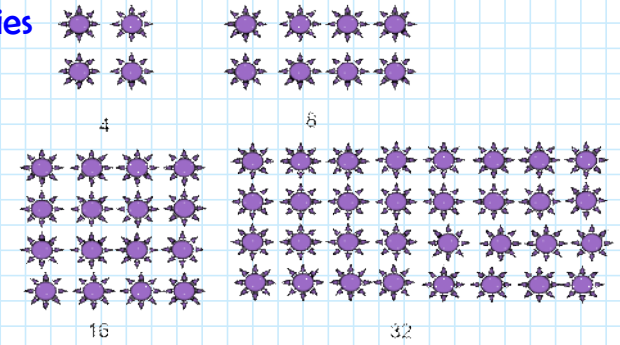
Thursday, Nov. 30th

Plan For Today:

1. Any questions from 9.3?
2. Finish 9.3 practice and word problems.
   - ✸ Do 9.3 Check-in Quiz
   - ✸ Finish working on project and practice questions
3. Start Topic 10: Geometric Sequences & Series
   - **10.1 Geometric Sequences**
   - 10.2 Geometric Series
   - 10.3 Infinite Geometric Series
   - 10.4 Sigma Notation
4. Work on Practice Questions



Plan Going Forward:

- CH9 PROJECT DUE TUESDAY, DEC. 5TH
- CH9 TEST ON TUESDAY, DEC. 5TH

1. Finish going through 10.1 practice questions handout #1-8 and extra review pages.

✸ 10.1 CHECK-IN QUIZ ON TUESDAY, DEC. 5TH

- ❖ TOPIC10 PROJECT DUE TUESDAY, DEC. 12TH
- ❖ UNIT 4 EXAM ON THURSDAY, DEC. 14TH
  - REWRITE UNIT 4 EXAM ON TUESDAY, DEC. 19TH (LAST DAY)

2. We will continue Topic 10 (Geometric Sequences & Series) next Tuesday.

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class. Anurita Dhiman = adhiman@sd35.bc.ca

2023

Topic 10: Geometric Sequences and Series

Geometric Sequences

Learning Outcomes:

- common ratio, first term, general term
- geometric sequences connecting to exponential functions
- infinite geometric series
- sigma notation

A geometric sequence is referring to a sequence of terms where by every term is multiplied by a given ***ratio (r)*** to determine the next term. The following is a geometric sequence:

2, 6, 18, 54, …… The ratio can be found by taking any term of the sequence and dividing it by the term just before it.

The ratio of this sequence is: $r = \frac{t_2}{t_1} = \frac{6}{2} \rightarrow r = 3$

The first term of a sequence is referred as **"*a*"**. The first term of the above sequence is 2. A specific term in a sequence is written as t_n. In the above sequence, $a = t_1$, the second term is t_2, the third term is t_3, any term *n* is t_n. In general, a geometric sequence is written as:

$$ar^0, ar^1, ar^2, ar^3, ar^4, ..., ar^{n-2}, ar^{n-1}, ar^n$$

Therefore any specific term of a sequence is:

$t_n = ar^{n-1}$ *n-1* because the first term is not multiplied by *r*, so we multiply *n* terms by *n-1* ratios.

Lesson Examples:

1) Is each sequence geometric? If it is, determine: r, a, t_4, t_{15} ?

a) 1, 3, 9, 27, ……

$r = \frac{3}{1} = 3$

$a = 1$

$t_4 = ar^{n-1} = 1(3)^{4-1} = 3^3 = 27$

$t_{15} = ar^{n-1} = 1(3)^{15-1} = 3^{14} = 4782969$

b) 1, 2, 3, 4, ……

not geometric

c) 0.1, -0.2, 0.4, -0.8, ……..

$r = \frac{-0.2}{0.1} = -2$

$a = 0.1$

$t_4 = ar^{n-1} = 0.1(-2)^{4-1} = 0.1(-8) = -0.8$

$t_{15} = ar^{n-1} = 0.1(-2)^{15-1} = 1638.4$

d) 100, 90, 80, 70……..

not geometric

e) $3w, 12w^2, 48w^3, ...$

$r = \frac{12w^2}{3w} = 4w$

$a = 3w$

$t_4 = ar^{n-1}$

$= 3w(4w)^{4-1}$

$= 3w(64w^3)$

$= 192w^4$

$t_{15} = ar^{n-1}$

$= 3w(4w)^{15-1}$

$= 3w(4w)^{14}$

$= 3w(268435456w^{14})$

$= 805306368w^{15}$

2) For the sequence: 3, 12, 48, 192,…….; which term is 12288?

$t_n = ar^{n-1} = 3(4)^{n-1}$

$12288 = 3(4)^{n-1}$

$\frac{12288}{3} = \frac{3(4)^{n-1}}{3}$

$4096 = 4^{n-1}$ 2 methods can be done to solve.

$4^6 = 4^{n-1}$ Method 1: common base and solve

$6 = n - 1$

$n = 7$

Method 2: Solve using logs.

$4096 = 4^{n-1}$

$\log 4096 = \log 4^{n-1}$

$\log 4096 = (n-1)\log 4$

$\frac{\log 4096}{\log 4} = n - 1$

$n = \frac{\log 4096}{\log 4} + 1$

$n = 6 + 1$

$n = 7$

Topic 10 Practice Review Questions

Geometric Sequences & Series

1. Determine the common ratio of the geometric sequence 8, 12, 18, 27, . . .

2. The general term of a geometric sequence is $t_n = 8(-3)^{n-1}$. Determine the common ratio.

3. Calculate the 12th term of the geometric sequence: 5, 15, 45, . . .

4. Which term of the geometric sequence 5, 15, 45, . . . is 885 735?

$t_n = ar^{n-1}$ $\quad a=5 \quad r=\frac{15}{5}$

$r=3$

$\frac{5(3)^{n-1}}{5} = \frac{885\,735}{5}$

$3^{n-1} = 177147$

log Method

$\log 3^{n-1} = \log 177147$

$\frac{(n-1)\log 3}{\log 3} = \frac{\log 177147}{\log 3}$

$n-1 = \frac{\log 177147}{\log 3}$ $\quad +1$

$n = \frac{\log 177147}{\log 3} + 1$

$(n=12)$

Common base method → $3^{n-1} = 3^{11} \rightarrow n-1=11$ $\quad +1$

$n=12 \therefore t_{12}$

5. Determine the number of terms in the geometric sequence:

$$\frac{1}{128}, \frac{1}{32}, \frac{1}{8}, \ldots, 2048$$

6. The second term of a geometric series is -16 and the seventh term is 512. Determine the first term.

$t_2 = -16$

$ar^{2-1} = -16$

① $ar = -16$

$t_7 = 512$

$ar^{7-1} = 512$

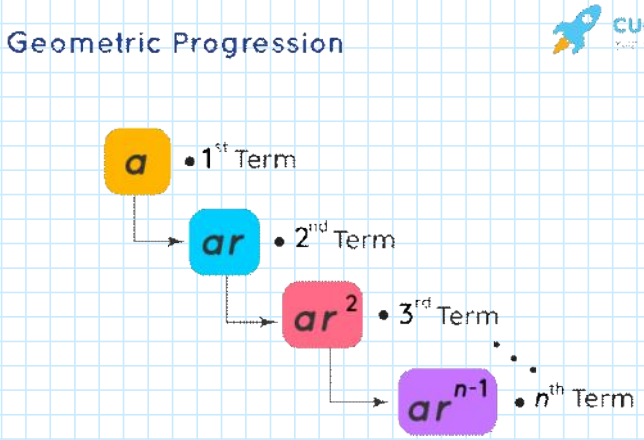
② $ar^6 = 512$

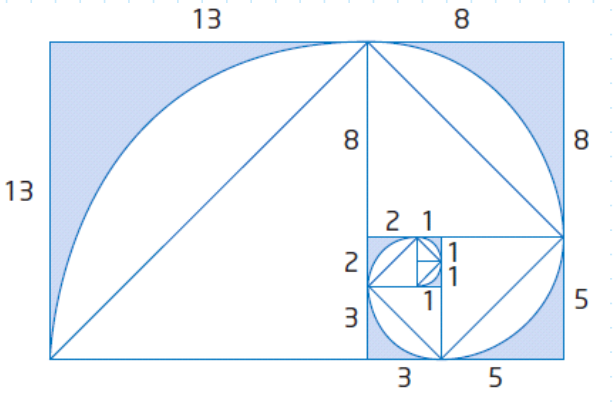
Ratio : $\frac{ar^6}{ar} = \frac{512}{-16}$

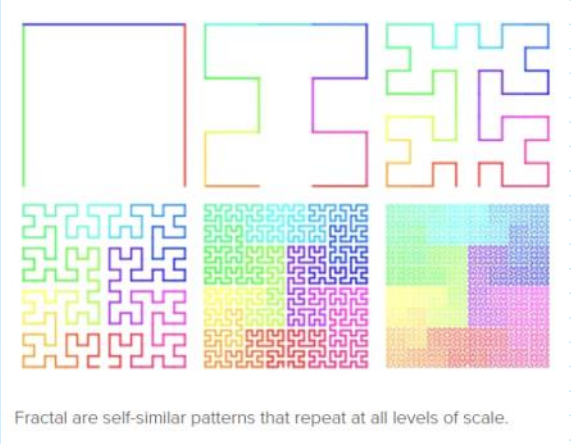
$r = -2$

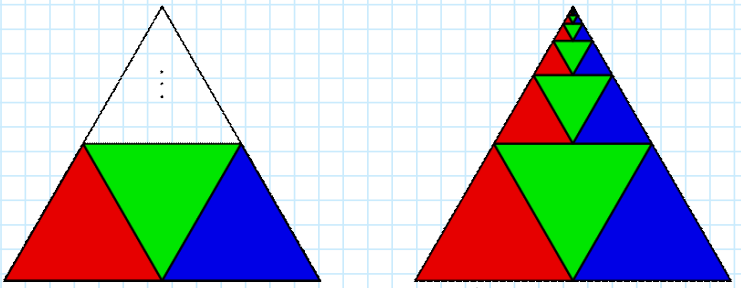
$a = 8$

10.1 Geometric Sequence






Fractal are self-similar patterns that repeat at all levels of scale.


Example: Area of the green triangle is reducing by 1/4 for each fractal level.

Geometric Sequence

A geometric sequence has a common ratio.

The formula for the n^{th} term is

$$a_n = ar^{n-1}$$

where a_n = n^{th} term of the sequence

a = first term of the sequence

r = common ratio

Geometric Sequences

- Ratio of consecutive terms is constant.
  - Called the "***common ratio.***"
- Examples:
- 1, 3, 9, 27, 81, … ratio= 3
- 64, -32, 16, -8, 4, … ratio = -1/2
- a, ar, ar^2, ar^3, ar^4, … ratio = r

WRITING A RULE FOR A GEOMETRIC SEQUENCE

$$a_n = a_1 r^{n-1}$$

$$3, 15, 75, 375, 1{,}875, \ldots$$

$$a_1 = 3$$

$$r = \frac{15}{3} = 5$$

determine the 9th term

$$a_9 = 3(5)^{(9-1)} \rightarrow a_9 = 3(5)^{(8)}$$

$$a_9 = 1{,}171{,}875$$

- Consider the geometric sequence:

$$3, 6, 12, 24, 48, \ldots$$

This sequence has $t_1 = 3$ and common ratio $r = 2$. Thus:

$$t_1 = 3$$
$$t_2 = 3 \cdot 2$$
$$t_3 = 3 \cdot 2 \cdot 2 = 3 \cdot 2^2$$
$$t_4 = 3 \cdot 2 \cdot 2 \cdot 2 = 3 \cdot 2^3$$
$$t_n = 3 \cdot 2^{n-1}$$

In the geometric sequence 6, 12, 24, …, determine each term. **a)** t_{10} **b)** t_n

Solution

The first term is 6 and the common ratio is $\frac{12}{6}$, or 2.

a) t_{10} can be found by starting with the first term and multiplying by the common ratio 9 times.

$t_{10} = 6 \times 2^9$
$= 3072$

b) An expression for t_n can be found by multiplying the first term by $(n - 1)$ common ratios.

$t_n = 6 \times 2^{n-1}$

Consider the geometric sequence 3, 6, 12, 24, ….

a) Determine the 14th term.

b) Which term is 384?

Solution

Use the formula for the general term: $t_n = ar^{n-1}$

a) Substitute $a = 3$, $r = 2$, and $n = 14$.

$t_{14} = 3 \times 2^{13}$
$= 24\,576$

The 14th term is 24 576.

b) Substitute $a = 3$, $r = 2$, and $t_n = 384$.

$384 = 3 \times 2^{n-1}$

Divide each side by 3.

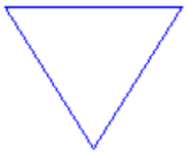
$128 = 2^{n-1}$

Since $128 = 2^7$, then $n - 1 = 7$

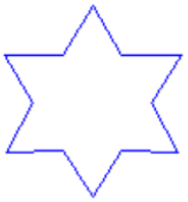
Hence, $n = 8$

384 is the 8th term of the sequence.

Example Topic 10 Project



2a. 3 line segments



2b.

Practise

1. Determine if the sequence is geometric. If it is, state the common ratio and the general term in the form $t_n = t_1 r^{n-1}$.

a) 1, 2, 4, 8, ...

b) 2, 4, 6, 8, ...

c) 3, −9, 27, −81, ...

d) 1, 1, 2, 4, 8, ...

e) 10, 15, 22.5, 33.75, ...

f) −1, −5, −25, −125, ...

2. Copy and complete the following table for the given geometric sequences.

	Geometric Sequence	Common Ratio	6th Term	10th Term
a)	6, 18, 54, ...			
b)	1.28, 0.64, 0.32, ...			
c)	$\frac{1}{5}, \frac{3}{5}, \frac{9}{5}, \cdots$			

3. Determine the first four terms of each geometric sequence.

a) $t_1 = 2, r = 3$ **b)** $t_1 = -3, r = -4$

c) $t_1 = 4, r = -3$ **d)** $t_1 = 2, r = 0.5$

4. Determine the missing terms, t_2, t_3, and t_4, in the geometric sequence in which $t_1 = 8.1$ and $t_5 = 240.1$.

5. Determine a formula for the nth term of each geometric sequence.

a) $r = 2, t_1 = 3$

b) 192, −48, 12, −3, ...

c) $t_3 = 5, t_6 = 135$

d) $t_1 = 4, t_{13} = 16\ 384$

Apply

6. Given the following geometric sequences, determine the number of terms, n.

Table A

	First Term, t_1	Common Ratio, r	nth Term, t_n	Number of Terms, n
a)	5	3	135	
b)	−2	−3	−1458	
c)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{48}$	
d)	4	4	4096	
e)	$-\frac{1}{6}$	2	$-\frac{128}{3}$	
f)	$\frac{p^2}{2}$	$\frac{p}{2}$	$\frac{p^9}{256}$	