Thursday, Jan. 25th

Plan For Today:

1. Go over Test 1. Any questions?
2. Any questions about material from last class? (Translations & Reflections)
   - Do 2.4 Translations & Reflections Check-in Quiz
3. Finish working through transformations in Chapter 2
   - ✓ 2.0 Graphing Review
   - ✓ 2.4 Horizontal and Vertical Translations
   - ✓ 2.4 Reflections
   - **2.4 Stretches**
   - **2.5 Inverse of a Relation**
   - **2.6 Combining Transformations**

$$f(x) = af(b(x-c)) + d$$

4. Work on practice questions in workbook and practice questions handout.
5. Work on Ch2 Transformations Desmos project.

   Project for Chapter 2 is **online**. Please join my PC12 Jan-Apr2024 Class in Desmos at the link I emailed you with your **FULL NAME** before starting the assignment. Here's a quick link to join the class in Desmos:
   http://tinyurl.com/PC12-Desmos-2024
   Here is a quick link to the first assignment:
   http://tinyurl.com/Jan24-Transformations

Plan Going Forward:

1. Work on practice questions from 2.4-2.6 in the workbook. Work on the Desmos project online.

   - **CH2 TEST ON THURSDAY, FEB. 1ST**
   - **CH2 ONLINE DESMOS PROJECT DUE THURSDAY, FEB. 1ST**

2. We will do some general review from Ch1 and Ch2 on Tuesday after the Ch2 test to prepare for the Unit 1 Exam.

   - **UNIT 1 EXAM ON CH1&2 ON THURSDAY, FEB. 1ST**
     - 10 Multiple Choice & 20 marks on the Written
     - ~1 hour - please prepare so you are not "learning" while doing the test
     - Closed-book - no notes
     - Rewrite is following Tuesday after class at 12:30pm
     - I'll email you when I post marks by Friday or Saturday

3. We will do a short intro to Chapter 3 Polynomials on Tuesday as well and continue with it next Thursday after the Unit 1 Exam.

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.

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2.4 Review Translations + Reflections. ✱ SKIP p.75

p.71 – 74

p.73 Ex: 4 Base Function

= square root function $y = \sqrt{x}$

or $f(x) = \sqrt{x}$

Note: $y = f(x)$ (they are the same)

Translations → $y = f(x-h)+k$.

h = right or left; k = up down.

Ex 2 4 right, 3 down.
+4, −3 → $y = f(x-4) - 3$

Ex 3 $y = f(x)$ → $y = f(x-2)+4$

2 right, up 4.

Ex 4 Recall: table of values are the points on the graph ∴ translation are applied to the points = mapping notation.

base: → transformed p.73

$(x, y) \rightarrow (x+h, y+k)$

Ex: 4 $(2, 2)$ on $y = f(x)$ (Base Function) → $y = f(x+3) - 2$ (transformed function.)

3 left, 2 down

$(2,2) \rightarrow (2-3, 2-2)$ → $\boxed{(-1, 0)}$

Reflections
p.74

Base Function → Reflections

$y = f(x)$ → $y = -f(x)$ ⇒ reflection over x-axis b/c multiply y-coordinate by −1

$(x, -y)$

→ $y = f(-x)$ ⇒ reflection over the y-axis b/c multiply x-coordinate by −1

$(-x, y)$

Ex 5: $y = x^2 + x$

a) reflect in x-axis.

∴ $y = -f(x)$ ✱ (−) in front of whole function → $y = -(x^2+x)$

$\boxed{y = -x^2 - x}$

or $-y = f(x)$ ✱ replace y with −y →

$\frac{-y}{-1} = \frac{x^2 + x}{-1}$

$\boxed{y = -x^2 - x}$

b) reflection in y-axis.

$y = f(-x)$ ✱ replace x → $y = x^2 + x$

b) reflection in y-axis.

$y = f(-x)$ ★ replace x with $-x$ → $y = x^2 + x$

↓

$y = (-x)^2 + (-x)$

$\boxed{y = x^2 - x}$

Ex: 6 $y = x^2 + 2x$ to $y = -(x^2 + 2x)$

since (−) in front of function as $y = -f(x)$ = reflection over/in x-axis

extra: Base

$y = \frac{1}{2}(x+1)^3 + 3$ → reflection in x-axis

NOT $y = -\frac{1}{2}(x+1)^3 + 3$

CORRECT: $y = -\left[\frac{1}{2}(x+1)^3 + 3\right]$

$\boxed{y = -\frac{1}{2}(x+1)^3 - 3}$

→ reflection in y-axis.

$y = \frac{1}{2}(-x+1)^3 + 3$ ✓ (factor)

$\boxed{y = \frac{1}{2}(-(x-1))^3 + 3}$

Ex 7 (3,2) is on $y = f(x)$ (Base)

a) $y = -f(x)$ → (3,−2)

b) $y = f(-x)$ → (−3,2)

c) $y = -f(-x)$ → (−3,−2)

#1, 2, 9, 10

2.4 Stretches.

↳ compressions + expansions.

Base Function → Transformed Function

$y = f(x)$ → $y = af(bx)$

a = vertical stretch by a factor of "a" = multiply y-coordinate by a = (x, ay)

b = horizontal stretch by a factor of "$\frac{1}{b}$" = multiply x-coord. by $\frac{1}{b}$ = $(\frac{1}{b}x, y)$ or $(\frac{x}{b}, y)$

Mapping

(x, y) → $(\frac{1}{b}x, ay)$

p.77 Ex 8 Base function $y = \sqrt{x}$

a) vert. exp. by 2 (VE of 2)

$y = 2\sqrt{x}$

b) vert. compression by $\frac{1}{2}$ (VC of $\frac{1}{2}$)

$y = \frac{1}{2}\sqrt{x}$

c) horiz. expansion by 2 (HE of 2)

$y = \sqrt{\frac{1}{2}x}$

★ reciprocal + replace ...

d) horiz. compression of $\frac{1}{2}$ (HC of $\frac{1}{2}$)

$y = \sqrt{2x}$

★ reciprocal of $\frac{1}{2}$ is 2 replace x ...

★ reciprocal & replace x with $\frac{1}{2}x$

★ reciprocal of $\frac{1}{2}$ is 2, replace x with $2x$

Ex: 9 $\quad y = f(x) \longrightarrow y = 3f(\frac{1}{4}x)$

Describe Transformations: VE of 3; $\frac{1}{4} \xrightarrow{\text{reciprocal}} \frac{4}{1} = 4$, HE of 4

Ex 10 $\quad (3,1)$ on $y = f(x) \rightarrow y = 2(4x)$

VE of 2 $\quad$ HC of $\frac{1}{4}$

★recall: $(x, y) \longrightarrow (\frac{1}{b}x, ay)$

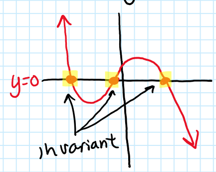
$(3,1) \longrightarrow (\frac{3}{4}, 2)$

$3(\frac{1}{4})$

Invariant Points: points which stay the same after a transformation.
p.78

∴ when $y = f(x) \longrightarrow y = af(x)$

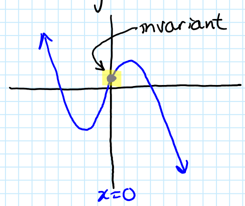
(x, ay)



∴ when $y = 0$ on the x-axis, the points are invariant.

∴ when $y = f(x) \longrightarrow y = f(bx)$

$(\frac{1}{b}x, y)$



∴ when $x = 0$ on the y-axis, the points are invariant

IF $y = af(bx) \longrightarrow$ only invariant point is $(0,0)$ (where both x + y are zero)

Practice #3-8 $\quad$ skip #11-17
#18, 19

Try 2.6 Combining #1, 2a,b,c,e,g,h, 6,7

2.6 Combining

↳ order for transformations.

Base $\quad y = f(x) \longrightarrow$ Transformed $\quad y = af(b(x-h)) + k$

- a = vert stretch
- b = horiz stretch by $\frac{1}{b}$
- h = right / left
- k = up / down

$(x, y) \longrightarrow (\frac{1}{b}x + h, ay + k)$

ORDER
1. stretch + reflections
2. translations

p.95 Ex 3 $y=f(x) \longrightarrow y=-4f(6-3x)+1$

(factor + reorder $b(x-h)$)

$y=-4f(-3x+6)+1$

$y=-4f(-3(x-2))+1$

- VE by 4, ref. in x-axis
- HC by $\frac{1}{3}$, ref. in y-axis
- 2 right
- 1 up

① Stretches + Reflections

② Translations

$(3,2) \rightarrow \left(-\frac{1}{3}(3), -4(2)\right)$

$(-1, -8) \rightarrow (-1+2, -8+1)$

$= \boxed{(1,-7)}$

2.4 Practice

p83 # 18 c)

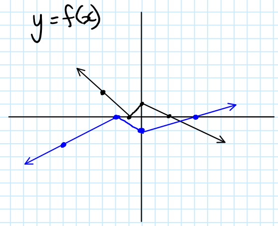
$y=-f\left(\frac{x}{2}\right)$

$y=-f\left(\frac{1}{2}x\right)$

- ref. in x-axis
- HE by 2

$(2x, -y)$

$y=f(x)$



Base $y=f(x)$

x	y	2x	-y
-3	2	-6	-2
-1	0	-2	0
0	1	0	-1
2	0	4	0

(×2, ×-1)

2.4 Compressions/Expansions

Vertical and Horizontal Stretches

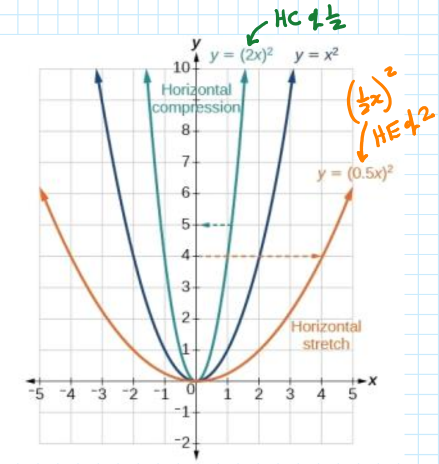
stretch

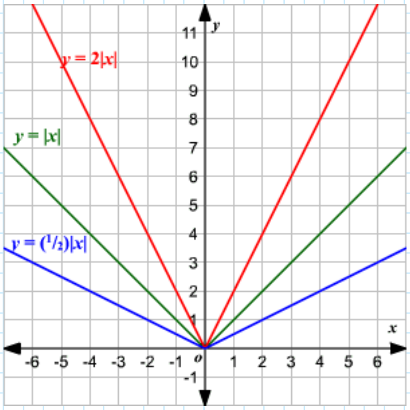
- a transformation in which the distance of each x-coordinate or y-coordinate from the line of reflection is multiplied by some scale factor
- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

$$y = f(x) \longrightarrow y = af(bx)$$





Transformation Rules for Functions		
Function Notation	Type of Transformation	Change to Coordinate Point
f(x) + d	Vertical translation up d units	(x, y) → (x, y + d)
f(x) − d	Vertical translation down d units	(x, y) → (x, y − d)
f(x + c)	Horizontal translation left c units	(x, y) → (x − c, y)
f(x − c)	Horizontal translation right c units	(x, y) → (x + c, y)
−f(x)	Reflection over x-axis	(x, y) → (x, −y)
f(−x)	Reflection over y-axis	(x, y) → (−x, y)
af(x)	Vertical stretch for |a| > 1 Vertical compression for 0 < |a| < 1	(x, y) → (x, ay)
f(bx)	Horizontal compression for |b| > 1 Horizontal stretch for 0 < |b| < 1	$(x, y) \rightarrow \left(\frac{x}{b}, y\right)$

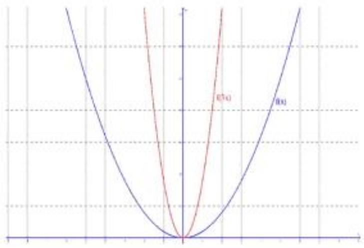
Horizontally Compressed

Example 1

The values and graph of the function $f(x)$ are shown in blue.
Make a table and a graph of the function $g(x) = f(3x)$.

Solution

x	f(x)	x	g(x)
-6	36	-2	36
-3	9	-1	9
-1	1	-1/3	1
0	0	0	0
1	1	1/3	1
3	9	1	9
6	36	2	36



2

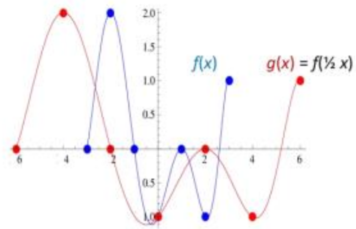
Horizontally Stretched

Example 1

The values and graph of the function $f(x)$ are shown in blue.
Make a table and a graph of the function $g(x) = f(\frac{1}{2}x)$.

Solution

x	f(x)	x	g(x)
-3	0	-6	0
-2	2	-4	2
-1	0	-2	0
0	-1	0	-1
1	0	2	0
2	-1	4	-1
3	1	6	1



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Vertical Stretches $y - k = af(x - h)$

In general, for any function $y = f(x)$, the graph of the function $y = a f(x)$ has been vertically stretched about the x-axis by a factor of $|a|$.

The point $(x, y) \rightarrow (x, ay)$. Only the y coordinates are affected.

Invariant points are on the line of stretch, the x-axis.
are the x-intercepts.

When $|a| > 1$, the points on the graph move farther away from the x-axis. $y = 3f(x)$ Vertical stretch by a factor of 3

When $|a| < 1$, the points on the graph move closer to the x-axis. $y = \frac{1}{3}f(x)$ Vertical stretch by a factor of ⅓

Vertical Stretching $y = af(x)$, $|a| > 1$

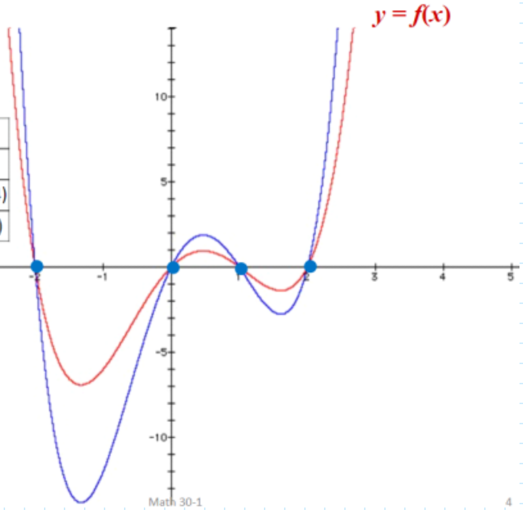
$y = 2f(x)$

A vertical stretch about the x-axis by a factor of 2.

$y = f(x)$

Key Points

(x, y)	→	(x, 2y)
(-2, 0)	→	(-2, 0)
(-1, -7)	→	(-1, -14)
(1/2, 1)	→	(1/2, 2)



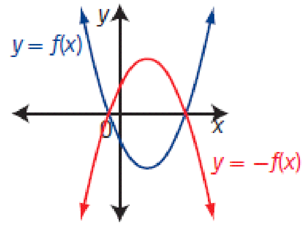
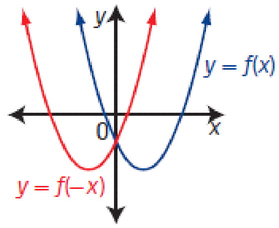
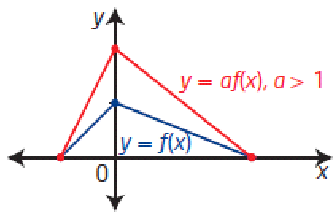
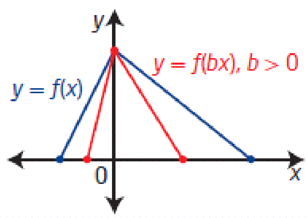
Invariant Points

(-2, 0)
(0, 0)
(1, 0)
(2, 0)

4

Key Ideas

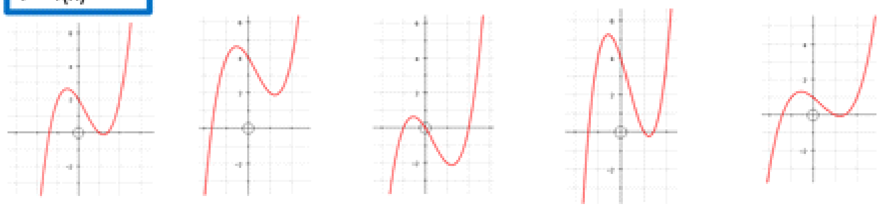
- Any point on a line of reflection is an invariant point.

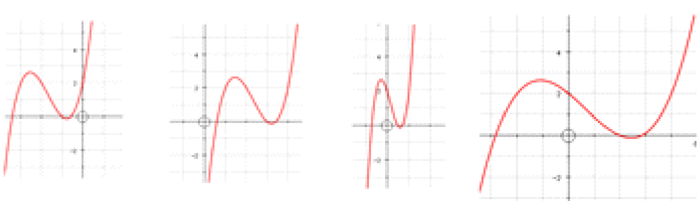
Function	Transformation from $y = f(x)$	Mapping	Example
$y = -f(x)$	A reflection in the x-axis	$(x, y) \rightarrow (x, -y)$	
$y = f(-x)$	A reflection in the y-axis	$(x, y) \rightarrow (-x, y)$	
$y = af(x)$	A vertical stretch about the x-axis by a factor of $\|a\|$; if $a < 0$, then the graph is also reflected in the x-axis	$(x, y) \rightarrow (x, ay)$	
$y = f(bx)$	A horizontal stretch about the y-axis by a factor of $\frac{1}{\|b\|}$; if $b < 0$, then the graph is also reflected in the y-axis	$(x, y) \rightarrow \left(\frac{x}{b}, y\right)$	

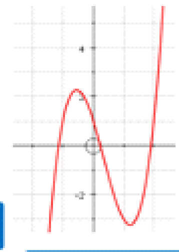
Transformations of Graphs

Using your knowledge of transformations of graphs match up the transformations of the function with the graph. The first one is done for you.

Y = f(x)







Y = f(x) + 2

Y = 2f(x)

Y = f(x+2)

Y = f(2x)

Y = 2f(x) – 3

This doubles in size and then moves down 3

Y = f(x) - 2

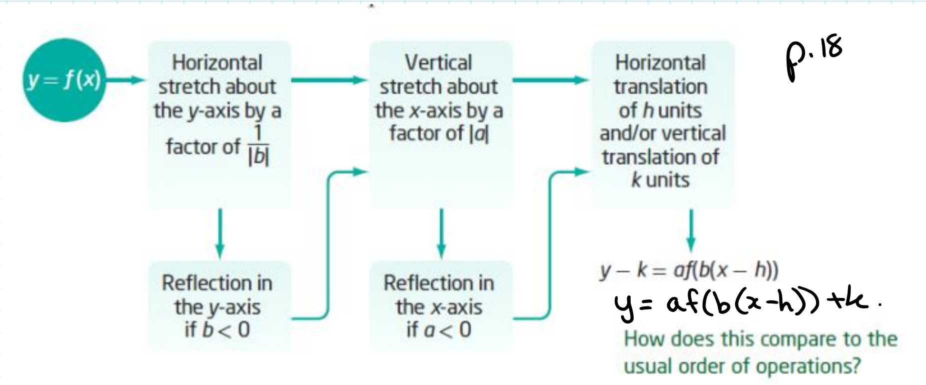
Y = ½ f(x)

Y = f(x-2)

Y = f(½ x)

Extension: The original graph has a peak at (-0.5, 2.5) Write the new location of this peak after the transformations for each graph. How has the peak moved and why has this happened?

2.6 Combining Transformations



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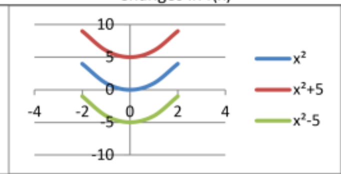
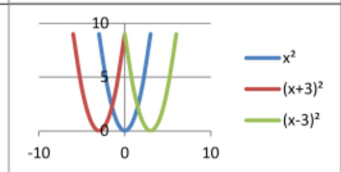
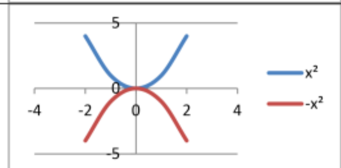
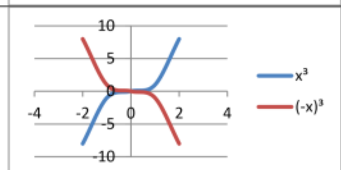
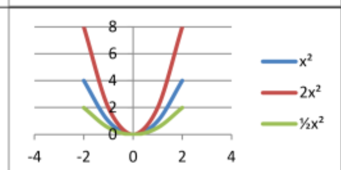
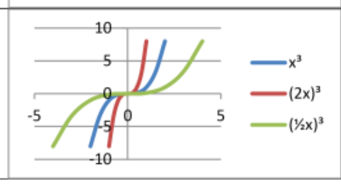
af(x)	Vertical stretch for |a| > 1	(x, y) → (x, ay)
	Vertical compression for 0 < |a| < 1	
f(bx)	Horizontal compression for |b| > 1	$(x, y) \rightarrow \left(\frac{x}{b}, y\right)$
	Horizontal stretch for 0 < |b| < 1	

Summary: standard form, mapping notation and order of performing transformations

Summary of Transformations

Graph	Draw the graph of f(x) and:	Changes in f(x)
Vertical shift $y = f(x) + c$ $y = f(x) - c$	Raise the graph of f(x) by c units -add c to y coordinate Lower the graph of f(x) by c units -subtract c from y coordinate	
Horizontal shift $y = f(x + c)$ $y = f(x - c)$	Shift the graph f(x) to the left c units -subtract c from x coordinate Shift the graph f(x) to the right c units -add c to x coordinate	
Reflection about the x-axis $y = -f(x)$	Reflect the graph of f(x) about the x-axis -multiply each y coordinate by -1	
Reflection about the y-axis $y = f(-x)$	Reflect the graph of f(x) about the y-axis -multiply each x coordinate by -1	
Vertical stretching and compression $y = cf(x), c > 1$ $y = cf(x), 0 < c < 1$	Vertically stretching the graph of f(x) $(c > 1)$ Vertically compressing the graph of f(x) $(0 < c < 1)$ -multiply each y coordinate by c	
Horizontal stretching and compression $y = f(cx), c > 1$ $y = f(cx), 0 < c < 1$	Horizontally compressing the graph of f(x) $(c > 1)$ Horizontally stretching the graph of f(x) $(0 < c < 1)$ -divide each x coordinate by c	
$y = \frac{1}{f(x)}$	Take the reciprocal of each y coordinate of f(x)	
Order of operations for transformations: 1) horizontal shifts 2) stretches/compressions 3) reflections 4) vertical shifts		

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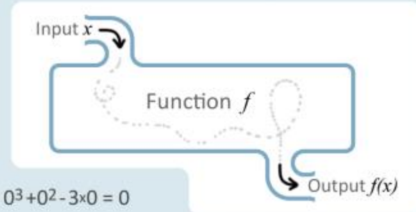
Functions & Graph Transformations

What is a Function?

A function describes a relation between two (or more) values. Each input value has one output.

For example: let $f(x) = x^3 + x^2 - 3x$

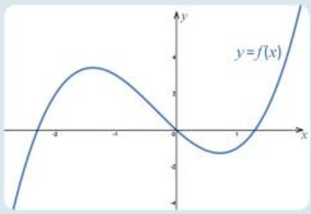
$f(2) = 2^3 + 2^2 - 3 \times 2 = 6, \quad f(0) = 0^3 + 0^2 - 3 \times 0 = 0$



A function is usually called f but can, like a variable, be called any letter or symbol.

Graphing a Function

A function is graphed the same as the "y = ..." equations you're used to, we just use "$f(x)$ = ..." instead!



Transformations

Transformations cause functions to change in some way. Constants are used to either **translate** or **stretch** a function's graph.

Translate: a shift, the graph moves:

up & down		left & right
$f(x)+a$	or	$f(x+b)$

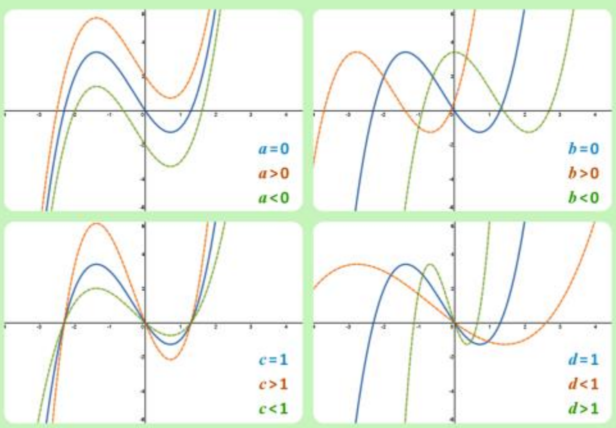
Stretch: (or squeeze) parallel to:

y axis		x axis
$cf(x)$	or	$f(dx)$

Top tip

If the constant is **inside** the function, in with the x, then it causes transformations in the direction of the x axis.

If it is **outside** the function, operating on $f(x)$, then it causes changes in the direction of the y axis.



$f(x)+a$

Shifts the graph **up for $a>0$** **down for $a<0$**

Think of moving the graph a units up the y axis

$f(x+b)$

Shifts the graph **left for $b>0$** **right for $b<0$**

Think of moving the graph "$-b$" units along the x axis

$cf(x)$

Stretches the graph parallel to the y axis **out for $c > 1$** (squeeze) **in for $c < 1$**

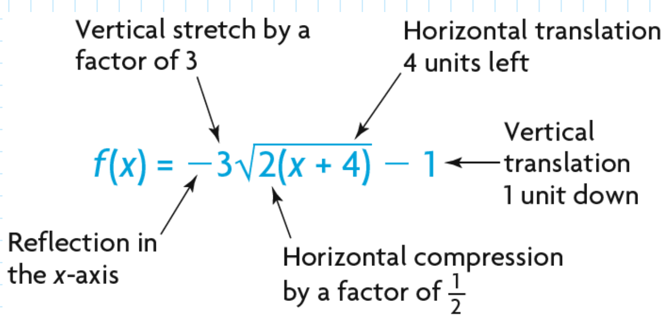
The scale factor is c, think of points being c times further from y=0

$f(dx)$

Stretches the graph parallel to the x axis **out for $d < 1$** **in for $d > 1$**

The scale factor is $1/d$, think of points being d times as close to x=0

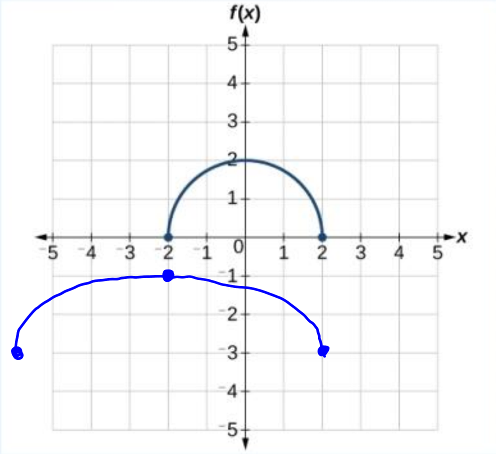
These bottom two graphs only use positive constants. **Exercise:** can you sketch what they'd look like with negative constants?



Use the graph of $f(x)$ to sketch a graph of $k(x) = f\left(\frac{1}{2}x + 1\right) - 3$. → factored form → $k(x) = f\left(\frac{1}{2}(x+2)\right) - 3$

① HE of 2

② 2 left, 3 down



$y = f(x)$

x	y
-2	0
0	2
2	0

$y = k(x)$

①

$2x$	y
-4	0
0	2
4	0

②

$2x-2$	$y-3$
-6	-3
-2	-1
2	-3