Name: Practice **DATE:** (no marks) **TOTAL = ____ / 6 marks**

Check-in Quiz Section 10.1 – Geometric Sequences

Complete the following questions SHOWING ALL WORK and steps where applicable.

1. Determine if the following sequence IS or IS NOT a geometric sequence.
   CIRCLE THE CORRECT ANSWER

0.25 each = 1 mark

a) $2, 6, 18, 54, \ldots$ ✓ IS GEOMETRIC NOT GEOMETRIC

$\frac{6}{2} = 3$

b) $8, -2, \frac{1}{2}, -\frac{1}{8}, \ldots$ IS GEOMETRIC NOT GEOMETRIC

$r = \frac{-2}{8}$

$r = -\frac{1}{4}$

c) $1, \frac{3}{2}, \cancel{\frac{12}{4}}, \frac{60}{8}, \ldots$ IS GEOMETRIC NOT GEOMETRIC

$\times\frac{3}{2}$

$\frac{9}{4}$

$r = \frac{3}{2}$

d) $\frac{1}{x}, \frac{2}{x^2}, \frac{4}{x^3}, \ldots$ ✓ IS GEOMETRIC NOT GEOMETRIC

$\times\frac{2}{x}$

$\frac{\frac{2}{x^2}}{\frac{1}{x}} \rightarrow \frac{2}{\cancel{x^2}} \times \frac{\cancel{x}}{1} \rightarrow r = \frac{2}{x}$

2. Algebraically determine the common ratio, r, in each of the following geometric sequences:

1 mark each = 2 marks

a) $-48, 192, -768, 3072, \ldots$

$r = \frac{192}{-48}$

$\boxed{r = -4}$

b) $2x, 5x^3, \frac{25}{2}x^5, \ldots$

$\frac{5x^{\cancel{3}\,2}}{2\cancel{x}} = \boxed{\frac{5x^2}{2}}$

3. Algebraically determine the 8[th] term, t_8, of the following geometric sequence:

1 mark

$t_n = ar^{n-1}$ $n=8$

18.1, 1.81, 0.181, 0.0181, ...

$a = 18.1$ $r = \frac{1.81}{18.1}$

$r = 0.1$

~~NOT 1.81E-6~~

$t_8 = 18.1(0.1)^{8-1} \rightarrow t_8 = 18.1(0.1)^7$

$\boxed{t_8 = 1.81 \times 10^{-6} = 0.00000181}$

4. Algebraically determine the 10[th] term, t_{10}, of the geometric sequence given the following information:

1 mark

EXACT form (fraction only) (no decimals)

$a = -32, r = \frac{3}{2}$ $n = 10$

$t_{10} = -32\left(\frac{3}{2}\right)^{10-1}$

$t_{10} = -32\left(\frac{3}{2}\right)^{9}$

$= -\cancel{32}^{1}\left(\frac{19683}{\cancel{512}_{16}}\right)$

$\boxed{t_{10} = -\frac{19683}{16}}$

5. Algebraically determine which term of the following geometric sequence is **14348907**:

t_n 1 mark

$t_n = ar^{n-1}$

9, 27, 81, ...

$a=9$ $r=3$

$\frac{14348907}{9} = \frac{\cancel{9}(3)^{n-1}}{9}$

$1594323 = 3^{n-1}$

$\cancel{3}^{13} = \cancel{3}^{n-1}$

$13 = n-1$

$+1$

$n = 14 \rightarrow \boxed{t_{14}}$

$\frac{\log(1594323)}{\log 3} = \frac{(n-1)\log 3}{\log 3}$

$\frac{\log 1594323}{\log 3} = n-1$

$+1$

$n = \frac{\log 1594323}{\log 3} + 1$

$n = 14 \rightarrow \boxed{t_{14}}$

Geometric Series

A geometric series, is the sum of the terms in a geometric sequence.

$$S_n = a + ar + ar^2 + ar^3 + ar^4 + ...+ ar^{n-2} + ar^{n-1} + ar^n$$

There are two ways we can find the sum of a series. One is to write out all the terms and add them together. This can be tedious, especially with complex sequences and long ones. The other way is by using the formulas below:

$$S_n = \frac{a(1-r^n)}{1-r} \text{ or } = \frac{a(r^n-1)}{r-1}$$

where a is the first term, r is the common ratio, l is the last term, and $r \neq 1$, n is the number of terms.

$$S_n = \frac{a-lr}{1-r}$$

ℓ : last term.

To understand how this formula is derived, read p. 122 of your text.

Lesson Examples:

1) Find S_9 of the sequence: 3, 6, 12,......

$$
\begin{aligned}
S_n &= \frac{a(1-r^n)}{1-r} \\
&= \frac{3(1-2^9)}{1-2} \\
&= \frac{3(1-2^9)}{-1} \\
&= -3(1-2^9) \\
&= -3(1-512) \\
&= -3(-511) \\
&= 1533
\end{aligned}
$$

BEDMAS

$= \frac{3(1-512)}{-1}$

$= \frac{3(-511)}{-1}$

$= \frac{-1533}{-1}$

$= \boxed{1533}$

3) a) Find the sum of the following series: 0.025 + 0.075 + ...+ 54.675

$$
\begin{aligned}
S_n &= \frac{a-lr}{1-r} \\
&= \frac{0.025-54.675(3)}{1-3} \\
&= \frac{0.025-164.025}{-2} \\
&= \frac{-164}{-2} = 82
\end{aligned}
$$

* l = last term

$r = \frac{0.075}{0.025}$

$r = 3$

$S_n = 82$

b) Determine, algebraically, how many terms are in this series above.

$$
\begin{aligned}
&t_n = ar^{n-1} = 0.025(3)^{n-1} \\
&54.675 = 0.025(3)^{n-1} \\
&\frac{54.675}{0.025} = \frac{0.025(3)^{n-1}}{0.025} \\
&2187 = 3^{n-1} \\
&3^7 = 3^{n-1} \\
&7 = n-1 \\
&n = 8
\end{aligned}
$$

t_8 = 8th term is the last one

∴ there are 8 terms in this series.

4) Extensive research has shown that when gum is chewed, 10% of the flavor is lost with each chomp. Answer the following questions to the nearest squirt of flavor:

100−10 = 90% remains

$r = 1 - \% \rightarrow r = 1 - 0.1 \rightarrow \boxed{0.9}$

a) If the first chop gives 40 squirts of flavor, how many squirts will the 10th chomp give?

term 10

$a = 40$

$r = 0.9$

$n = 10$

You are trying to find the 10th term.

$$t_{10} = ar^{n-1}$$
$$t_{10} = 40(0.9)^{10-1}$$
$$t_{10} = 40(0.9)^{9}$$
$$t_{10} = 15.50$$

15.50 squirts of flavour at 10th chomp.

b) What is the total amount of flavour that will be received in the first 10 chomps?

sum to term 10

Here the word total means find the sum of all the chomps up to 10:

$$S_n = \frac{a(1-r^n)}{1-r}$$
$$S_{10} = \frac{40(1-0.9^{10})}{1-0.9}$$
$$S_{10} = \frac{40(1-0.9^{10})}{0.1}$$
$$S_{10} = 400(1-0.9^{10})$$
$$S_{10} = 260.53$$

260.53 squirts of flavor total

5) Determine the nth term, and the sum of the first n terms of the geometric sequence which has 2, 6, and 18 as its first three numbers.

no 'n' ∴ simplify, but answer will still have 'n' in it.

$a = 2$

$r = \frac{6}{2} = 3$

$t_n = ar^{n-1} = 2(3)^{n-1}$

$\boxed{t_n = 2(3)^{n-1}}$ any term

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{2(1-3^n)}{1-3} = \frac{2(1-3^n)}{-2} = -(1-3^n) = 3^n - 1$$

$\boxed{S_n = 3^n - 1}$ sum of n terms.

=

=

=

6) a) While training for a race, a runner increases her distance by 4% each day. If she runs 2 km on the first day, what will be distance on the 26th day of training?

$r = 1 + 0.04$

$r = 1.04$

$a = 2$

t_{26}

$t_n = ar^{n-1}$

b) What will be the total distance she has ran by the 26th day of training?

$$S_{26} = \frac{a(1-r^n)}{1-r} \rightarrow = \frac{2(1-1.04^{26})}{1-1.04} = \frac{2(1-1.04^{26})}{-0.04}$$

$$= -50(1-1.04^{26})$$

$$\boxed{S_{26} = 88.62\text{ km}}$$

[redacted]

7) A school has a telephone tree to contact people for the upcoming field trip. The first student calls two students, each of whom call two more, each of whom call two more, and so on.

$a = 1$

$r = 2$

a) How many are contacted at the 8th level of calling?

$t_8 =$

b) By the 8th level, how many students, in total, have been contacted?

$S_8 =$

c) If there are 300 students in total, by what level will all have been contacted?

$S_n =$

$300 = \frac{1(1-2^n)}{1-2}$

$-300 = 1 - 2^n$

$+2^n \quad +300$

$\log 2^n = \log 301 \rightarrow$

$n \log 2 = \log 301$

$n = \frac{\log 301}{\log 2}$

$n = 8.23 \rightarrow$ ∴ by the 9th level

$t_n \rightarrow 300 = ar^{n-1}$

$300 = 1(2)^{n-1}$

$\log 300 = \log 2^{n-1}$

$\log 300 = (n-1)\log 2$

solve for n

no common base ∴ log.

$\frac{\log 300}{\log 2} = n - 1$

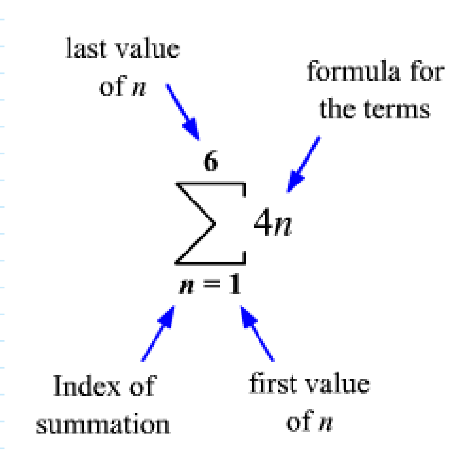
$n = \frac{\log 300}{\log 2} + 1 \rightarrow n = 9.23$

∴ $n = 10$

Thursday, Dec. 7th

Plan For Today:

1. Any questions from 10.2 Sum Formulae?
   - ✸ Do 10.2 Check-in Quiz
2. Finish Topic 10: Geometric Sequences & Series
   - ✓ 10.1 Geometric Sequences
   - ✓ 10.2 Geometric Series
   - ◆ **10.3 Infinite Geometric Series**
   - ◆ **Word Problems**
   - ◆ **10.4 Sigma Notation**
3. Work on Practice Questions



Plan Going Forward:

1. Finish going through Topic 10 practice questions in handout.

- ◆ **TOPIC10 PROJECT DUE TUESDAY, DEC. 12TH**
- **UNIT 4 EXAM ON CH9 & T10 ON THURSDAY, DEC. 14TH**
  - Rewrite next Tuesday during class (last class) on Tuesday, Dec. 19th
  - I'll email you when I post marks by Friday or Saturday

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.
Anurita Dhiman = adhiman@sd35.bc.ca

10.3 · Infinite Sum

$$S_\infty = \frac{a}{1-r} \qquad 0<|r|<1$$

2. Find the value of the infinite geometric series $4+2+1+\underbrace{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}}_{\text{fractions}}+\ldots$

in other words, find the total or find the finite sum.

$a=4$

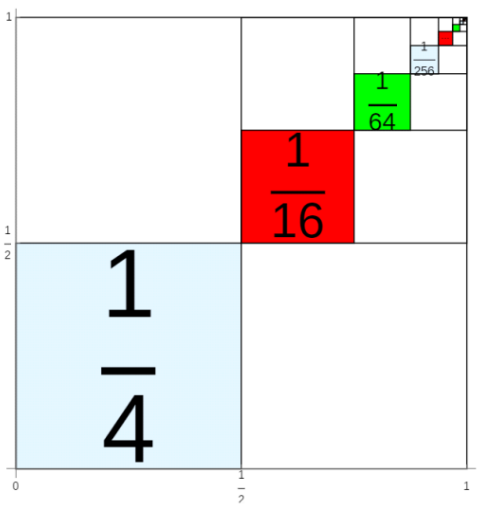
$r=\frac{1}{2}$

$$S_\infty = \frac{4}{1-\frac{1}{2}}$$

$$= \frac{4}{\frac{1}{2}}$$

$$= 4 \times \frac{2}{1}$$

$$\boxed{S_\infty = 8}$$



$a=\frac{1}{4}$

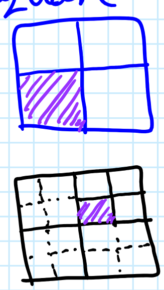
$t_2=\frac{1}{16}$

$$r=\frac{\frac{1}{16}}{\frac{1}{4}}$$

$$= \frac{1}{\cancel{16}_{4}} \times \frac{\cancel{4}^{1}}{1}$$

$r=\frac{1}{4}$

area of 1st shade square.



1st iteration (t_1)

2nd iteration (t_2)

$$S_\infty = \frac{\frac{1}{4}}{1-\frac{1}{4}}$$

$$= \frac{1}{\ldots}$$

S_∞ ... 4

$r = \frac{1}{4}$

$= \dfrac{\frac{1}{4}}{\frac{4}{4} - \frac{1}{4}}$

$= \dfrac{\frac{1}{4}}{\frac{3}{4}}$

$= \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{3}$

$\boxed{S_\infty = \frac{1}{3}}$

Consider the infinite geometric series $4 - \frac{4}{5} + \frac{4}{25} - \ldots$.

b) Determine the sum to infinity.

$a = 4$

$r = \dfrac{-\frac{4}{5}}{4}$

$= -\frac{\cancel{4}}{5} \times \frac{1}{\cancel{4}}$

$r = -\frac{1}{5}$

$S_\infty = \dfrac{a}{1-r}$

$= \dfrac{4}{1-(-\frac{1}{5})}$

$= \dfrac{4}{\frac{5}{5} + \frac{1}{5}}$

$= \dfrac{4}{\frac{6}{5}}$

$= \cancel{4}^{2} \times \frac{5}{\cancel{6}_{3}}$

$\boxed{S_\infty = \frac{10}{3}}$

Your Turn

Determine whether each infinite geometric series converges or diverges. Calculate the sum, if it exists.

a) $1 + \frac{1}{5} + \frac{1}{25} + \cdots$ **b)** $4 + 8 + 16 + \cdots$

Series is convergent (converges) when r is b/w 0 + 1 → $0 < |r| < 1$

series is convergent (converges) when
r is b/w 0 + 1 $\rightarrow$ $0 < |r| < 1$

for infinite series $\therefore$ use $S_\infty = \frac{a}{1-r}$

series is divergent (diverges) when r is
greater than 1 $\longrightarrow$ $|r| > 1$

for increasing series $\rightarrow$ need to know 'n'
number of terms
+ this is not an
infinite sum forma

$$S_n = \frac{a(1-r^n)}{1-r}$$

Infinite Geometric Series

Compare the series $1+2+4+8+16+.........$ and the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+.....$

As the number of terms increase, the sum of the first series will get larger and larger; the sum of the second series will also increase, but not as quickly, and the terms will continue to decrease approaching zero. At a certain number of terms later, the sum will no longer increase significantly; this is the infinite sum.

$$S_\infty = \frac{a}{1-r}$$ where a is the first term, r is the common ratio, $o<|r|<1$

Lesson Examples:

1) Find the sum of the series $2+\frac{2}{5}+\frac{2}{25}+...$

$a=2$

$r=\frac{\frac{2}{5}}{2}=\frac{2}{5}\times\frac{1}{2}=\boxed{\frac{1}{5}}$ since this ratio is between 0 and 1, this is an infinite series

Use: $S_\infty=\frac{a}{1-r}$

$S_\infty=\frac{2}{1-\frac{1}{5}}=\frac{2}{\frac{5}{5}-\frac{1}{5}}=\frac{2}{\frac{4}{5}}=\cancel{2}^{1}\times\frac{5}{\cancel{4}_{2}}=\boxed{\frac{5}{2}}$

2) An oil well produces 25000 barrels of oil during its first month of production. If its production drops 5% each month, estimate the total production before the well runs dry.

decrease

$a=25000$

$r=0.95$

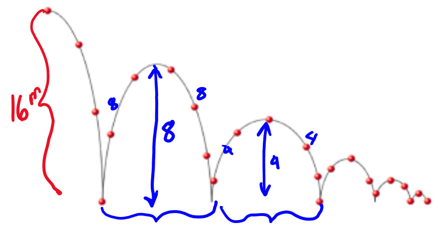
$r=1-0.05$

$S_\infty=\frac{a}{1-r}$

$S_\infty=\frac{25000}{1-0.95}=\frac{25000}{0.05}=500000$

3) A ball is dropped from a height of 16 metres. The ball rebounds a half of the height after each bounce. Calculate the total vertical distance the ball travels before coming to rest.

Since the ball is falling from 16 metres, it will rise to only 8 metres then fall 8 metres. The next bounce it will rise 4 metres then fall 4 metres. Use the complete up and down distance to begin your first term.



$16m + 8m \ldots\ldots$

$a = 16 \quad r = \frac{1}{2}$

$a = 8 + 8 = 16$

$r = \frac{8}{16} = \frac{1}{2}$ or 0.5

$S_\infty = \frac{a}{1-r} = \frac{16}{1-0.5} = \frac{16}{0.5} = 32$

Add the 16 metres from the original drop

$32 + 16 = 48m$

$S_\infty = \frac{16}{1-\frac{1}{2}} = \frac{16}{\frac{1}{2}}$

$16 \times \frac{2}{1}$

$S_\infty = 32$

+ initial 16m drop

$\boxed{\text{TOTAL} = 48m}$

Sigma Notation

Sigma means sum, therefore geometric series are often expressed in this sigma notation.

A particular series can be condensed into sigma notation, or any given sigma can be expanded to a geometric series.

$$\sum_{k=1}^{n} ar^{k-1}$$

where n is the total number of terms in the series, a is the first term, r is the common ratio, and k is the term number.

Sigma can be written in any form that may be expanded, however for condensing a series into sigma notation, the above with the term equation is used.

Lesson Examples:

1) Expand the following:

a) $\sum_{i=1}^{4}(-1)^{i-1}4i$

substitue 1 in for i, then 2, then 3, then 4.

$=(-1)^{1-1}4(1)+(-1)^{2-1}4(2)+(-1)^{3-1}4(3)+(-1)^{4-1}4(4)$

$=(-1)^{0}4+(-1)^{1}8+(-1)^{2}12+(-1)^{3}16$

$=(1)4+(-1)8+(1)12+(-1)16$

$=4-8+12-16$

* to evaluate, simply add the terms of the series together

$\therefore \sum_{i=1}^{4}(-1)^{i-1}4i = \boxed{-8}$

b) $\sum_{k=4}^{7}2(3)^{k+2}$

$=2(3)^{4+2}+2(3)^{5+2}+2(3)^{6+2}+2(3)^{7+2}$

$=2(3)^{6}+2(3)^{7}+2(3)^{8}+2(3)^{9}$

$=2(729)+2(2187)+2(6561)+2(19683)$

$=1458+4374+13122+39366$

1   2   3   4

$k=4$ $k=5$ $k=6$ $k=7$

total # of terms

$7-4+1=4$ terms

c) $\sum_{k=1}^{\infty}\frac{1}{3^k}$

$=\frac{1}{3^1}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\frac{1}{3^5}+\ldots$

$=\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\frac{1}{243}+\ldots$

If evaluate, determine sum $\rightarrow S_\infty=\frac{a}{1-r}$

2) Evaluate each of the above.

This means find the sum since sigma represents a series written in condensed form, you can find the sum by determining the first and second term then finding the value of a, r, and n to calculate the sum.

a) $\sum_{i=1}^{4}(-1)^{i-1}4i$

$=4-8+12-16$

since this is not geometric, simply add the 4 terms

$S_4=-8$

b) $\sum_{k=4}^{7}2(3)^{k+2}$ $a=1458$ $r=\frac{4374}{1458}=3$ $n=4$

$=1458+4374+13122+39366$

$S_4=\frac{a(1-r^n)}{1-r}=\frac{1458(1-3^4)}{1-3}=\frac{1458(1-81)}{-2}=-729(-80)=\boxed{58320}$

c) $\sum_{k=1}^{\infty}\frac{1}{3^k}$ $a=\frac{1}{3}$ $r=\frac{\frac{1}{9}}{\frac{1}{3}}=\frac{1}{9}\times\frac{3}{1}=\frac{1}{3}$ $n=4$

$=\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\frac{1}{243}+\ldots$

since $0<r<1$ use the infinite sum formula

$S_\infty=\frac{a}{1-r}=\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{\frac{1}{3}}{\frac{3}{3}-\frac{1}{3}}=\frac{\frac{1}{3}}{\frac{2}{3}}=\frac{1}{{}_1\cancel{3}}\times\frac{\cancel{3}^1}{2}=\boxed{\frac{1}{2}}$

3) How many terms are there in each of the following:

a) $\sum_{k=1}^{50} 3^{k-1}$

b) $\sum_{k=4}^{11} 3^{k-1}$

1 to 50 is 50 terms.

4 to 11

To calculate it is:

$50-1+1=50$ terms

11 – 4 + 1 = 8 terms

You have to add back the one you subtracted to get the correct number.

4) Convert the following into sigma notation:

a) 3 + 12 + 48 + 192 + 768 + 3072

b) $8 - 2 + \frac{1}{2} - \frac{1}{8} + ...$

Use the formula: $\sum_{k=1}^{n} ar^{k-1}$

this is just the sum notation with the term formula for geometric sequences

$a=3$
$r=4$
$n=6$
$\sum_{k=1}^{6} 3(4)^{k-1}$

$a=8$
$r=\frac{-2}{8}=-\frac{1}{4}$
$n=\infty$
$\sum_{k=1}^{\infty} 8\left(-\frac{1}{4}\right)^{k-1}$

10.2-10.3 Geometric Series

Geometric Series: *An indicated sum of terms in a geometric sequence.*

Example:

Geometric Sequence	VS	Geometric Series
3, 6, 12, 24, 48		3 + 6 + 12 + 24 + 48

Geometric Sequence and Geometric Series

A geometric sequence is a sequence of numbers in which the ratio between consecutive terms is constant.

The formula for the n^{th} term of a geometric sequence is

$$a_n = ar^{n-1}$$

where *a* is the first term and *r* is the ratio

A geometric series results from adding the terms of a geometric sequence.

The formula for the sum of a finite geometric series is

$$S_n = \sum_{i=1}^{n} ar^{i-1} = \frac{a(1-r^n)}{1-r}, r \neq 1$$

where *n* is the number of terms, *a* is the first term and *r* is the ratio

The formula for the sum of an infinite geometric series is

$$S_n = \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}, |r| < 1$$

where *a* is the first term and *r* is the ratio

If $|r| \geq 1$, then the infinite series does not have a sum

Geometric Series

A geometric sequence is a sequence of numbers in which the ratio between consecutive terms is constant.

A geometric series results from adding the terms of a geometric sequence.

The formula for the partial sum of the first n terms of a geometric series is

$$S_n = \sum_{i=1}^{n} ar^{i-1} = \frac{a\left(1-r^n\right)}{1-r}, r \neq 1$$

where a is the first term and r is the ratio

The sum of an infinite geometric series is given by

$$a + ar + ar^2 + \ldots + ar^{n-1} + \ldots = \sum_{i=1}^{\infty} ar^{i-1}$$

where a is the first term and r is the ratio

The sum converges to $\dfrac{a}{1-r}$ if |r|< 1

The sum diverges if |r| ≥ 1

Example #2: Find the sum of the following series.

$7 + 14 + 28 + \ldots$ for 12 terms

Step #1: Identify the variables.

$\frac{14}{7} = 2$ and $\frac{28}{14} = 2$, so $r = 2$.

$a = 7,\ r = 2,\ n = 12$

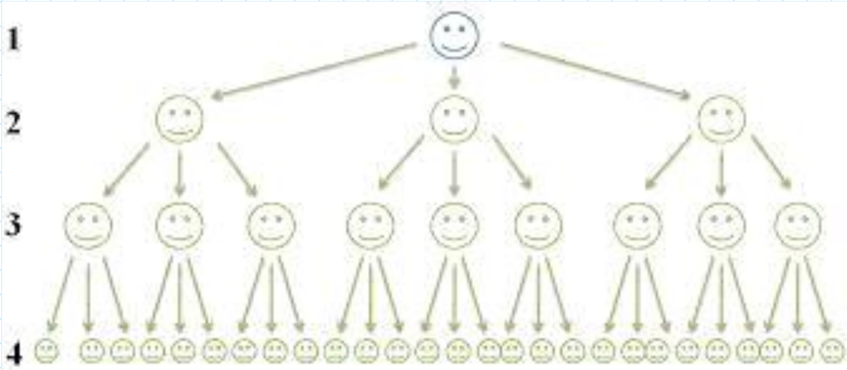
Step #2: Substitute and evaluate.

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

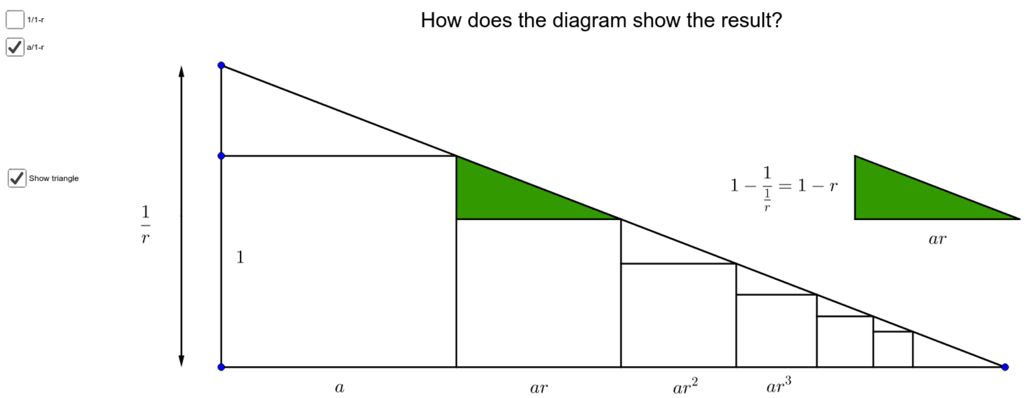
$$S_n = \frac{7(1-2^{12})}{1-2}$$

$$S_n = \frac{7(-4095)}{-1}$$

$$S_n = 28665$$



How does the diagram show the result?



$$\frac{a+ar+ar^2+ar^3+\ldots}{\frac{1}{r}}=\frac{ar}{1-r}$$

Sum to infinity of a geometric series

$$S_\infty = \frac{a_1}{1-r}$$

exists for $-1 < r < 1$

a_1 is the first term r is the ratio

$r = a_n \div a_{n-1}$

2. Find the value of the infinite geometric series $4+2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$

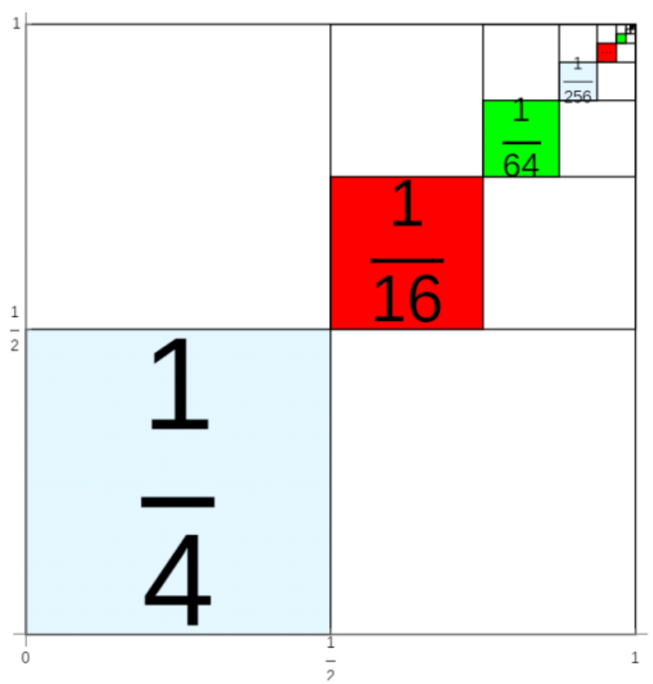
$$a_1 = 4, \quad r = \frac{1}{2}$$

2. Find the value of the infinite geometric series $4+2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$

$$a_1 = 4, \quad r = \frac{1}{2}$$

$$S_\infty = \frac{a_1}{1-r}$$

$$S_\infty = \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8$$



$a = \frac{1}{4}$

Determine the sum of the first 9 terms of the geometric series $2 + 6 + 18 + 54 + \ldots$.

Solution

Think …

To use the method, we need to know the 9th term.

Determine t_9 using $t_n = ar^{n-1}$.

In this series, $a = 2, r = 3$, and $n = 9$

$$t_9 = 2 \times 3^8$$
$$= 13\ 122$$

Let S represent the sum: $S = 2 + 6 + 18 + \ldots + 13\ 122$ ①

Multiply by the common ratio 3: $3S = 6 + 18 + \ldots + 13\ 122 + 39\ 366$ ②

Subtract ① from ②: $2S = -2 + 39\ 366$

$= 39\ 364$

$S = 19\ 682$

The sum of the first 9 terms of the series is 19 682.

Determine the sum of the first 10 terms of each geometric series.

a) $4 + 12 + 36 + 108 + \ldots$

b) $6 + 3 + 1.5 + 0.75 + \ldots$

Solution

Use the formula $S_n = \frac{a(r^n - 1)}{r - 1}$.

a) Substitute $a = 4,\ r = 3,\ n = 10$.

$$S_{10} = \frac{4(3^{10} - 1)}{3 - 1}$$
$$= 2(3^{10} - 1)$$
$$= 118\ 096$$

b) Substitute $a = 6,\ r = 0.5,\ n = 10$.

$$S_{10} = \frac{6(0.5^{10} - 1)}{0.5 - 1}$$
$$= -12(0.5^{10} - 1)$$
$$= 11.988\ 281\ 25$$

Consider the infinite geometric series $4 - \frac{4}{5} + \frac{4}{25} - \dots$.

a) Explain why the series has a sum to infinity.

b) Determine the sum to infinity.

Solution

a) For this series, $a = 4$ and $r = -\frac{1}{5}$ → use S_∞ formula

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{4\left(\left(-\frac{1}{5}\right)^n - 1\right)}{-\frac{1}{5} - 1} \times \frac{-1}{-1}$$

$$= \frac{4\left(1 - \left(-\frac{1}{5}\right)^n\right)}{\frac{6}{5}}$$

$$= \frac{10}{3}\left(1 - \left(-\frac{1}{5}\right)^n\right)$$

extra

The series has a sum to infinity because the expression $\left(-\frac{1}{5}\right)^n$ gets closer to 0 as n gets larger.

b) The sum to infinity is $\frac{10}{3}$. S

Which infinite geometric series has a sum? What is the sum?

a) $4 - 6 + 9 - 13.5 + \dots$

b) $6 + 2 + \frac{2}{3} + \frac{2}{9} + \dots$

Solution

a) $4 - 6 + 9 - 13.5 + \dots$

For this series, $r = -1.5$; since $|r| > 1$, the series has no sum.

b) $6 + 2 + \frac{2}{3} + \frac{2}{9} + \dots$

For this series, $r = \frac{1}{3}$; since $|r| < 1$, the series has a sum. Use the formula.

$$S = \frac{a}{1 - r}$$

$$= \frac{6}{1 - \frac{1}{3}}$$

$$= \frac{6}{\frac{2}{3}}$$

$$= 9$$

The sum of the series is 9.

Your Turn

Determine the sum of the following geometric series.

a) $\frac{1}{64} + \frac{1}{16} + \frac{1}{4} + \cdots + 1024$ **b)** $-2 + 4 - 8 + \cdots - 8192$

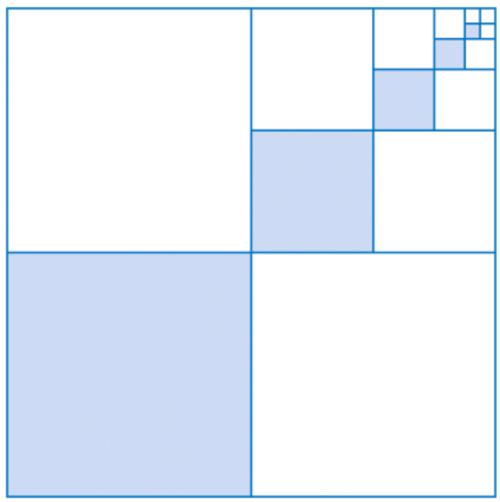
Your Turn

Determine whether each infinite geometric series converges or diverges. Calculate the sum, if it exists.

a) $1 + \frac{1}{5} + \frac{1}{25} + \cdots$ **b)** $4 + 8 + 16 + \cdots$

Apply the Sum of an Infinite Geometric Series

Assume that each shaded square represents $\frac{1}{4}$ of the area of the larger square bordering two of its adjacent sides and that the shading continues indefinitely in the indicated manner.



a) Write the series of terms that would represent this situation.

b) How much of the total area of the largest square is shaded?

Solution