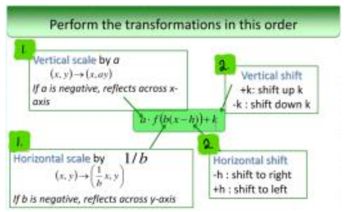
Tuesday, Jan. 30th

Plan For Today:

1. Go over Test 1. Any questions?
2. Any questions about material from last class? (Translations & Reflections)
   - **Do 2.4 Check-in Quiz**

$$f(x) = af(b(x-c)) + d$$

3. Finish working through transformations in Chapter 2
   - ✓ 2.0 Graphing Review
   - ✓ 2.4 Horizontal and Vertical Translations
   - ✓ 2.4 Reflections
   - ✓ 2.4 Stretches
   - **2.6 Combining Transformations**
   - **2.5 Inverse of a Relation**



4. Work on practice questions in workbook and practice questions handout.
5. Work on Ch2 Transformations Desmos project.

   Project for Chapter 2 is **online**. Please join my PC12 Jan-Apr2024 Class in Desmos at the link I emailed you with your **FULL NAME** before starting the assignment. Here's a quick link to join the class in Desmos:
   http://tinyurl.com/PC12-Desmos-2024
   Here is a quick link to the first assignment:
   http://tinyurl.com/Jan24-Transformations
6. Start Chapter 3: Polynomial Functions
   - **3.1: Characteristics of Polynomial Functions**
   - 3.2: The Remainder Theorem
   - 3.3: The Factor Theorem
   - 3.4: Equations & Graphs of Polynomials Functions
7. Work on Practice Questions

Plan Going Forward:

1. Work on practice questions from 2.5-2.6 in the workbook. Finish the Desmos project online.

   - **CH2 TEST ON THURSDAY, FEB. 1ST**
   - **CH2 ONLINE DESMOS PROJECT DUE THURSDAY, FEB. 1ST**

2. We will do some general review from Ch1 and Ch2 on Tuesday after the Ch2 test to prepare for the Unit 1 Exam.

   - **UNIT 1 EXAM ON CH1&2 ON TUESDAY, FEB. 6TH**
     - 10 Multiple Choice & 20 marks on the Written
     - ~1 hour - please prepare so you are not "learning" while doing the test
     - Closed-book - no notes
     - Rewrite is following Tuesday after class at 12:30pm

- I will go over the marked exam on Thursday

3. We will continue Chapter 3 Polynomials on Thursday after the Ch1 Test. Work on 3.1 questions in the workbook.

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.
Anurita Dhiman = adhiman@sd35.bc.ca

f) The shape $f(x) = |x|$ moved 3 units to the left and reflected in the x-axis.

$-y \rightarrow y = -f(x)$

$f(x) = -|x+3|$

$y = f(x)$ general

$y = af(b(x-h)) + k$ factored.

h) $y = -f(x+2)$ → Describe.
ref. in x-axis
2 left.

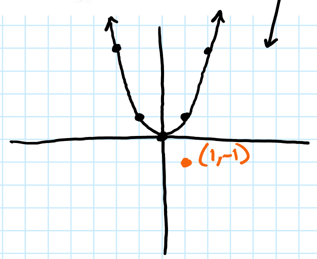
$-y$ or $-f(x)$

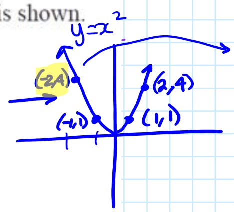
$(-3, 1)$ $x-1$ -2

$(-5, -1)$

(a, b)
↓
$(a-2, -b)$ Mapping

4. Use the graph of $f(x) = x^2$ to write an equation for each function whose graph is shown.





$(-2, 4)$ ×2 ×½

$(-4, 2)$

HE of 2
VC of ½

$y = \frac{1}{2}\left(\frac{1}{2}x\right)^2 = y = \frac{1}{8}x^2$

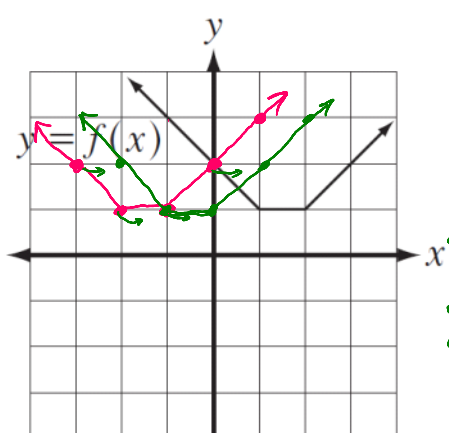
$y = x^2 \rightarrow y = -x^2$ $y = -f(x)$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9
4	16

$4y \rightarrow$ vert. exp. of 4.

$y = -4x^2$

$y = -(2x)^2$ HC of ½



$\{x \mid x \in R\}$

$\{y \mid y \geq 1, y \in R\}$

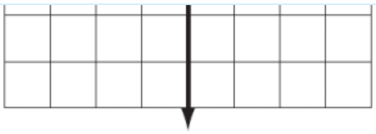
$y = f(1-x)$

$f(b(x-h))$

$y = f(-x+1)$

factor

$y = f(-(x-1))$



$y = f(-(x-1))$

ref. over y-axis $(-x)$ — 1 unit right $x+1$

x	y
-1	3
0	2
1	1
2	1
3	2

→

-x	y
1	3
0	2
-1	1
-2	1
-3	2

+1 →

-x+1	y
2	3
1	2
0	1
-1	1
-2	2

2.6 Combing

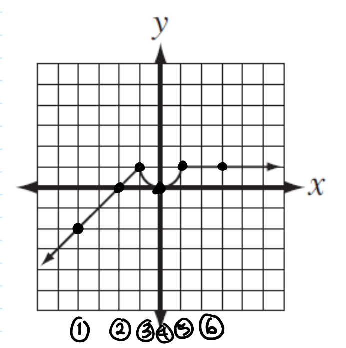
$y = a f(b(x-h)) + k$

$a \to ay$, $b \to \frac{1}{b}x$ — ① ; $h \to (\frac{1}{b}x) \pm h$, $k \to (ay) \pm k$ — ②

① Stretches + reflections
② translations.

$\frac{-1}{-\frac{1}{2}} \to -1 \times -\frac{2}{1} = 2$

p.99 #7



y=f(x)

	x	y
①	-4	-2
②	-2	0
③	-1	1
④	0	0
⑤	1	1
⑥	3	1

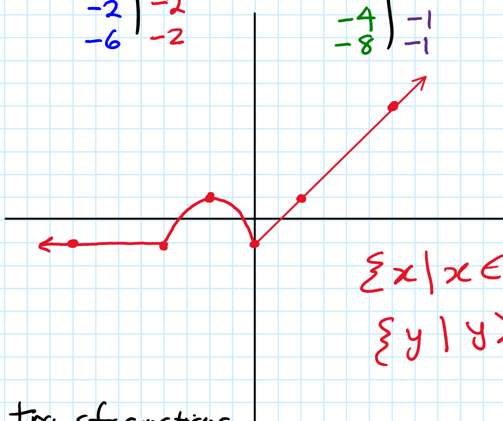
$y = -2f\left(-\frac{1}{2}x - 1\right) + 1$

$y = -2f\left(-\frac{1}{2}(x+2)\right) + 1$ — factored transformation form

-2y = ref in x-axis + VE of 2; -2x = ref. in y-axis HE of 2; 2 left $(-2x) - 2$; 1 up $(-2y) + 1$

-2x	-2y
8	4
4	0
2	-2
0	0
-2	-2
-6	-2

-2x-2	-2y+1
6	5
2	1
0	-1
-2	1
-4	-1
-8	-1



$\{x \mid x \in R\}$
$\{y \mid y \geq -1, y \in R\}$

p.98 #6

d) $f(x) = -\frac{1}{4}(x+2)^3 + 1$

① Determine base function → $y = x^3$

③ Describe transformations.

$y = -\frac{1}{4}(x+2)^3 + 1$

① base function $y = x^3$

③ Describe transformations.

$y = -\frac{1}{4}(x+2)^3 + 1$

- ref. in x-axis, VC of $\frac{1}{4}$: $-\frac{1}{4}y$
- 2 left: $x-2$
- 1 up: $(-\frac{1}{4}y)+1$

② Graph/table of values of base

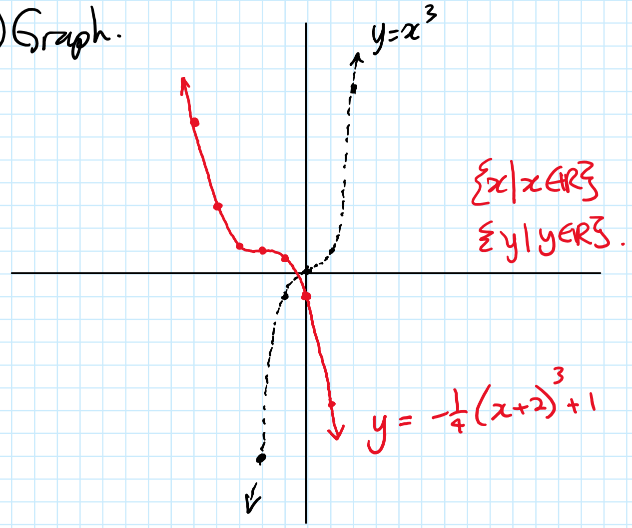
x	y
−3	−27
−2	−8
−1	−1
0	0
1	1
2	8
3	27

④ transform points.

x	$-\frac{1}{4}y$
−3	27/4 (6¾ = 6.75)
−2	2
−1	1/4
0	0
1	−1/4
2	−2
3	−27/4 (−6¾ = −6.75)

$x-2$	$(-\frac{1}{4}y)+1$	+1
−5	31/4 (7.75)	$\frac{27}{4}+\frac{4}{4}$
−4	3	$\frac{1}{4}+\frac{4}{4}$
−3	5/4 (1.25)	
−2	1	
−1	3/4 (0.75)	$-\frac{1}{4}+\frac{4}{4}=$
0	−1	
1	$-\frac{23}{4}$ (−5.75)	$-\frac{27}{4}+\frac{4}{4}$

⑤ Graph.



2.5 Inverse Functions.

Base Function → Inverse Function

$y = f(x)$ → $y = f^{-1}(x)$

(x, y) → (y, x)

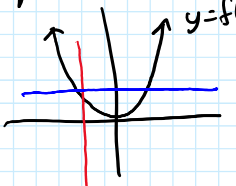
switch/swap x and y for equation + each coordinate

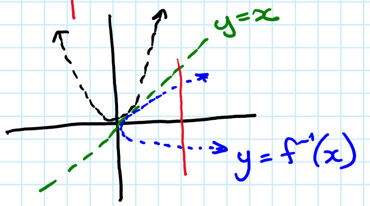
☆ domain + range also inverse

☆ if the original function passes the vertical line test, you can do a horizontal line test to see if the inverse is/isn't a function.

2.5 #1 p.90.

a)





SKIP Checks → #2 p. 90 → if by substitution the equation simplifies to $\boxed{x}$ then they are inverses.

SKIP #3 (restrictions)

#4 Equations.

f) $f(x) = \frac{2x-1}{3x+2}$ → $f^{-1}(x)$

(y)

① switch x + y

$$x = \frac{2y-1}{3y+2}$$

② solve for y

$$x(3y+2) = 2y-1$$

$$3xy + 2x = 2y - 1$$

($-2x$, $-2y$)

$$3xy - 2y = -2x - 1$$

common factor y

$$\frac{y(3x-2)}{(3x-2)} = \frac{-2x-1}{(3x-2)}$$

$$y = \frac{-2x-1}{3x-2}$$

$$\boxed{f^{-1}(x) = \frac{-2x-1}{3x-2}}$$

$$\boxed{= \frac{-2x-1}{3x-2}}$$

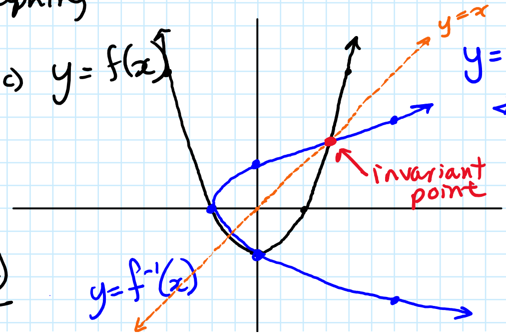
−0.5

$\boxed{y = \square}$

SKIP #5, 7, 9-12.

Graphing

#6c) y = f(x)



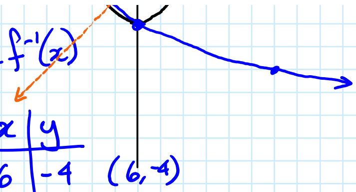
* switch each (x, y) coordinate to (y, x)

y=f(x)

$y=f(x)$

x	y
-4	6
-2	0
0	-2
2	0
4	6

$y=f^{-1}(x)$

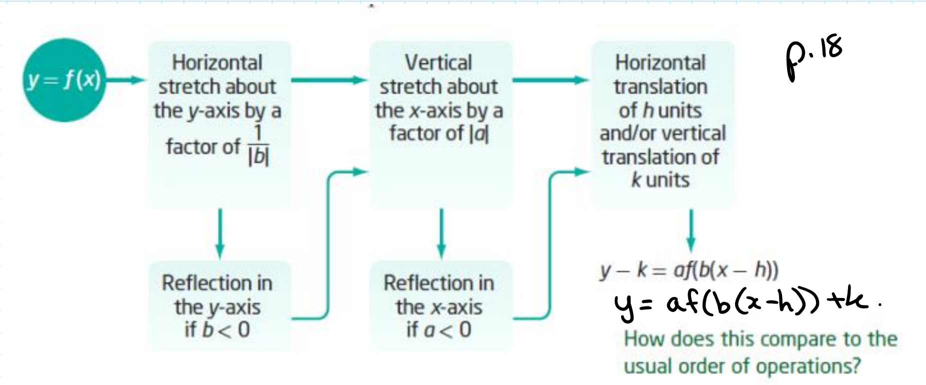


x	y
6	-4
0	-2
-2	0
0	2
6	4

$\{x \mid x \in R\}$ → $\{x \mid x \geq -2, x \in R\}$

$\{y \mid y \geq -2, y \in R\}$ → $\{y \mid y \in R\}$.

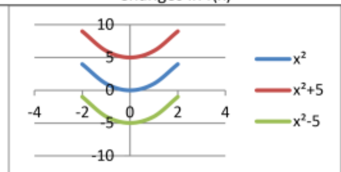
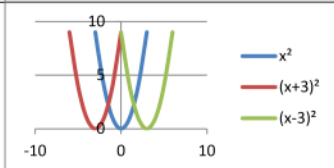
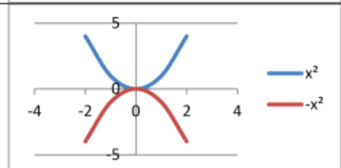
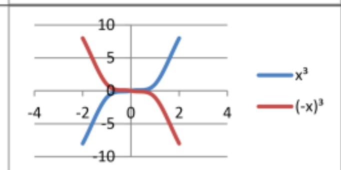
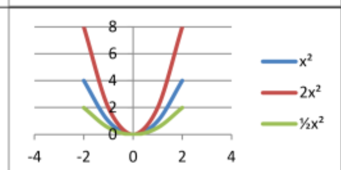
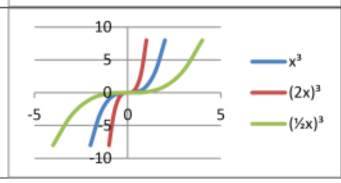
2.6 Combining Transformations



Transformation Rules for Functions		
Function Notation	**Type of Transformation**	**Change to Coordinate Point**
f(x) + d	Vertical translation up d units	(x, y) → (x, y + d)
f(x) − d	Vertical translation down d units	(x, y) → (x, y − d)
f(x + c)	Horizontal translation left c units	(x, y) → (x − c, y)
f(x − c)	Horizontal translation right c units	(x, y) → (x + c, y)
−f(x)	Reflection over x-axis	(x, y) → (x, −y)
f(−x)	Reflection over y-axis	(x, y) → (−x, y)
af(x)	Vertical stretch for |a|> 1	(x, y) → (x, ay)
	Vertical compression for 0 < |a| < 1	
f(bx)	Horizontal compression for |b| > 1	$(x, y) \rightarrow \left(\frac{x}{b}, y\right)$
	Horizontal stretch for 0 < |b| < 1	

Summary: standard form, mapping notation and order of performing transformations

Summary of Transformations

Graph	Draw the graph of f(x) and:	Changes in f(x)
Vertical shift $y = f(x) + c$ $y = f(x) - c$	Raise the graph of f(x) by c units -add c to y coordinate Lower the graph of f(x) by c units -subtract c from y coordinate	
Horizontal shift $y = f(x + c)$ $y = f(x - c)$	Shift the graph f(x) to the left c units -subtract c from x coordinate Shift the graph f(x) to the right c units -add c to x coordinate	
Reflection about the x-axis $y = -f(x)$	Reflect the graph of f(x) about the x-axis -multiply each y coordinate by -1	
Reflection about the y-axis $y = f(-x)$	Reflect the graph of f(x) about the y-axis -multiply each x coordinate by -1	
Vertical stretching and compression $y = cf(x), c > 1$ $y = cf(x), 0 < c < 1$	Vertically stretching the graph of f(x) $(c > 1)$ Vertically compressing the graph of f(x) $(0 < c < 1)$ -multiply each y coordinate by c	
Horizontal stretching and compression $y = f(cx), c > 1$ $y = f(cx), 0 < c < 1$	Horizontally compressing the graph of f(x) $(c > 1)$ Horizontally stretching the graph of f(x) $(0 < c < 1)$ -divide each x coordinate by c	
$y = \frac{1}{f(x)}$	Take the reciprocal of each y coordinate of f(x)	
Order of operations for transformations: 1) horizontal shifts 2) stretches/compressions 3) reflections 4) vertical shifts		

March 2017

MVCC Learning Commons Math Lab

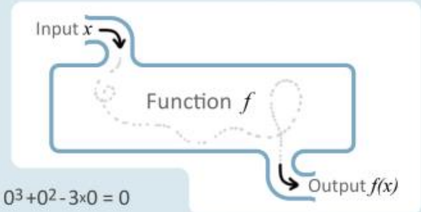
Functions & Graph Transformations

What is a Function?

A function describes a relation between two (or more) values. Each input value has one output.

For example: let $f(x) = x^3 + x^2 - 3x$

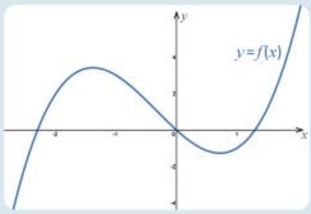
$f(2) = 2^3 + 2^2 - 3{\times}2 = 6$, $\quad f(0) = 0^3 + 0^2 - 3{\times}0 = 0$



A function is usually called f but can, like a variable, be called any letter or symbol.

Graphing a Function

A function is graphed the same as the "y = ..." equations you're used to, we just use "$f(x)$ = ..." instead!



Transformations

Transformations cause functions to change in some way. Constants are used to either **translate** or **stretch** a function's graph.

Translate: a shift, the graph moves:

up & down $f(x)+a$ or left & right $f(x+b)$

Stretch: (or squeeze) parallel to:

y axis $cf(x)$ or x axis $f(dx)$

Top tip

If the constant is **inside** the function, in with the x, then it causes transformations in the direction of the x axis.

If it is **outside** the function, operating on $f(x)$, then it causes changes in the direction of the y axis.

$f(x)+a$

Shifts the graph **up for $a>0$** **down for $a<0$**

Think of moving the graph a units up the y axis

$f(x+b)$

Shifts the graph **left for $b>0$** **right for $b<0$**

Think of moving the graph "$-b$" units along the x axis

$cf(x)$

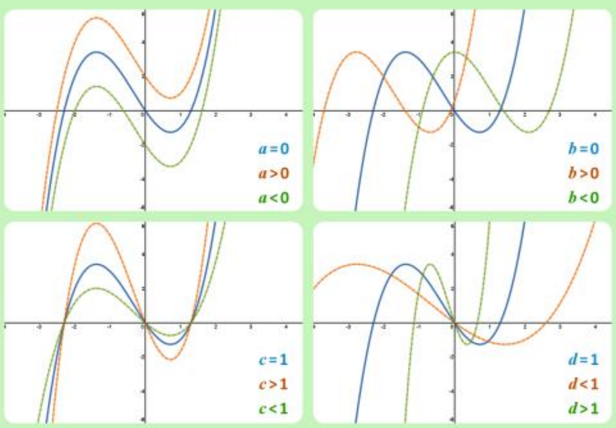
Stretches the graph parallel to the y axis **out for $c>1$** (squeeze) **in for $c<1$**

The scale factor is c, think of points being c times further from $y=0$

$f(dx)$

Stretches the graph parallel to the x axis **out for $d<1$** **in for $d>1$**

The scale factor is $1/d$, think of points being d times as close to $x=0$

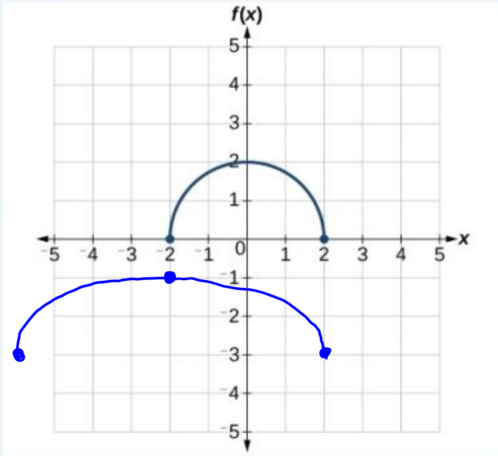


These bottom two graphs only use positive constants. **Exercise:** can you sketch what they'd look like with negative constants?

$$f(x) = -3\sqrt{2(x+4)} - 1$$

- Reflection in the x-axis: $-$
- Vertical stretch by a factor of 3: 3
- Horizontal compression by a factor of $\frac{1}{2}$: 2
- Horizontal translation 4 units left: $+4$
- Vertical translation 1 unit down: -1

Use the graph of $f(x)$ to sketch a graph of $k(x) = f\left(\frac{1}{2}x + 1\right) - 3$. → factored form → $k(x) = f\left(\frac{1}{2}(x+2)\right) - 3$



① HE of 2

② 2 left, 3 down

$y = f(x)$

x	y
-2	0
0	2
2	0

$y = k(x)$

①

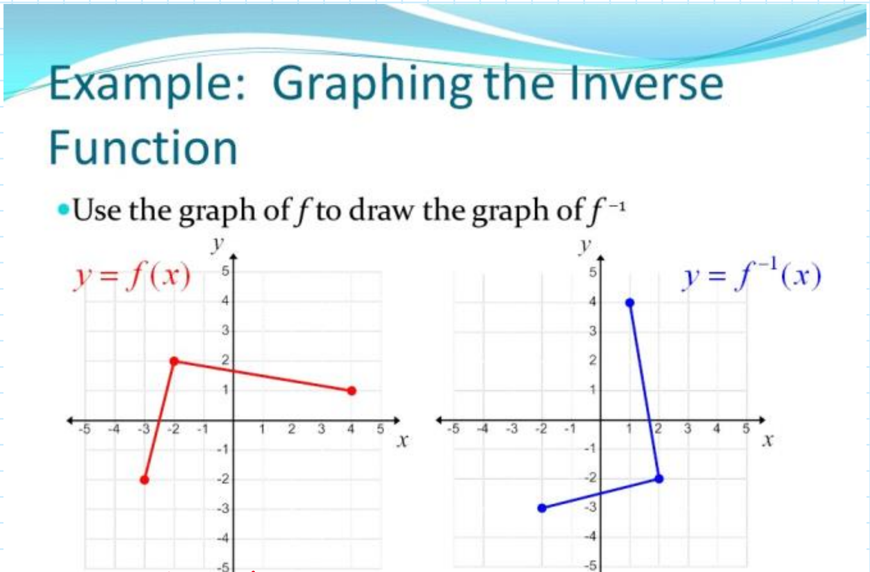
2x	y
-4	0
0	2
4	0

②

2x−2	y−3
-6	-3
-2	-1
2	-3

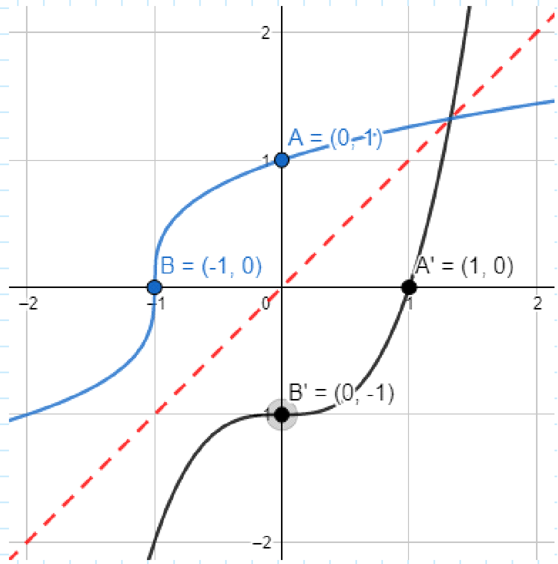
2.5 Inverse

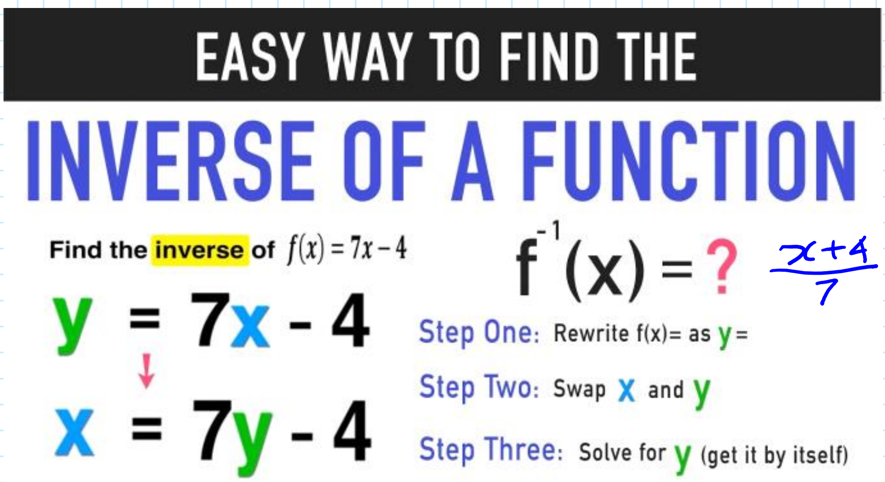
$$f^{-1}(x)$$



$-3 \leq x \leq 4$
$-2 \leq y \leq 2$

$-2 \leq x \leq 2$
$-3 \leq y \leq 4$

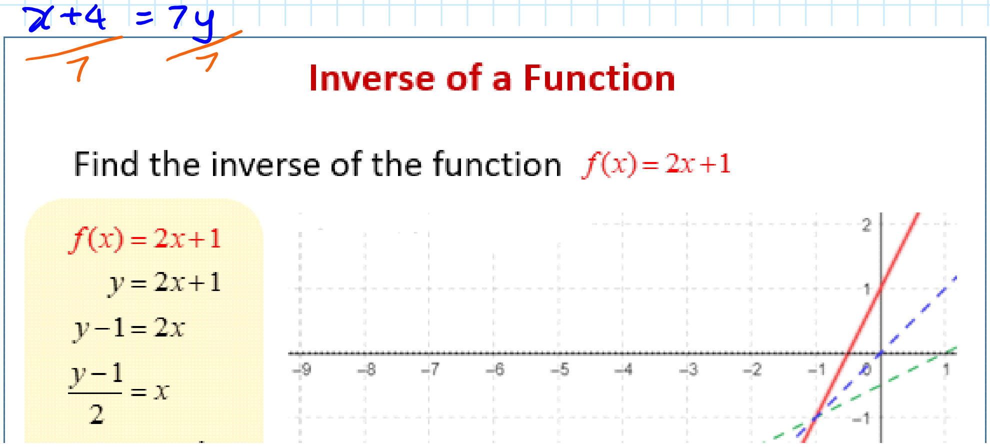




$\frac{x+4}{7} \rightarrow f^{-1}(x) = \frac{1}{7}(x+4)$

$+4$

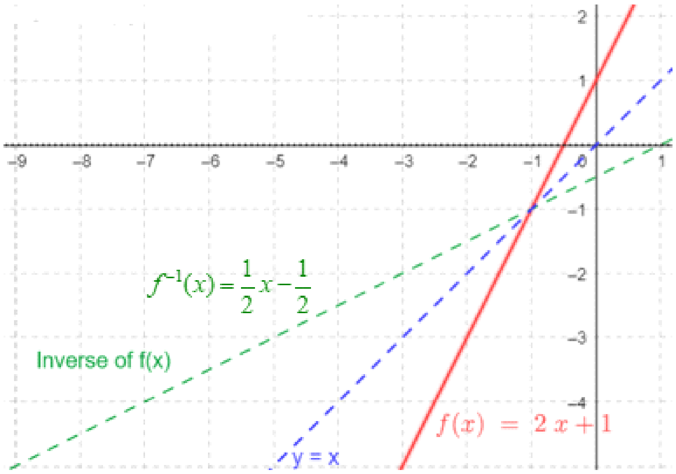
$\frac{x+4}{7} = \frac{7y}{7}$



Inverse of a Function

Find the inverse of the function $f(x)=2x+1$

$$f(x)=2x+1$$
$$y=2x+1$$
$$y-1=2x$$
$$\frac{y-1}{2}=x$$
$$y=\frac{x-1}{2}$$
$$y=\frac{1}{2}x-\frac{1}{2}$$
$$f^{-1}(x)=\frac{1}{2}x-\frac{1}{2}$$



$f(x)$ and $f^{-1}(x)$ are mirror images about the line $y=x$

Original Function	Inverse Function
$f(x)$	$f^{-1}(x)$
Domain: $x \geq 5$	**Domain:** $x \leq 0$
Range: $y \leq 0$	**Range:** $y \geq 5$

Find the Inverse of a Function

1. Replace f(x) with y
2. Interchange x and y
3. Solve the equation for y
4. Replace y with $f^{-1}(x)$

Example:

Given $f(x)=\frac{4x+2}{5}$ find the inverse of $f(x)$

$f(x)=\frac{4x+2}{5}$

$y=\frac{4x+2}{5}$ Replace f(x) with y

$x=\frac{4y+2}{5}$ Interchange x and y

$5x=4y+2$

$5x-2=4y$ Solve the equation for y

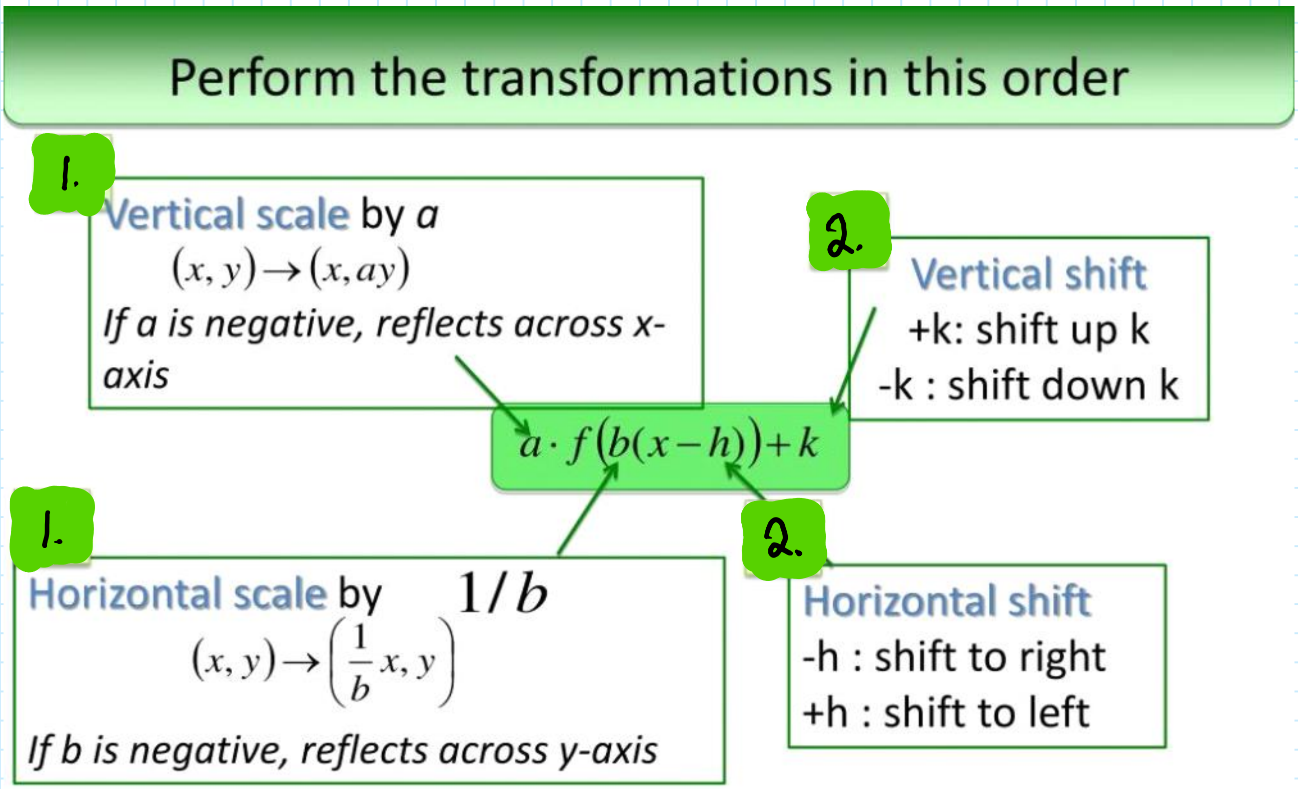
$y=\frac{5x-2}{4}$

$f^{-1}(x)=\frac{5x-2}{4}$ Replace y with $f^{-1}(x)$

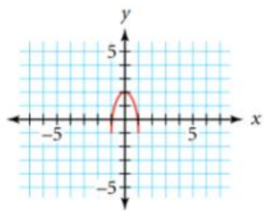
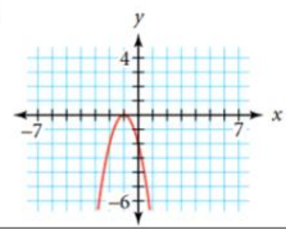
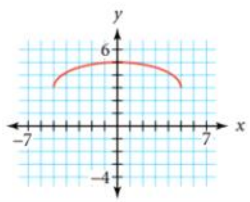
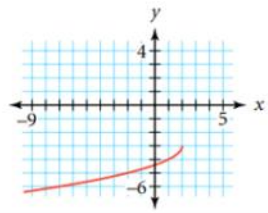
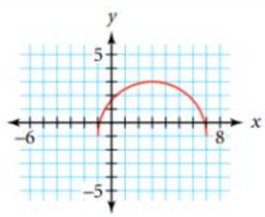
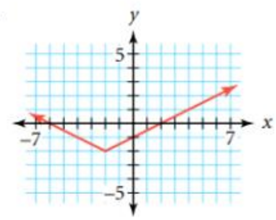
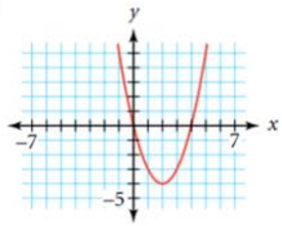
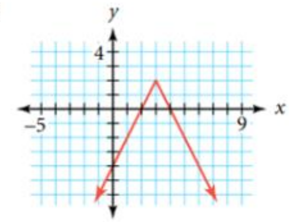
Summary & Practice

Key Ideas

- Write the function in the form $y = af(b(x - h)) + k$ to better identify the transformations.
- Stretches and reflections may be performed in any order before translations.
- The parameters a, b, h, and k in the function $y = af(b(x - h)) + k$ correspond to the following transformations:
  - a corresponds to a vertical stretch about the x-axis by a factor of $|a|$. If $a < 0$, then the function is reflected in the x-axis.
  - b corresponds to a horizontal stretch about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the function is reflected in the y-axis.
  - h corresponds to a horizontal translation.
  - k corresponds to a vertical translation.



Try this matching:

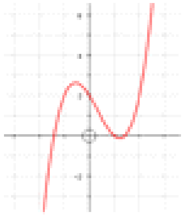
a. 	1. $y = (x-2)^2 - 4$ 2. $y = -\sqrt{-(x-2)} - 3$	e. 
b. 	3. $y = 0.5\lvert x+2 \rvert - 2$ 4. $y = -2(x+1)^2$	f. 
c. 	5. $y = 3\sqrt{1-x^2} - 1$ 6. $y = 4\sqrt{1-\left(\frac{x-3}{4}\right)^2} - 1$	g. 
d. 	7. $y = -2\lvert x-3 \rvert + 2$ 8. $y = 2\sqrt{1-\left(\frac{x}{5}\right)^2} + 3$	h. 

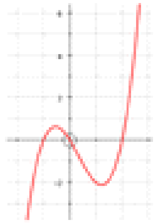
Answers : 1d, 2f, 3g, 4e, 5a, 6c, 7h, 8b

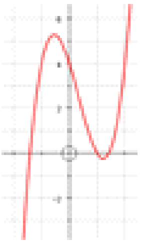
Transformations of Graphs

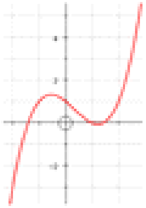
Using your knowledge of transformations of graphs match up the transformations of the function with the graph. The first one is done for you.

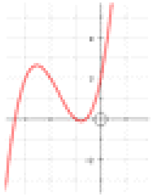
Y = f(x)

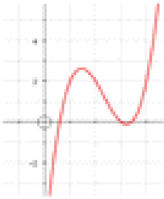


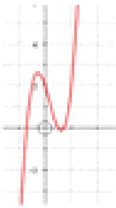


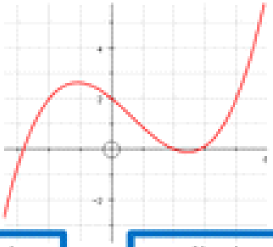


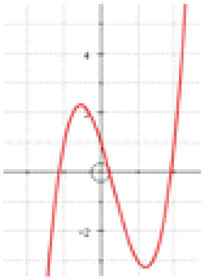












Y = f(x) + 2

Y = 2f(x)

Y = f(x+2)

Y = f(2x)

Y = f(x) - 2

Y = ½ f(x)

Y = f(x-2)

Y = f(½ x)

Y = 2f(x) – 3

This doubles in size and then moves down 3

Extension: The original graph has a peak at (-0.5, 2.5) Write the new location of this peak after the transformations for each graph. How has the peak moved and why has this happened?