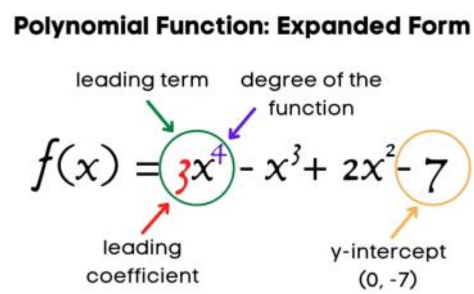
Thursday, Feb. 1st

Plan For Today:

1. Questions from Chapter 2 or the Ch2 Project?
   - Do Chapter 2 Test
   - Note: Chapter 2 Project Due TODAY - please complete by tonight at the latest.
2. Any review from Ch1?
3. Start Chapter 3: Polynomial Functions
   - **3.1: Characteristics of Polynomial Functions**
   - **3.2: Equations & Graphs of Polynomials Functions**
   - 3.3: The Remainder Theorem
   - 3.4: Factoring Review & The Factor Theorem
   - 3.5: Applications & Word Problems
4. Work on practice questions from Workbook



Plan Going Forward:

1. I will hand back the Chapter 2 marked test at the start of next class.
2. Review for the Unit 1 Exam. Do Chapter reviews for 1 and 2. Re-do all practice questions from handouts.

- **UNIT 1 EXAM ON CH1&2 ON TUESDAY, FEB. 6TH**
  - 10 Multiple Choice & 20 marks on the Written
  - ~1 hour - please prepare so you are not "learning" while doing the test
  - Closed-book - no notes
  - Rewrite is following Tuesday after class at 12:30pm
  - I will go over the marked exam on Thursday

3. We will continue Chapter 3 Polynomials on Tuesday after the U1 Test. Work on 3.1-3.2 questions in the workbook.

- **NO CHECK-IN QUIZ ON TUESDAY, FEB. 6TH**

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class. Anurita Dhiman = adhiman@sd35.bc.ca

3.1 Polynomials p.113

$$a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0 x^0$$

constant term = y-intercept $(0, a_0)$

$x^0 = 1$

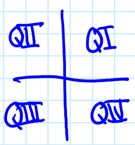
a = leading coefficient = the coefficient in front of term with highest degree.

n = degree of function (highest exponent)

ex: $x^2 - 5x + 10x^4 + 3x^3$

$a = 10$ $\quad n = 4$ (degree = 4)

Summary: refer to chart



EVEN DEGREES

ex: x^2, x^4, x^6

+a (+ leading coefficient)

end behaviour = up into QII + QI

−a → end behaviour = down into QIII + QIV

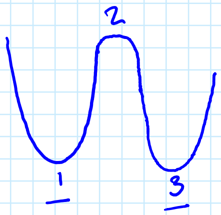
x-intercepts: 0 to n

turning points: 1, 3, 5, ... up to n−1

$\{x \mid x \in \mathbb{R}\}$

+a: $\{y \mid y \geq \text{minimum}, y \in \mathbb{R}\}$.

−a: $\{y \mid y \leq \text{maximum}, y \in \mathbb{R}\}$.



ODD DEGREES.

ex: x, x^3, x^5

+a → end behaviour = down into QIII + up into QI

−a → end behaviour = up into QII + down into QIV

x-intercepts: 1 to n

turning points: 0, 2, 4, ... up to n−1

$\{x \mid x \in \mathbb{R}\}$

$\{y \mid y \in \mathbb{R}\}$

Note: x-intercepts = zeros = roots = solutions

Multiplicity = how often an x-intercept is counted.

p.118 = depends on shape of graph.

ex: if a graph has a solution (x-intercept) repeat 'r' number of times, it has a multiplicity of "r".

ex: b) $g(x) = x^5 + x^4 - 2x^3 - 2x^2 + x + 1$

review: degree = 5 ($n=5$)

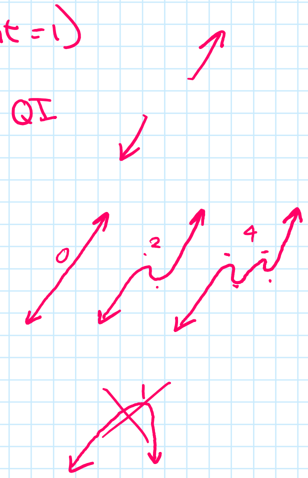
$a = 1$ (leading coefficient = 1)

(+)

behaviour: down into QIII up into QI

possible # of x-intercepts: 1 – 5

possible # of turning points = 0, 2, 4



D: $\{x \mid x \in \mathbb{R}\}$.

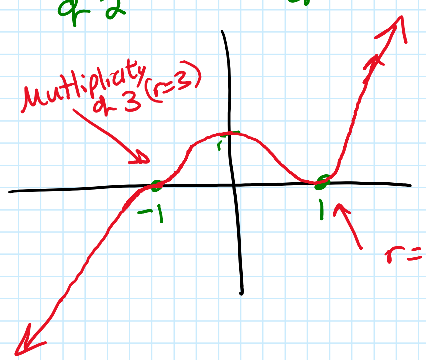
R: $\{y \mid y \in \mathbb{R}\}$.

y-int: (0,1)

$g(x) = (x-1)^2 (x+1)^3$

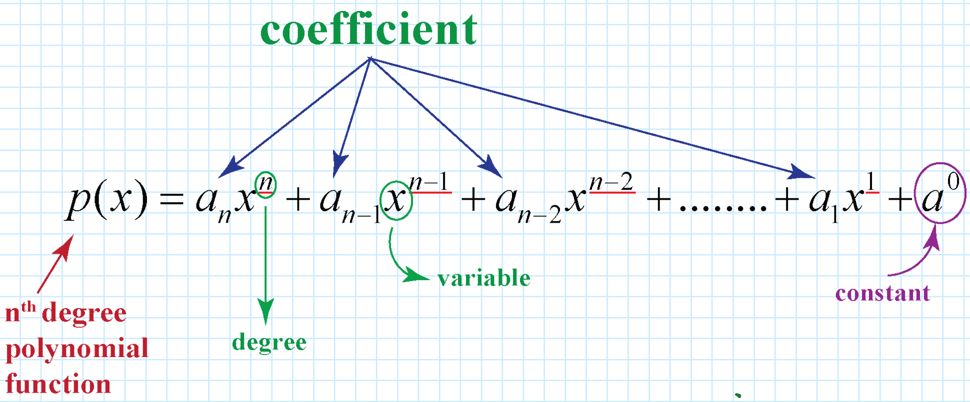
solutions = $x = 1$ $x = -1$ ← x-intercepts.

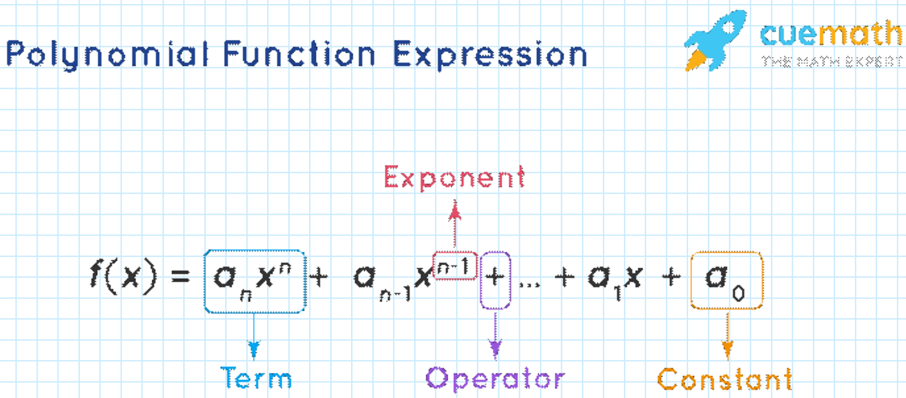
multiplicity of 2 multiplicity of 3.

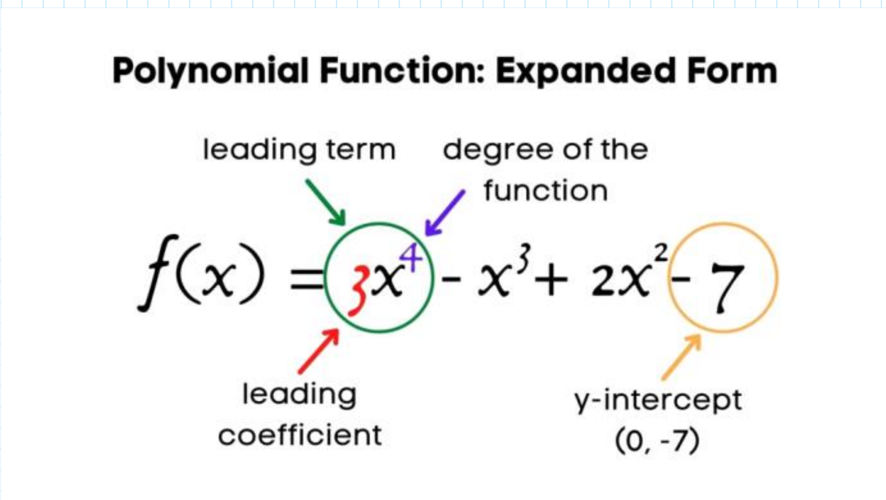


3.1 What's a Polynomial?

What is a polynomial function?







monomial	$\underset{1}{\underline{2x}}$
binomial	$\underset{1}{\underline{2x}} + \underset{2}{\underline{3y}}$
trinomial	$\underset{1}{\underline{2x^2}} + \underset{2}{\underline{3x}} + \underset{3}{\underline{5}}$
polynomial	$\underset{1}{\underline{3x^3}} + \underset{2}{\underline{2x^2}} - \underset{3}{\underline{6x}} + \underset{4}{\underline{2}}$

Non-Examples of Polynomials

Remember... these are NOT polynomials!

- $\frac{a^2}{b^3}$ Fractions, Division
- Square Roots $\sqrt{x^2 + 2x + 2}$
- 9^{2x} Variables as the exponent
- Negatives as the exponent $8x^{-3} = \frac{8}{x^3}$

POLYNOMIAL OR NOT?

Shade each polynomial. If it is not a polynomial, explain why.

① $x^1 + x^2 + x^3 + \ldots$	$x^{1/2} + 2$ NOT ⑦
② $x^4 - \frac{1}{8}x$	$4x^{-2} + 2x - 3$ ⑧
③ $3/x$ NOT	12 ⑨
④ $9\sqrt{x} + 2x$ NOT	$5x + 1$ ⑩
⑤ $x^{4/2}$	$x^3 + x^4 + x^5$ ⑪
⑥ $5 - \frac{4}{x^2}$ NOT	$5 - x^{11}$ ⑫

1, 2, 5, 9, 10, 11, 12

Types of Polynomial

Types of Polynomial (Number of Terms)

Monomials (one term)	Binomials (two terms)	Trinomials (three terms)	Polynomials (many terms)
6	$6x+2$	$3x^2-5x+8$	$2x^3-6x^2-5x+8$
$4x^3$	ab^4-5	a^3+4y-7	$2a^3+3y^2+4y-8a-7$
$-5a^2b^3$	$y+2f$	$\frac{w}{2}-2s+t$	$\frac{w}{2}-2s+t+9$

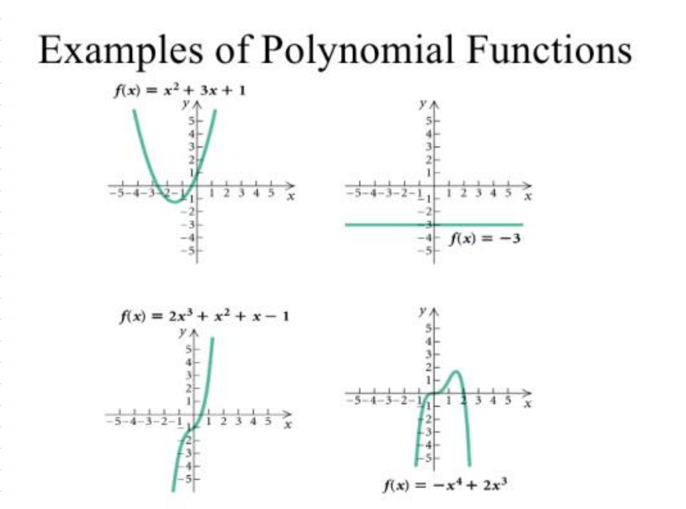
Types of Polynomial (Degree)

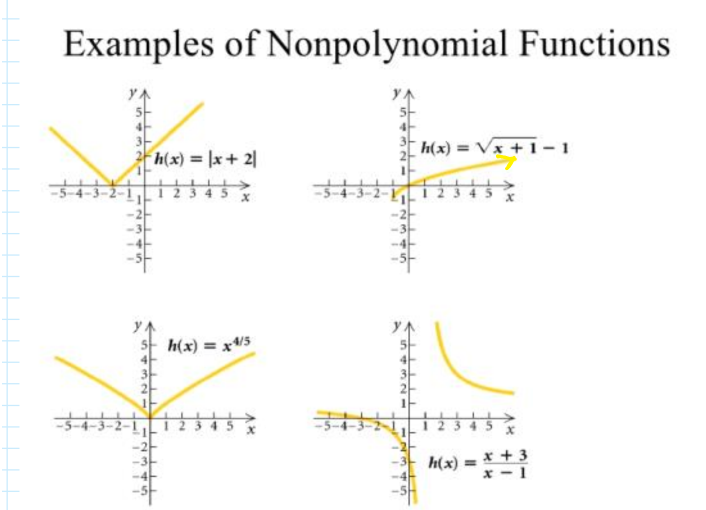
Constant Polynomial (Degree 0)	Linear Polynomial (Degree 1)	Quadratic Polynomial (Degree 2)	Cubic Polynomial (Degree 3)
8	$x+8$	$3x^2-2x+7$	$5x^3$
$-\frac{2}{3}$	$\frac{3}{4}x-6$	$5y^2-\frac{1}{4}$	$2y^3-y+4$

$2x^3 + 8x^2 - 17x - 3$

TERM	$2x^3$	$8x^2$	$-17x$	-3
DEGREE OF TERM	3	2	1	0
DEGREE OF POLYNOMIAL	3			
LEADING TERM	$2x^3$			
LEADING COEFFICIENT	2			
CONSTANT TERM	-3			

3.2 Characteristics of Polynomial Functions



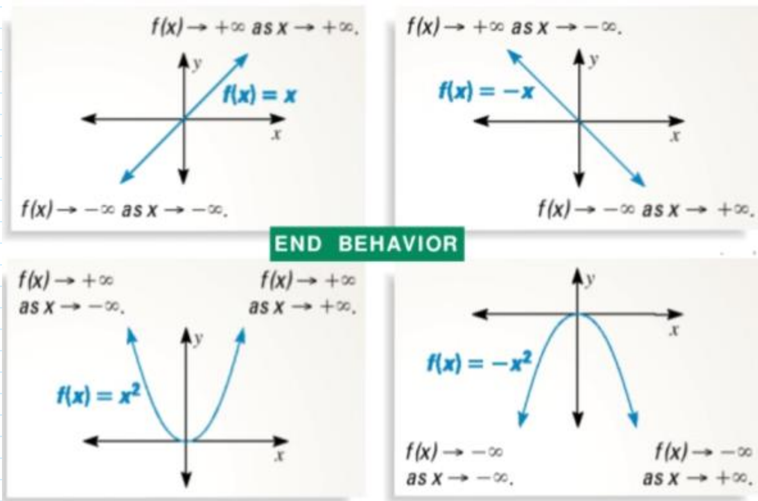


If "n" is even, the graph of the polynomial is "U-shaped" meaning it is parabolic (the higher the degree, the more curves the graph will have in it).

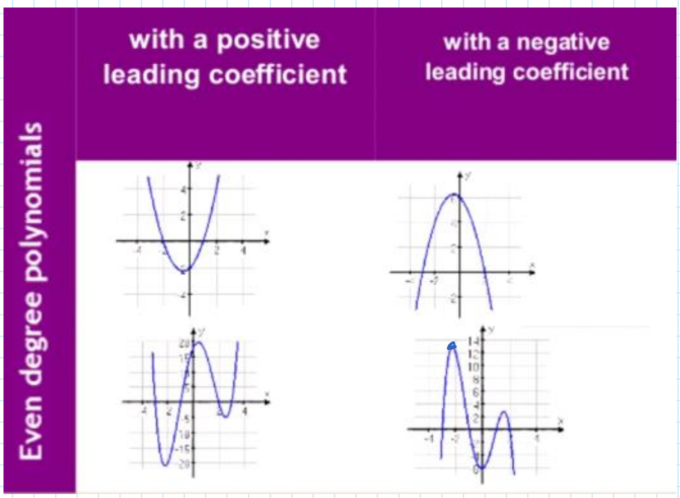
If "n" is odd, the graph of the polynomial is "snake-like" meaning looks like a snake (the higher the degree, the more curves the graph will have in it).

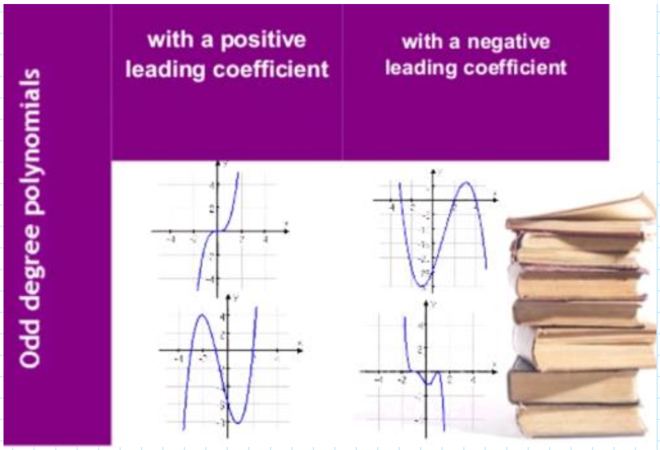
https://www.slideshare.net/morrobea/63-evaluatingandgraphingpolynomilafunctions

The **end behavior** of a polynomial function's graph is the behavior of the graph as x approaches infinity ($+\infty$) or negative infinity ($-\infty$). The expression $x \longrightarrow +\infty$ is read as "x approaches positive infinity."



Degree is odd		Degree is even	
Leading coefficient is positive	Start low, End high	Leading coefficient is positive	Start high, End high
Leading coefficient is negative	Start high, End low	Leading coefficient is negative	Start low, End low





Degree and Leading coefficient tells us the behaviour of the graph.

$y = -x^4 + 2x^3 - x^2 + 3x + 20$ Negative even

$y = -10x^5 - 3x^3 + 2x - 4$ Negative odd

$y = (x+5)(x-3)(2x+4)$ Positive odd

$x(x)(2x) = 2x^3$

$y = x(x-4)^2(3x+1)$ Positive even

$x(x)^2(3x) = 3x^4$

$y = x^2(3-x)(x+4)^3$ Negative even

$x^2(-x)(x)^3 = -x^6$

Determine the left and right behavior of the graph of each polynomial function.

$f(x) = x^4 + 2x^2 - 3x$

Even, Leading coefficient 1 (positive) , starts high ends high

$f(x) = -x^5 + 3x^4 - x$

Odd, Leading coefficient 1 (negative) , starts high ends low

$f(x) = 2x^3 - 3x^2 + 5$

ODD , Leading coefficient 2 (positive) , starts LOW ends HIGH

Degree tells us the number of possible x-intercepts (roots or zeros)

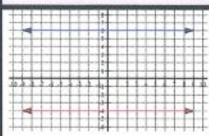
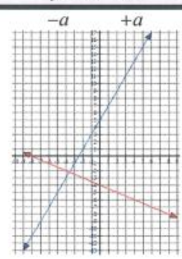
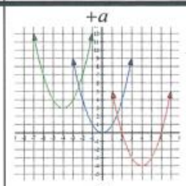
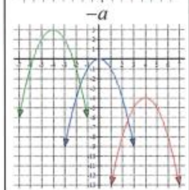
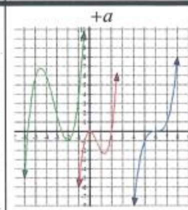
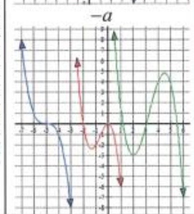
Degree tells us the possible number of turns in the graph

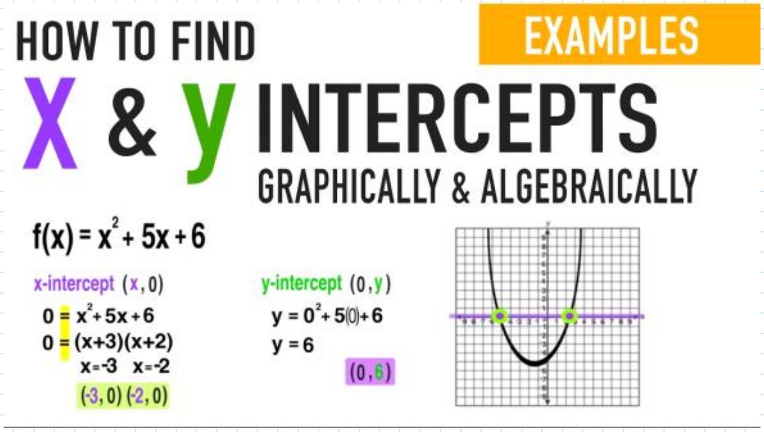
Degree tells us the number of possible x-intercepts (roots or zeros)
Degree tells us the possible number of turns in the graph

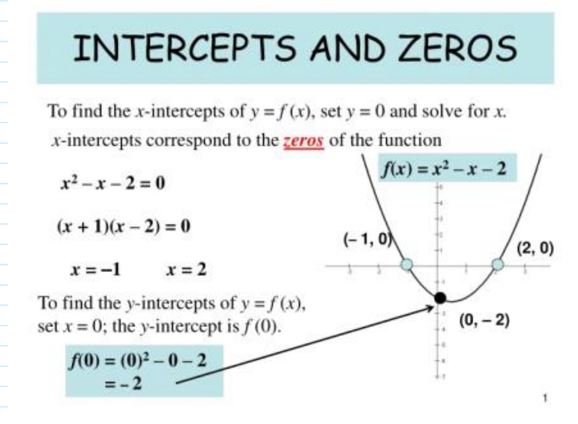
$$\underset{x-intercept}{\overset{(\mathbf{x}, 0)}{\uparrow}} \quad \underset{y-intercept}{\overset{(0, \mathbf{y})}{\uparrow}}$$

https://slideplayer.com/slide/9143093/

3.1 Characteristics of Polynomials Summary Sheet

Type of Function	Constant $y = x^0 + C$	Linear $y = ax + C \rightarrow y = ax^1 + C$	Quadratic $y = ax^2 + bx + C$	Cubic $y = ax^3 + bx^2 + cx + C$
			 	 
Degree, *n*	0, zero	1	2	3
End Behaviour	If constant is positive → Quadrant II to I If constant is negative → Quadrant III to IV	If leading coefficient is positive → Quadrant III to I If leading coefficient is negative → Quadrant II to IV	If leading coefficient is positive → Quadrant II to I If leading coefficient is negative → Quadrant III to IV	If leading coefficient is positive → Quadrant III to I If leading coefficient is negative → Quadrant II to IV
Number of *x*-intercepts	0 unless the constant functions lies on the y-axis, then all points are the *x*-intercepts	1	0,1, or 2	1,2, or 3
Number of *y*-intercepts	1	1	1	1
Number of turning points	0	0	1	0 or 2
Domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
Range	$\{y \mid y = C, y \in R\}$	$\{y \mid y \in R\}$	$\{y \mid y \geq vertex, y \in R\}$ or $\{y \mid y \leq vertex, y \in R\}$	$\{y \mid y \in R\}$





Example: How many roots will the following functions have?

MIN – MAX

$y = -x^{4} + 2x^3 - x^2 + 3x + 20$ — 4 roots — 0–4

$y = -10x^{5} - 3x^3 + 2x - 4$ — 5 roots — 1–5

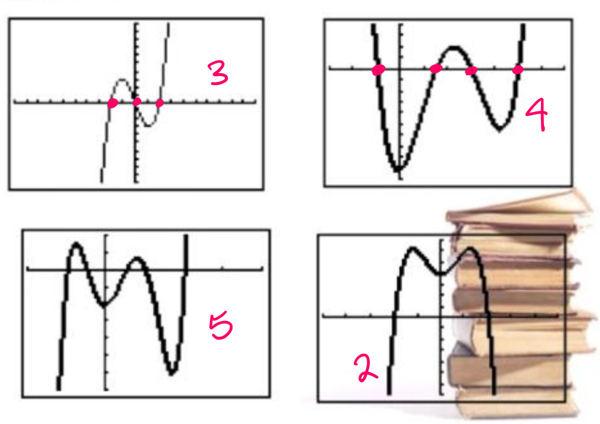
$y = x(x-4)^2(3x+1)$ — 4 roots — 0–4

$x(x)^2(3x) = 3x^{4}$

$y = x^2(3-x)(x+4)^3$ — 6 roots — 0–6

$x^2(-x)(x)^3 = -x^{6}$

How many zeros do these graphs have????



3.4 Multiplicity

Multiplicity is the number of times an x-intercept occurs in a graph:

Roots

Multiplicity of roots

Single root Example: Factor (x+2)

root: -2

graph goes straight through root

double root Example: Factor $(x-1)^2$

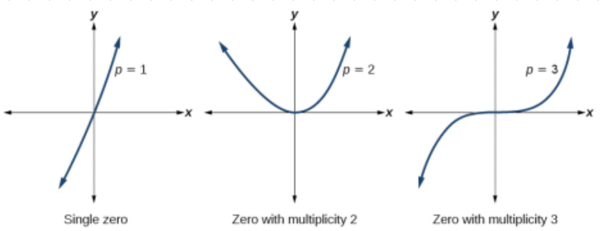
root: 1 (M2)

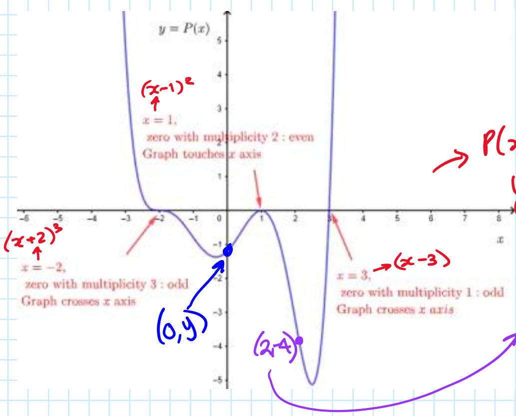
graph goes through the root like a quadratic

triple root Example: Factor $(x+4)^3$

root: -4 (M3)

graph goes through the root like a cubic



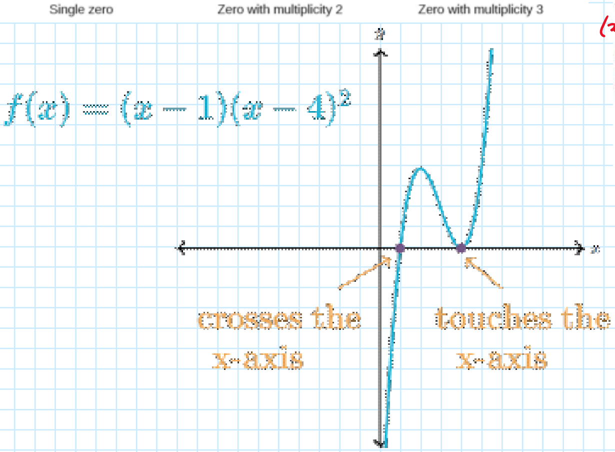


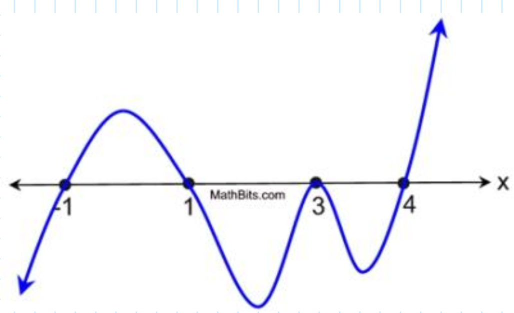
$P(x) = a(x+2)^3(x-1)^2(x-3)$

$y = a(x+2)^3(x-1)^2(x-3)$

a?

$-4 = a(2+2)^3(2-1)^2(2-3)$





If you factor the polynomial, you can see the value of the x-intercepts (roots) and the corresponding multiplicity

Real roots are x-intercepts.

To find the roots, we let y = 0 and solve for x.

Example: Find the roots for the following.

$y=(x+5)(x-3)(2x+4)$ Roots: -5, 3, -2

$y=x(x-4)^2(3x+1)$ Roots: 0, 4 (M2), $-\frac{1}{3}$

$y=x^2(3-x)(x+4)^3$ Roots: 0 (M2), 3, -4 (M3)

To find the y-intercept, make x=0 and solve for y:

Example: Find the y-intercept for each

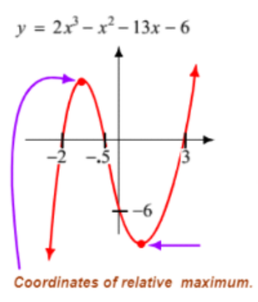
$y=(x+5)(x-3)(2x+4)$ y-intercept: (0, -60)

$y=-x^4+2x^3-x^2+3x+20$ y-intercept: (0, 20)

$y=x^2(3-x)(x+4)^3$ y-intercept: (0,0)

To graph a polynomial function:

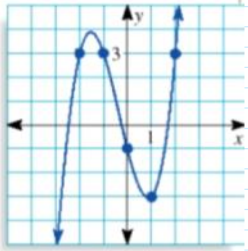
1. Determine the y-intercept by making x=0
2. Determine the x-intercepts by solving the polynomial when y=0
(or you can use your graphing calculator to find these characteristics)
3. Use the degree and leading coefficient to determine behaviour.
4. Use the table of values to estimate the relative and/or absolute Max's and Min's and get other points.
5. Draw the curve with the correct behaviour.



Graph $f(x) = x^3 + x^2 - 4x - 1$.

SOLUTION

To graph the function, make a table of values and plot the corresponding points. Connect the points with a smooth curve and check the end behavior.

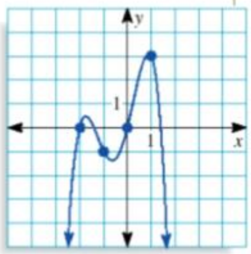


x	−3	−2	−1	0	1	2	3
$f(x)$	−7	3	3	−1	−3	3	23

Graph $f(x) = -x^4 - 2x^3 + 2x^2 + 4x$.

SOLUTION

To graph the function, make a table of values and plot the corresponding points. Connect the points with a smooth curve and check the end behavior.



x	−3	−2	−1	0	1	2	3
$f(x)$	−21	0	−1	0	3	−16	−105

Now we are ready to graph.

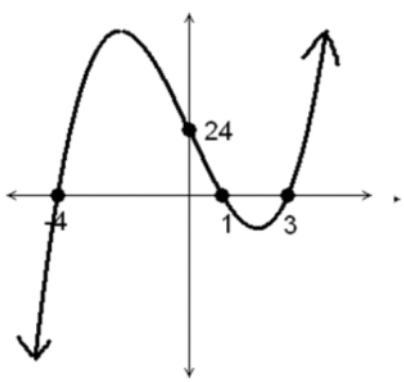
State the type, roots, y-intercept and graph.

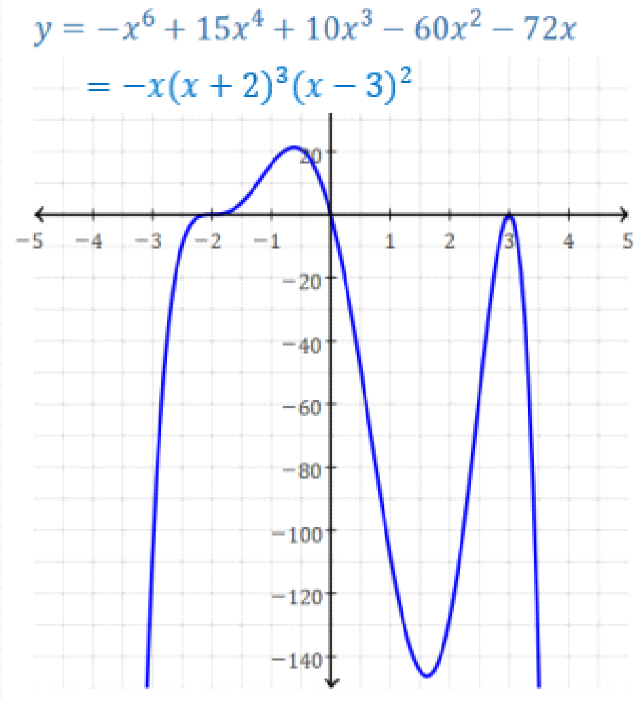
$$y = 2(x-3)(x+4)(x-1)$$

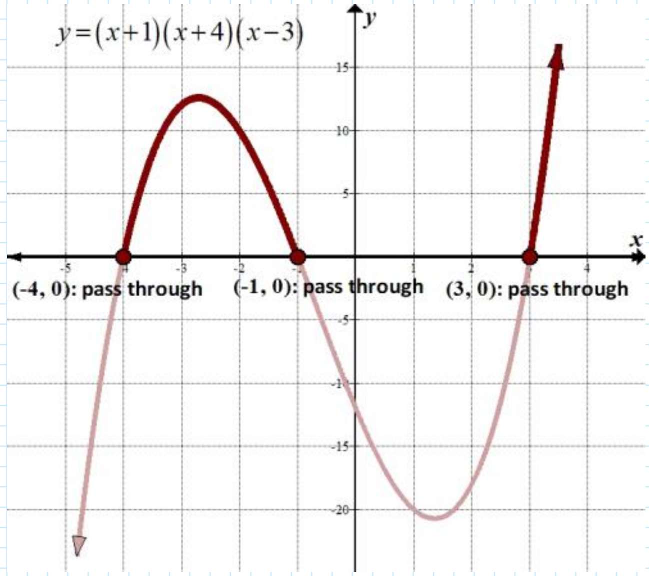
Type: positive odd

roots: 3, -4, 1

y-intercept: (0, 24)




$y = -x^6 + 15x^4 + 10x^3 - 60x^2 - 72x$
$= -x(x+2)^3(x-3)^2$
−5 −4 −3 −2 −1 1 2 3 4 5
20 −20 −40 −60 −80 −100 −120 −140


$y = (x+1)(x+4)(x-3)$
y
x
(-4, 0): pass through
(-1, 0): pass through
(3, 0): pass through