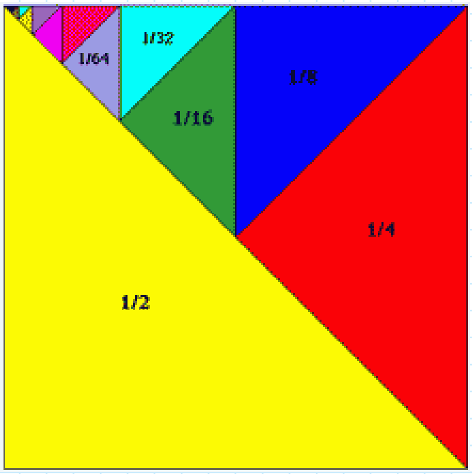
Tuesday, Dec. 5th

Plan For Today:

1. Any questions from Chapter 9 Rational Functions?
   - Hand-in Chapter 9 Project
   - Do Ch9 Test
2. Any questions from 10.1 Term Formula?
   - Do 10.1 Check-in Quiz
3. Continue Topic 10: Geometric Sequences & Series
   - ✓ 10.1 Geometric Sequences
   - **10.2 Geometric Series**
   - **10.3 Infinite Geometric Series**
   - 10.4 Sigma Notation
4. Work on Practice Questions



Plan Going Forward:

1. Finish going through 10.2-10.3 practice questions in handout.

   - **10.2-10.3 CHECK-IN QUIZ ON THURSDAY, DEC. 7TH**

2. We will try to do majority of Topic 10 (Geometric Sequences & Series) on Thursday and finish it next Tuesday with some review for the Unit 4 exam.

   - **TOPIC10 PROJECT DUE TUESDAY, DEC. 12TH**
   - **UNIT 4 EXAM ON CH9 & T10 ON THURSDAY, DEC. 14TH**
     - Rewrite next Tuesday during class (last class) on Tuesday, Dec. 19th
     - I'll email you when I post marks by Friday or Saturday

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.
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Tuesday, Dec. 5th In-Class Notes

Name: Practice **DATE:** (no marks) **TOTAL = ____ / 6 marks**

Check-in Quiz Section 10.1 – Geometric Sequences

Complete the following questions SHOWING ALL WORK and steps where applicable.

1. Determine if the following sequence IS or IS NOT a geometric sequence.
   CIRCLE THE CORRECT ANSWER

0.25 each = 1 mark

a) $2, 6, 18, 54, \ldots$ ✓ — IS GEOMETRIC — NOT GEOMETRIC

$\frac{6}{2} = 3$

b) $8, -2, \frac{1}{2}, -\frac{1}{8}, \ldots$ — IS GEOMETRIC — NOT GEOMETRIC

$r = \frac{-2}{8}$

$r = -\frac{1}{4}$

c) $1, \frac{3}{2}, \cancel{\frac{12}{4}}, \frac{60}{8}, \ldots$ — IS GEOMETRIC — NOT GEOMETRIC

$\times\frac{3}{2}$

$r = \frac{3}{2}$

$\frac{9}{4}$

d) $\frac{1}{x}, \frac{2}{x^2}, \frac{4}{x^3}, \ldots$ ✓ — IS GEOMETRIC — NOT GEOMETRIC

$\times\frac{2}{x}$

$$\frac{\frac{2}{x^2}}{\frac{1}{x}} \rightarrow \frac{2}{x^{\cancel{2}}} \times \frac{\cancel{x}}{1} \rightarrow r = \frac{2}{x}$$

2. Algebraically determine the common ratio, r, in each of the following geometric sequences:

1 mark each = 2 marks

a) $-48, 192, -768, 3072, \ldots$

$r = \frac{192}{-48}$

$\boxed{r = -4}$

b) $2x, 5x^3, \frac{25}{2}x^5, \ldots$

$$\frac{5x^{\cancel{3}\,2}}{2\cancel{x}} = \boxed{\frac{5x^2}{2}}$$

3. Algebraically determine the 8th term, t_8, of the following geometric sequence:

1 mark

$t_n = ar^{n-1}$ $n=8$

$$18.1, 1.81, 0.181, 0.0181, \ldots$$

$a = 18.1$ $r = \frac{1.81}{18.1}$

$r = 0.1$

~~NOT 1.81E-6~~

$t_8 = 18.1(0.1)^{8-1} \rightarrow t_8 = 18.1(0.1)^7$

$$\boxed{\begin{aligned} t_8 &= 1.81 \times 10^{-6} \\ &= 0.00000181 \end{aligned}}$$

4. Algebraically determine the 10th term, t_{10}, of the geometric sequence given the following information:

1 mark

EXACT form (fraction only) (no decimals)

$$a = -32, r = \frac{3}{2}$$ $n = 10$

$t_{10} = -32\left(\frac{3}{2}\right)^{10-1}$

$t_{10} = -32\left(\frac{3}{2}\right)^{9}$

$= -\cancel{32}^{\,1}\left(\frac{19683}{\cancel{512}_{\,16}}\right)$

$$\boxed{t_{10} = -\frac{19683}{16}}$$

5. Algebraically determine which term of the following geometric sequence is **14348907**:

t_n 1 mark

$t_n = ar^{n-1}$

$$9, 27, 81, \ldots$$

$a=9$ $r=3$

$\frac{14348907}{9} = \frac{\cancel{9}(3)^{n-1}}{9}$

$1594323 = 3^{n-1}$

$\cancel{3}^{13} = \cancel{3}^{n-1}$

$13 = n-1$

$+1$

$n = 14 \rightarrow \boxed{t_{14}}$

$\frac{\log(1594323)}{\log 3} = \frac{(n-1)\log 3}{\log 3}$

$\frac{\log 1594323}{\log 3} = n-1$

$+1$

$n = \frac{\log 1594323}{\log 3} + 1$

$n = 14 \rightarrow \boxed{t_{14}}$

Geometric Series

A geometric series, is the sum of the terms in a geometric sequence.

$$S_n = a + ar + ar^2 + ar^3 + ar^4 + \ldots\ldots + ar^{n-2} + ar^{n-1} + ar^n$$

There are two ways we can find the sum of a series. One is to write out all the terms and add them together. This can be tedious, especially with complex sequences and long ones. The other way is by using the formulas below:

$$S_n = \frac{a(1-r^n)}{1-r} \text{ or } = \frac{a(r^n-1)}{r-1}$$

where a is the first term, r is the common ratio, l is the last term, and $r \neq 1$, n is the number of terms.

$$S_n = \frac{a-lr}{1-r}$$

l : last term.

To understand how this formula is derived, read p. 122 of your text.

Lesson Examples:

1) Find S_9 of the sequence: 3, 6, 12,......

$$S_n = \frac{a(1-r^n)}{1-r}$$
$$= \frac{3(1-2^9)}{1-2}$$
$$= \frac{3(1-2^9)}{-1}$$
$$= -3(1-2^9)$$
$$= -3(1-512)$$
$$= -3(-511)$$
$$= 1533$$

BEDMAS

$$= \frac{3(1-512)}{-1}$$
$$= \frac{3(-511)}{-1}$$
$$= \frac{-1533}{-1}$$
$$= \boxed{1533}$$

3) a) Find the sum of the following series: 0.025 + 0.075 + ...+ 54.675

$$S_n = \frac{a-lr}{1-r}$$
$$= \frac{0.025-54.675(3)}{1-3}$$
$$= \frac{0.025-164.025}{-2}$$
$$= \frac{-164}{-2} = 82$$

$\star$ l = last term

$r = \frac{0.075}{0.025}$

$r = 3$

$S_n = 82$

b) Determine, algebraically, how many terms are in this series above.

$$t_n = ar^{n-1} = 0.025(3)^{n-1}$$
$$54.675 = 0.025(3)^{n-1}$$
$$\frac{54.675}{0.025} = \frac{0.025(3)^{n-1}}{0.025}$$
$$2187 = 3^{n-1}$$
$$3^7 = 3^{n-1}$$
$$7 = n-1$$
$$n = 8$$

t_8 = 8^{th} term is the last one

$\therefore$ there are 8 terms in this series.

4) Extensive research has shown that when gum is chewed, 10% of the flavor is lost with each chomp. Answer the following questions to the nearest squirt of flavor:

100−10 = 90% remains

$r = 1 - \% \rightarrow r = 1 - 0.1 \Rightarrow \boxed{0.9}$

a) If the first chop gives 40 squirts of flavor, how many squirts will the 10th chomp give?

term 10

$a = 40$

$r = 0.9$

$n = 10$

You are trying to find the 10th term.

$t_{10} = ar^{n-1}$

$t_{10} = 40(0.9)^{10-1}$

$t_{10} = 40(0.9)^{9}$

$t_{10} = 15.50$ → 15.50 squirts of flavour at 10^{th} chomp.

b) What is the total amount of flavour that will be received in the first 10 chomps?

sum to term 10

Here the word total means find the sum of all the chomps up to 10:

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{10} = \frac{40(1-0.9^{10})}{1-0.9}$$

$$S_{10} = \frac{40(1-0.9^{10})}{0.1}$$

$$S_{10} = 400(1-0.9^{10})$$

$S_{10} = 260.53$ → 260.53 squirts of flavor total

5) Determine the nth term, and the sum of the first n terms of the geometric sequence which has 2, 6, and 18 as its first three numbers.

no 'n' ∴ simplify, but answer will still have 'n' in it.

$a = 2$

$r = \frac{6}{2} = 3$

$t_n = ar^{n-1} = 2(3)^{n-1}$ → $\boxed{t_n = 2(3)^{n-1}}$ any term

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{2(1-3^n)}{1-3} = \frac{2(1-3^n)}{-2} = -(1-3^n) = 3^n - 1$$

$\boxed{S_n = 3^n - 1}$ sum of n terms.

=

=

=

6) a) While training for a race, a runner increases her distance by 4% each day. If she runs 2 km on the first day, what will be distance on the 26th day of training?

$r = 1 + 0.04$

$r = 1.04$

$a = 2$

t_{26}

$t_n = ar^{n-1}$

b) What will be the total distance she has ran by the 26th day of training?

$$S_{26} = \frac{a(1-r^n)}{1-r} \rightarrow = \frac{2(1-1.04^{26})}{1-1.04} = \frac{2(1-1.04^{26})}{-0.04} \rightarrow = -50(1-1.04^{26})$$

$$\boxed{S_{26} = 88.62 \text{ km}}$$

[redacted]

7) A school has a telephone tree to contact people for the upcoming field trip. The first student calls two students, each of whom call two more, each of whom call two more, and so on.

$a = 1$

$r = 2$

a) How many are contacted at the 8th level of calling?

$t_8 =$

b) By the 8th level, how many students, in total, have been contacted?

$S_8 =$

c) If there are 300 students in total, by what level will all have been contacted?

solve for n

$S_n =$

$$300 = \frac{1(1-2^n)}{1-2}$$

$-300 = 1 - 2^n$

$+2^n \quad +300$

$\log 2^n = \log 301 \rightarrow$

$n \log 2 = \log 301$

$n = \frac{\log 301}{\log 2}$

$n = 8.23 \rightarrow$ ∴ by the 9th level

$t_n \rightarrow 300 = ar^{n-1}$

$300 = 1(2)^{n-1}$

$\log 300 = \log 2^{n-1}$

$\log 300 = (n-1)\log 2$

no common base ∴ log.

$\frac{\log 300}{\log 2} = n-1$

$n = \frac{\log 300}{\log 2} + 1 \rightarrow n = 9.23$

∴ $n = 10$

10.2-10.3 Geometric Series

Geometric Series: *An indicated sum of terms in a geometric sequence.*

Example:

Geometric Sequence	VS	Geometric Series
3, 6, 12, 24, 48		3 + 6 + 12 + 24 + 48

Geometric Sequence and Geometric Series

A geometric sequence is a sequence of numbers in which the ratio between consecutive terms is constant.

The formula for the n^{th} term of a geometric sequence is

$$a_n = ar^{n-1}$$

where a is the first term and r is the ratio

A geometric series results from adding the terms of a geometric sequence.

The formula for the sum of a finite geometric series is

$$S_n = \sum_{i=1}^{n} ar^{i-1} = \frac{a(1-r^n)}{1-r}, r \neq 1$$

where n is the number of terms, a is the first term and r is the ratio

The formula for the sum of an infinite geometric series is

$$S_n = \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}, |r| < 1$$

where a is the first term and r is the ratio

If $|r| \geq 1$, then the infinite series does not have a sum

Geometric Series

A geometric sequence is a sequence of numbers in which the ratio between consecutive terms is constant.

A geometric series results from adding the terms of a geometric sequence.

The formula for the partial sum of the first n terms of a geometric series is

$$S_n = \sum_{i=1}^{n} ar^{i-1} = \frac{a\left(1-r^n\right)}{1-r}, r \neq 1$$

where a is the first term and r is the ratio

The sum of an infinite geometric series is given by

$$a + ar + ar^2 + \ldots + ar^{n-1} + \ldots = \sum_{i=1}^{\infty} ar^{i-1}$$

where a is the first term and r is the ratio

The sum converges to $\dfrac{a}{1-r}$ if $|r| < 1$

The sum diverges if $|r| \geq 1$

Example #2: Find the sum of the following series.

$7 + 14 + 28 + \ldots$ for 12 terms

Step #1: Identify the variables.

$\frac{14}{7} = 2$ and $\frac{28}{14} = 2$, so $r = 2$.

$a = 7,\ r = 2,\ n = 12$

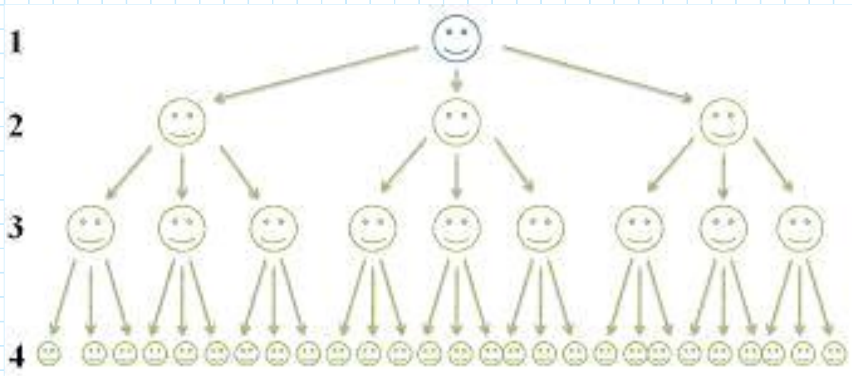
Step #2: Substitute and evaluate.

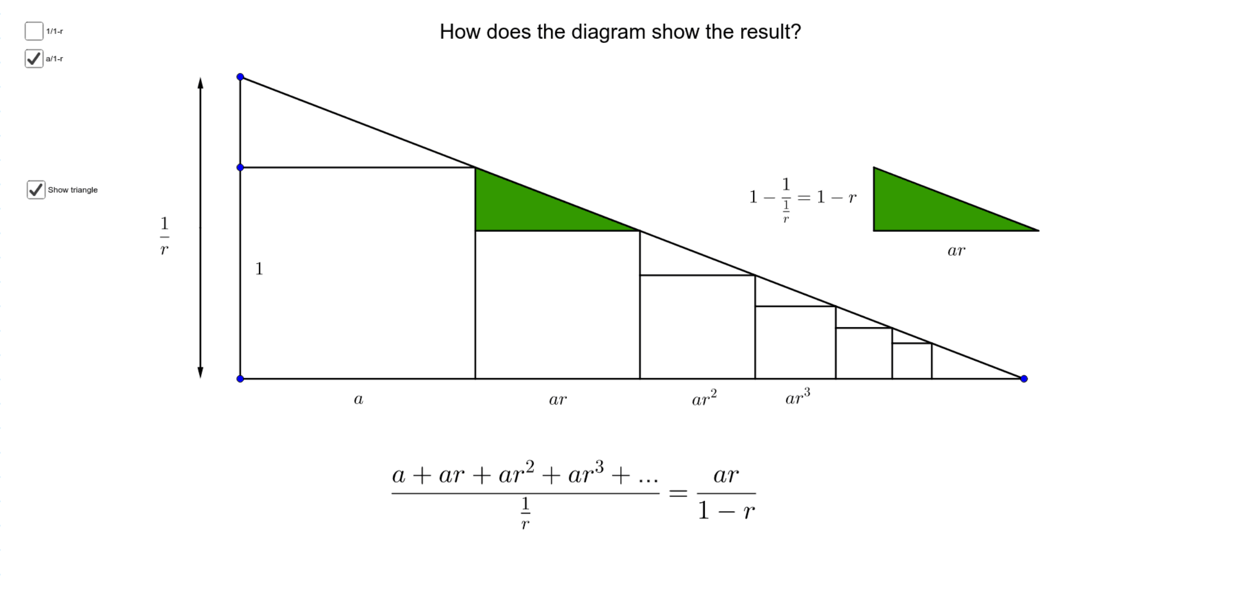
$$S_n = \frac{a(1-r^n)}{(1-r)}$$

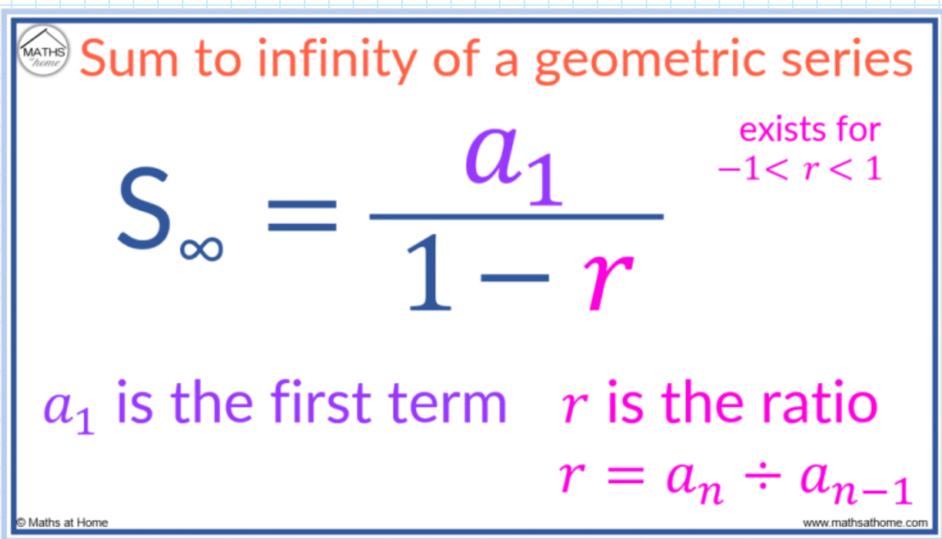
$$S_n = \frac{7(1-2^{12})}{1-2}$$

$$S_n = \frac{7(-4095)}{-1}$$

$$S_n = 28665$$







2. Find the value of the infinite geometric series $4+2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$

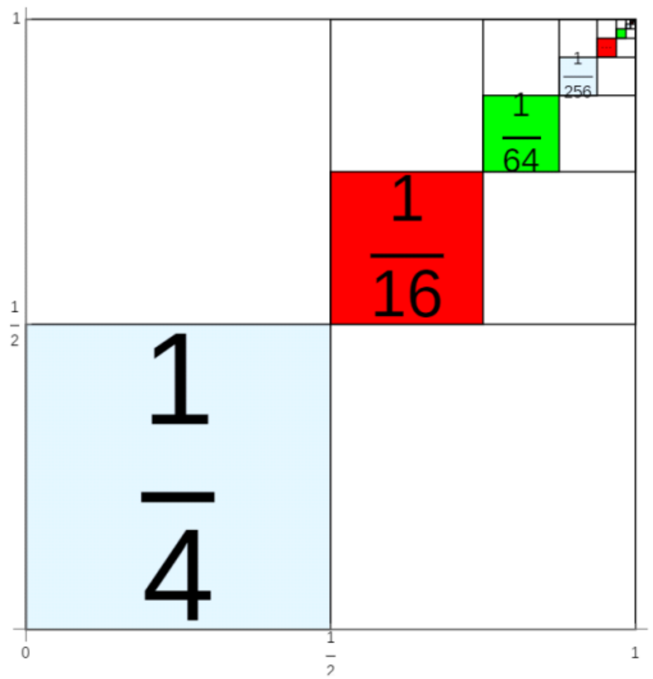
$$a_1=4, \quad r=\frac{1}{2}$$

2. Find the value of the infinite geometric series $4+2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots$

$$a_1 = 4, \quad r = \frac{1}{2}$$

$$S_\infty = \frac{a_1}{1-r}$$

$$S_\infty = \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8$$



Determine the sum of the first 9 terms of the geometric series $2 + 6 + 18 + 54 + \ldots$.

Solution

Think …

To use the method, we need to know the 9th term.

Determine t_9 using $t_n = ar^{n-1}$.

In this series, $a = 2, r = 3$, and $n = 9$

$$\begin{aligned} t_9 &= 2 \times 3^8 \\ &= 13\ 122 \end{aligned}$$

Let S represent the sum: $S = 2 + 6 + 18 + \ldots + 13\ 122$ ①

Multiply by the common ratio 3: $3S = 6 + 18 + \ldots + 13\ 122 + 39\ 366$ ②

Subtract ① from ②:

$$\begin{aligned} 2S &= -2 + 39\ 366 \\ &= 39\ 364 \\ S &= 19\ 682 \end{aligned}$$

The sum of the first 9 terms of the series is 19 682.

Determine the sum of the first 10 terms of each geometric series.

a) $4 + 12 + 36 + 108 + \ldots$ **b)** $6 + 3 + 1.5 + 0.75 + \ldots$

Solution

Use the formula $S_n = \dfrac{a(r^n - 1)}{r - 1}$.

a) Substitute $a = 4,\ r = 3,\ n = 10$.

$$\begin{aligned} S_{10} &= \frac{4(3^{10} - 1)}{3 - 1} \\ &= 2(3^{10} - 1) \\ &= 118\ 096 \end{aligned}$$

b) Substitute $a = 6,\ r = 0.5,\ n = 10$.

$$\begin{aligned} S_{10} &= \frac{6(0.5^{10} - 1)}{0.5 - 1} \\ &= -12(0.5^{10} - 1) \\ &= 11.988\ 281\ 25 \end{aligned}$$

Consider the infinite geometric series $4 - \frac{4}{5} + \frac{4}{25} - \ldots$.

a) Explain why the series has a sum to infinity.

b) Determine the sum to infinity.

Solution

a) For this series, $a = 4$ and $r = -\frac{1}{5}$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$
$$= \frac{4\left(\left(-\frac{1}{5}\right)^n - 1\right)}{-\frac{1}{5} - 1} \times \frac{-1}{-1}$$
$$= \frac{4\left(1 - \left(-\frac{1}{5}\right)^n\right)}{\frac{6}{5}}$$
$$= \frac{10}{3}\left(1 - \left(-\frac{1}{5}\right)^n\right)$$

The series has a sum to infinity because the expression $\left(-\frac{1}{5}\right)^n$ gets closer to 0 as n gets larger.

b) The sum to infinity is $\frac{10}{3}$.

Which infinite geometric series has a sum? What is the sum?

a) $4 - 6 + 9 - 13.5 + \ldots$

b) $6 + 2 + \frac{2}{3} + \frac{2}{9} + \ldots$

Solution

a) $4 - 6 + 9 - 13.5 + \ldots$

For this series, $r = -1.5$; since $|r| > 1$, the series has no sum.

b) $6 + 2 + \frac{2}{3} + \frac{2}{9} + \ldots$

For this series, $r = \frac{1}{3}$; since $|r| < 1$, the series has a sum. Use the formula.

$$S = \frac{a}{1 - r}$$
$$= \frac{6}{1 - \frac{1}{3}}$$
$$= \frac{6}{\frac{2}{3}}$$
$$= 9$$

The sum of the series is 9.

Your Turn

Determine the sum of the following geometric series.

a) $\frac{1}{64} + \frac{1}{16} + \frac{1}{4} + \cdots + 1024$

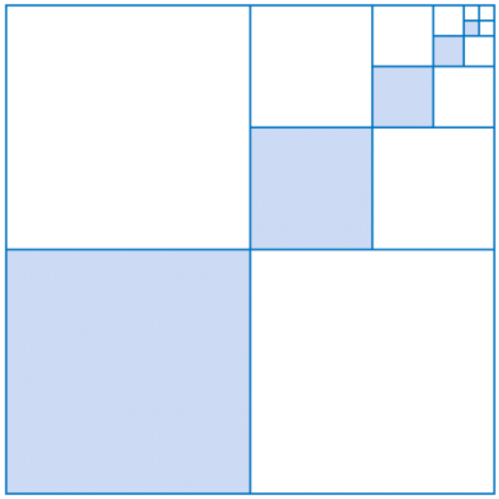
b) $-2 + 4 - 8 + \cdots - 8192$

Your Turn

Determine whether each infinite geometric series converges or diverges. Calculate the sum, if it exists.

a) $1 + \frac{1}{5} + \frac{1}{25} + \cdots$ **b)** $4 + 8 + 16 + \cdots$

Apply the Sum of an Infinite Geometric Series



Assume that each shaded square represents $\frac{1}{4}$ of the area of the larger square bordering two of its adjacent sides and that the shading continues indefinitely in the indicated manner.

a) Write the series of terms that would represent this situation.

b) How much of the total area of the largest square is shaded?

Solution

Your Turn

You can express $0.\overline{584}$ as an infinite geometric series.

$0.\overline{584} = 0.584\ 584\ 584\ldots$

$= 0.584 + 0.000\ 584 + 0.000\ 000\ 584 + \cdots$

Determine the sum of the series.