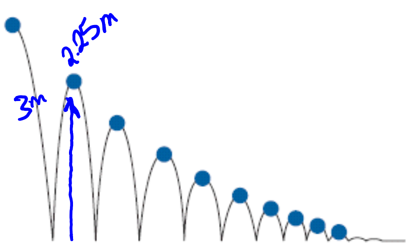
a) 1, 2, 4, 8, 16, 32 ... 25^{th} 26^{th}

b) $t_n = 1(2)^{n-1}$

c) $n = 26$ $\quad t_n = 1(2)^{26-1}$

$\boxed{t_n = 33{,}554{,}432 \text{ yeast}}$

A ball is dropped from a height of 3.0 m. After each bounce it rises to 75% of its previous height.



a) Write the first term and the common ratio of the geometric sequence.

b) Write the general term of the sequence that relates the height of the bounce to the number of bounces.

c) What height does the ball reach after the 6th bounce?

d) After how many bounces will the ball reach a height of approximately 40 cm?

0.4m

a) $a = 2.25\text{ m}$

b) $t_n = 2.25(0.75)^{n-1}$

c) $t_6 = 2.25(0.75)^{6-1}$

$= 2.25(0.75)^5$

$\boxed{t_6 = 0.53\text{ m}}$

d) $\frac{0.4\text{m}}{2.25} = \frac{2.25(0.75)^{n-1}}{2.25}$

$\log \frac{0.4}{2.25} = \log (0.75)^{n-1}$

$\frac{\log\left(\frac{0.4}{2.25}\right)}{\log 0.75} = \frac{(n-1)\log 0.75}{\log 0.75}$

$\frac{\log\left(\frac{0.4}{2.25}\right)}{\log 0.75} = n - 1$

+1

$n = \frac{\log\left(\frac{0.4}{2.25}\right)}{\log 0.75} + 1$

$n = 7.00$

by the 7th bounce.

0.4m

$\boxed{t_6 = 0.53\,m}$

$n = 7.00$

by the 7th bounce.

The common ratio of a geometric series is $\frac{1}{3}$ and the sum of the first 5 terms is 121.

a) What is the value of the first term? a?

b) Write the first 5 terms of the series.

$r = \frac{1}{3}$

$S_5 = 121$

$n = 5$

$$121 = \frac{a\left(1-\left(\frac{1}{3}\right)^5\right)}{1-\frac{1}{3}}$$

$$\frac{2}{3} \times 121 = \frac{a\left(1-\frac{1}{243}\right)}{\frac{2}{3}}$$

$$\frac{\frac{242}{3}}{\frac{242}{243}} = \frac{a\left(\frac{242}{243}\right)}{\frac{242}{243}}$$

$$\frac{\cancel{242}}{\cancel{3}} \times \frac{\cancel{243}^{81}}{\cancel{242}}$$

$81 = a \rightarrow \boxed{a = 81}$

What is the second term of a geometric series in which the third term is $\frac{9}{4}$ and the sixth term is $-\frac{16}{81}$? Determine the sum of the first 6 terms. Express your answer to the nearest tenth.

$t_2 = ?$

$S_6 = ?$

1

$t_3 = \frac{9}{4}$

$t_6 = -\frac{16}{81}$

$$\underbrace{t_3 \xrightarrow{r} t_4 \xrightarrow{r} t_5 \xrightarrow{r} t_6}_{r^3}$$

2

Ratio: $\frac{t_6}{t_3} = r^3$ or $t_3 r^3 = t_6$

$$\frac{-\frac{16}{81}}{\frac{9}{4}} = r^3$$

$$-\frac{16}{81} \times \frac{4}{9} = r^3$$

$$\sqrt[3]{\frac{-64}{729}} = \sqrt[3]{r^3}$$

$r = -\frac{4}{9}$

3

$t_3 = \frac{9}{4}$

$ar^{n-1} = \frac{9}{4}$

$a\left(-\frac{4}{9}\right)^{3-1} = \frac{9}{4}$

$$\frac{a\left(\frac{16}{81}\right)}{\frac{16}{81}} = \frac{\frac{9}{4}}{\frac{16}{81}}$$

$a = \frac{729}{64}$

$r = -\frac{4}{9}$ $\quad a = \frac{729}{64}$

$t_2 = \frac{729}{64}\left(-\frac{4}{9}\right)^{2-1}$

$= \frac{\cancel{729}^{81}}{\cancel{64}_{16}}\left(-\frac{\cancel{4}}{\cancel{9}}\right)$

$t_2 = -\frac{81}{16}$

4

$$S_6 = \frac{\frac{729}{64}\left(1-\left(-\frac{4}{9}\right)^6\right)}{1-\frac{4}{9}}$$

$$= \frac{729}{64}\left(1 - \frac{4096}{531441}\right)$$

$$= \frac{\frac{729}{64}\left(1 - \frac{4096}{531441}\right)}{\frac{5}{9}}$$

$$= \frac{\cancel{729}^{\,1}}{64}\left(\frac{\cancel{527345}^{\,105469}}{\cancel{531441}_{\,\cancel{59049}\,81}}\right) \times \frac{\cancel{9}^{\,1}}{\cancel{5}_{\,1}}$$

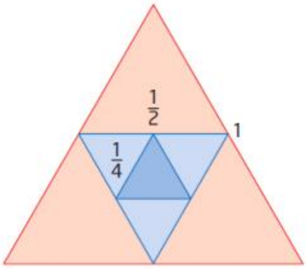
$$\boxed{S_6 = \frac{105469}{5184}} \rightarrow \boxed{S_6 = 20.3}$$

$$\frac{a\left(\frac{16}{81}\right)}{\frac{16}{81}} = \frac{4}{\frac{16}{81}}$$

$$a = \frac{9}{4} \times \frac{81}{16}$$

$$a = \frac{729}{64}$$

Each side of an equilateral triangle has length of 1 cm. The midpoints of the sides are joined to form an inscribed equilateral triangle. Then, the midpoints of the sides of that triangle are joined to form another triangle. If this process continues forever, what is the sum of the perimeters of the triangles?



Perimeter $\rightarrow a = 3$

$\rightarrow t_2 = \frac{3}{2}$

$r = \frac{1}{2}$

$$S_\infty = \frac{3}{1-\frac{1}{2}}$$

$$= \frac{3}{\frac{1}{2}}$$

$$= 3 \times \frac{2}{1}$$

$$\boxed{S_\infty = 6}$$

A hot air balloon rises 25 m in its first minute of flight. Suppose that in each succeeding minute the balloon rises only 80% as high as in the previous minute. What would be the balloon's maximum altitude?

Hot air balloon rising over Calgary.

$a = 25$

$r = 0.8$

$$S_\infty = \frac{25}{1-0.8}$$

$$= \frac{25}{0.2}$$

$$\boxed{S_\infty = 125\text{ m}}$$

2. The sum of the first two terms of a geometric sequence is 15, and the sum of the second and third terms is 60. Determine the first three terms of the sequence algebraically.

$S_2 = 15$

$t_1 + t_2 = 15$

$S_{2^{nd}+3^{rd}} \Rightarrow t_2 + t_3 = 60$

$t_n = ar^{n-1}$

1

$$\text{Ratio} = \frac{t_2 + t_3}{t_1 + t_2} = \frac{60}{15}$$

$$\frac{ar + ar^2}{a + ar} = \frac{60}{15}$$

common factor a

$$\frac{\cancel{a}(r + r^2)}{\cancel{a}(1 + r)} = \frac{60}{15}$$

$$\frac{r + r^2}{1 + r} = \frac{60}{15}$$

$15r + 15r^2 = 60 + 60r$

$-60r \quad -60$

$15r^2 + 9r - 60 = 0$

$15r^2 - 45r - 60 = 0$

2

$15(r^2 - 3r - 4) = 0$

$15(r+1)(r-4) = 0$

$r \neq -1 \quad r = 4$

3

$a + ar = 15$

$a + a(-1) = 15$

$\cancel{a - a} = 15 \quad r \neq \pm 1$

$a + a(4) = 15$

$5a = 15$

$\boxed{a = 3}$

4

$1^{st} = 3$

$2^{nd} = 12$

$3^{rd} = 48$

3. The third term of a geometric series is 24 and the fourth term is 36. Determine the sum of the first 10 terms.

$t_3 = 24 \qquad t_4 = 36$

① $r = \frac{36}{24}$

$r = \frac{3}{2}$

② $a\left(\frac{3}{2}\right)^{3-1} = 24$

$a\left(\frac{3}{2}\right)^2 = 24$

$a\left(\frac{9}{4}\right) = 24$

$a = \cancel{24}^{8} \times \frac{4}{\cancel{9}_{3}}$

$a = \frac{32}{3}$

③ $S_{10} = \frac{\frac{32}{3}\left(1 - \left(\frac{3}{2}\right)^{10}\right)}{1 - \frac{3}{2}}$

$= \frac{\frac{32}{3}\left(1 - \frac{59049}{1024}\right)}{-\frac{1}{2}}$

$\frac{\cancel{32}^{1}}{3}\left(\frac{60073}{\cancel{1024}_{32\,16}}\right) \times \left(-\frac{\cancel{2}^{1}}{1}\right)$

$\boxed{S_{10} = -\frac{60073}{96}}$

4. 4. A virus goes through a computer, infecting files. If one file was infected initially and the total number of files infected doubles every minute, how many total files will be infected in 20 minutes?

$a = 1$

$r = 2$

$S_{20} = \frac{1(1 - 2^{20})}{1 - 2}$

$S_{20} = \frac{1 - 1048576}{-1}$

$\boxed{S_{20} = 1048575 \text{ files}}$

5. A new shopping mall is gaining in popularity. Every day since it opened, the number of shoppers is 20% more than the number of shoppers the day before. The total number of shoppers over the first 4 days is 671. How many shoppers were at the mall on the first day? Round your final answer to the nearest integer

$r = 1 + 0.2$

$r = 1.2$

$n = 4$

$a = ?$

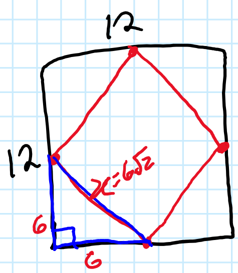
$S_4 = 671 \rightarrow 671 = \frac{a(1 - 1.2^4)}{1 - 1.2}$

$671 = \frac{a(1 - 2.0736)}{-0.2}$

$\frac{671}{5.368} = \frac{5.368a}{5.368}$

$\boxed{a = 125 \text{ shoppers}}$

6. A side of a square is 12 cm. The midpoints of its sides are joined to form an inscribed square, and this process is continued. Find the sum of the perimeters of the squares if this process is continued without end (round answer to two decimal places).



1^{st} Perimeter : $a = 48\text{cm}$

2^{n} Perimeter : $t_2 = 4(6\sqrt{2})$

$t_2 = 24\sqrt{2}$

$x^2 = 6^2 + 6^2$

$x^2 = 36 + 36$

$\sqrt{x^2} = \sqrt{72}$

$x = 6\sqrt{2}$

$r = \frac{24\sqrt{2}}{48}$

$r = \frac{\sqrt{2}}{2}$

$S_\infty = \frac{48}{1 - \frac{\sqrt{2}}{2}}$

$S_\infty = 48 \times \frac{2}{2 - \sqrt{2}}$

$x = 6\sqrt{2}$

$S_\infty = \frac{}{1 - \frac{\sqrt{2}}{2}}$

$= \frac{48}{\frac{2}{2} - \frac{\sqrt{2}}{2}}$

$= \frac{48}{\frac{2-\sqrt{2}}{2}}$

$S_\infty \qquad 2-\sqrt{2}$

$= \frac{96}{2-\sqrt{2}}$

$\boxed{S_\infty = 163.88\text{ cm}}$

2. Express each of these sums using sigma notation.

a. $1^2 + 2^2 + 3^2 + 4^2 + 5^2$

$\boxed{\sum_{k=1}^{5} k^2}$

b. . $13 + 20 + 27 \ldots$

$+7 \quad +7$

$\boxed{\sum_{k=1}^{\infty} 13 + 7(k-1)}$

c. $3 + 6 + 9 + 12 + 15$

$+3 \quad +3$

$\sum_{k=1}^{5} 3 + 3(k-1)$

d. $x^3 + x^5 + x^7 + x^9 + \ldots + x^{199}$

$\cdot x^2 \quad \cdot x^2$

$a = x^3$

$r = x^2$

② $\boxed{\sum_{k=1}^{99} x^3 (x^2)^{n-1}}$

or $\sum_{k=1}^{99} x^3 x^{2n-2}$

① $\frac{x^3 (x^2)^{n-1}}{x^3} = \frac{x^{199}}{x^3}$

$x^{2n-2} = x^{196}$

$2n - 2 = 196$

$+2$

$\frac{2n}{2} = \frac{198}{2}$

$n = 99$

3. Find the sum of these series.

a. $\sum_{n=1}^{25} 5n-2$

$\overset{1}{5(1)-2} + \overset{2}{5(2)-2} + \overset{3}{5(3)-2}$

$5-2 + 10-2 + 15-2$

$3 \xrightarrow{+5} 8 \xrightarrow{+5} 13 \xrightarrow{+5} + \ldots$

Arithmetic Series → diff formula not on test

b. $\sum_{k=1}^{\infty} 3\left(\frac{2}{5}\right)^k$

$a = 3\left(\frac{2}{5}\right)^1 \quad r = \frac{2}{5}$

$a = \frac{6}{5}$

$S_\infty = \frac{\frac{6}{5}}{1-\frac{2}{5}}$

$= \frac{\frac{6}{5}}{\frac{3}{5}}$

$\rightarrow \frac{\cancel{6}^{2}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{3}}$

$\boxed{S_\infty = 2}$

c. $\sum_{k=7}^{32} 2k+3$

Arithmetic

- not on test

d. $\sum_{k=1}^{10} 2^{k-1}$

1. $a = 2^{1-1}$
   $a = 1$
2. $t_2 = 2^{2-1}$
   $t_2 = 2$
3. $r = 2$
4. $S_{10} = \frac{1(1-2^{10})}{1-2}$
   $= \frac{1-1024}{-1}$
   $= \frac{-1023}{-1}$
   $\boxed{S_\infty = 1023}$