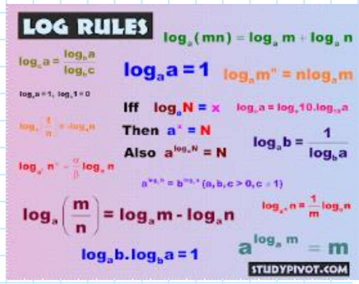
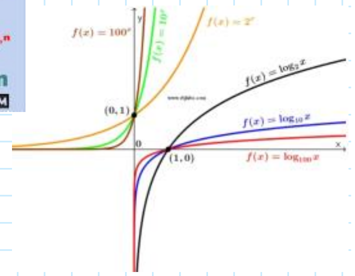
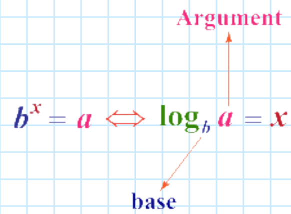
Tuesday, Feb. 27th

Plan For Today:







1. Questions from Chapter 3 or 4?
   - Do 5.1 Check-in Quiz
2. Start Chapter 5: Exponents & Logarithms
   - 5.1: Exponents
   - **5.2: Logarithmic Functions and Graphs**
   - **5.3: Properties of Logarithms**
   - 5.4: Exponential and Logarithmic Equations
   - 5.5: Applications of Exponential and Log Equations
3. Work on Practice Questions from Workbook

UNIT 2 REWRITE AFTER CLASS TODAY

Plan Going Forward:

1. Finish going through 5.2-5.3 and chapter practice questions in workbook and start working on review handout.

- **CHECK-IN QUIZ ON 5.2-5.3 ON THURSDAY, FEB. 29TH**

2. We will finish in Chapter 5 on Tuesday.

- **CHAPTER 5 PROJECT (PART A&B) DUE THURSDAY, MAR. 7TH**
  - **PART A IS IN DESMOS:** http://tinyurl.com/PC12-Feb2024-Ch5PartA
  - **PART B IS ON HANDOUT**
- **CHAPTER 5 TEST ON THURSDAY, MAR. 7TH**

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.
Anurita Dhiman = adhiman@sd35.bc.ca

Feb. 27, 2024 **Name:** ______________________________ **TOTAL =** _____ **/ 8 marks**

Check-in Quiz Section 5.1: Solving & Transformations of Exponential Functions

Complete the following questions SHOWING ALL WORK and steps where applicable.

1. Simplify the following exponential expression:

(2 marks)

a) $\dfrac{125^{3x-1}}{25^{x+4}}$

$\dfrac{(5^3)^{3x-1}}{(5^2)^{x+4}}$

$= \dfrac{5^{9x-3}}{5^{2x+8}}$

$= 5^{9x-3-(2x+8)}$

$= 5^{9x-3-2x-8} \rightarrow \boxed{5^{7x-11}}$

b) $\dfrac{\left(\frac{1}{3}\right)^{5x}(27)^{x-2}}{\left(\frac{1}{9}\right)^{1-x}} = \dfrac{(3^{-1})^{5x}(3^3)^{x-2}}{(3^{-2})^{1-x}}$

$= \dfrac{(3)^{(-5x} + (3)^{3x-6)}}{3^{-2+2x}}$

$= \dfrac{3^{-2x-6}}{3^{-2+2x}} \rightarrow 3^{-2x-6-(-2+2x)}$

$= 3^{-2x-6+2-2x}$

$= \boxed{3^{-4x-4}}$

2. Solve the following exponential equation:

2 marks

a) $4^x(16)^{x+3} = \left(\frac{1}{64}\right)^{-2x-1}$

$4^x(4^2)^{x+3} = (4^{-3})^{-2x-1}$

$4^x \cdot 4^{2x+6} = 4^{6x+3}$

$\cancel{4}^{3x+6} = \cancel{4}^{6x+3}$

if the bases are equal then the exponents are equal.

$3x+6 = 6x+3$

$-6x \quad\quad -6$

$\dfrac{-3x}{-3} = \dfrac{-3}{-3}$

$\boxed{x = 1}$

b) $36^{x-7} = \left(\frac{1}{6}\right) \bullet (6)^{-2x-1}$

$(6^2)^{x-7} = (6)^{-1} \cdot (6)^{-2x-1}$

$6^{2x-14} = 6^{-2x-2}$

$2x-14 = -2x-2$

$+2x \quad\quad +14$

$\dfrac{4x}{4} = \dfrac{12}{4}$

$\boxed{x = 3}$

3. Graph the following function and answer the questions below:

$$y = \frac{1}{2}(2)^{x+3} - 5$$

a) Describe/list the transformations on the base function.

(1 mark)

original base function

$y = 2^x \longrightarrow$ VC of $\frac{1}{2}$, 3 left, down 5

b) Sketch the graph of the transformed function. Show mapping notation.

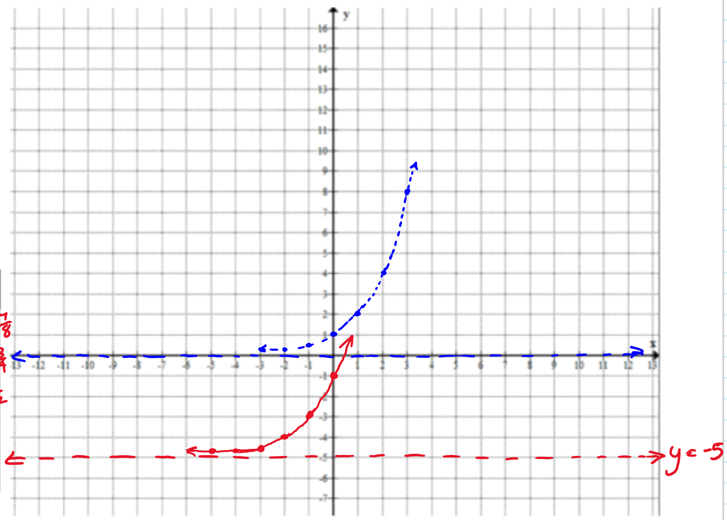
(2 marks)

$y = 2^x$

x	y	
-2	1/4	$2^{-2} \to \frac{1}{2^2} = \frac{1}{4}$
-1	1/2	2^{-1}
0	1	2^0
1	2	2^1
2	4	2^2
3	8	2^3

x	$\frac{1}{2}y$	
-2	$\frac{1}{8}$	$-\frac{40}{8}$
-1	$\frac{1}{4}$	$-\frac{20}{4}$
0	$\frac{1}{2}$	$-\frac{10}{2}$
1	1	-5
2	2	-5
3	4	

$x-3$	$\frac{1}{2}y-5$
-5	$-\frac{39}{8} = -4\frac{7}{8}$
-4	$-\frac{19}{4} = -4\frac{3}{4}$
-3	$-\frac{9}{2}, -4\frac{1}{2}$
-2	-4
-1	-3
0	-1



c) Determine the domain and range of the transformed function and the equation of the asymptote.

(1 mark)

$\{x \mid x \in \mathbb{R}\}$

$\{y \mid y > -5, y \in \mathbb{R}\}$.

$y = -5$

5.1 Graphing Summary.

Base Function

$y = c^x \longrightarrow$

Transformed

$y = a(c)^{b(x-h)} + k$

Vert. Stretch of a

$-a$ = ref. in x-axis.

Horizontal Stretch of $\frac{1}{b}$

$-b$ = ref. in y-axis.

h = right/left

$(x+2)$ 2 left

$(x-2)$ 2 right

k up/down

HA $\to$ $y = k$

x

$-2(x-5)$

x-axis. in y-axis.

ex: $y = \left(\frac{1}{2}\right)^{x} \longrightarrow y = -3\left(\frac{1}{2}\right)^{-2(x-5)} + 10$

x	y	
-3	8	$\left(\frac{1}{2}\right)^{-3}$
-2	4	$\left(\frac{1}{2}\right)^{-2}$
-1	2	
0	1	
1	$\frac{1}{2}$	
2	$\frac{1}{4}$	
3	$\frac{1}{8}$	

① VE of 3 + ref. in x-axis
HC of $\frac{1}{2}$ + ref. in y-axis.

② 5 right
10 up. ← HA $\Rightarrow y = 10$

Mapping.

$-\frac{1}{2}x$	$-3y$

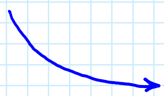
→

$-\frac{1}{2}x + 5$	$-3y + 10$

$y = 2^{x}$ vs $y = \left(\frac{1}{2}\right)^{x}$

growth.
(increasing)

base $= c > 1$



decay
(decreasing)

base $0 < c < 1$

base $= b$

$y = b^{x}$ $b > 1$ or $0 < b < 1$

p.207 5.2 Logs.

$y = b^{x}$ (exponential function) $\xrightarrow[\text{(switch } x + y)]{\text{inverse}}$ $x = b^{y}$

↓ solve for y
rewrite as a log.
$\Rightarrow$ log = exponent

$\log_b x = y$

$y = \log_b x$ log base b of x

summary

$y = b^{x} \longleftarrow \text{inverse} \longrightarrow y = \log_b x$

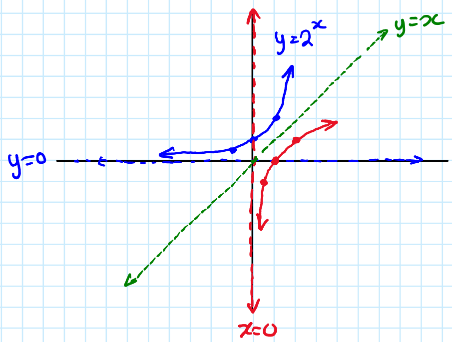
Summary

$y = b^x \longleftarrow \text{inverse} \longrightarrow y = \log_b x$

p.205

$y = 2^x$

x	y
-1	$\frac{1}{2}$
0	1
1	2



$y = \log_2 x$

x	y
$\frac{1}{2}$	-1
1	0
2	1

Log graph transformations:

Base Function → Transformed

$y = \log_b x \longrightarrow y = a\log_b b(x-h)+k$

- a: vert. stretch of a; $-a$ = refl in x-axis
- b: horiz. stretch of $\frac{1}{b}$; $-b$ = refl in y-axis
- h: right/left; VA: $x=h$
- k: up/down

1. base points on $y=\log_b x$ are the inverse of $y=b^x$
2. apply transformations & graph.

ex: $y = -2\log_2\left(\frac{1}{2}(x+5)\right) - 3$

1. HE of 2; VE of 2 & refl in x-axis
2. 5 left; 3 down.

$y = \log_2 x$ inverse of $y = 2^x$

$y=2^x$ $\xrightarrow{\text{inverse}}$ $y=\log_2 x$ $\xrightarrow{\text{transform}}$

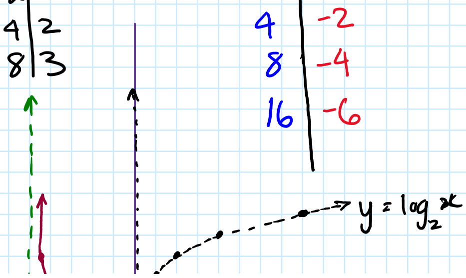
x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

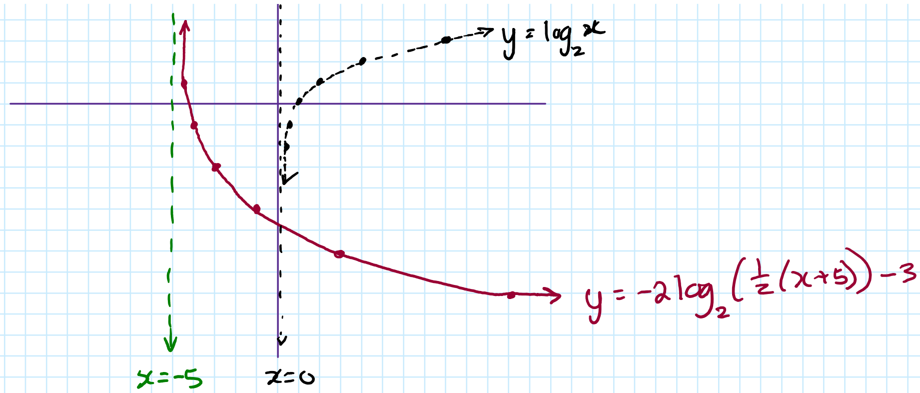
x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3

$2x$	$-2y$
$\frac{1}{2}$	4
1	2
2	0
4	-2
8	-4
16	-6

mapping notation.

$2x-5$	$-2y-3$
$-4\frac{1}{2}$	1
-4	-1
-3	-3
-1	-5
3	-7
11	-9





Exponential Equation	inverse of exponential	logarithmic equation
$y = b^x$	$x = b^y$	$y = \log_b x$

inverses of each other.

p. 209

$x = b^y$ — x: result, b: base, y: exponent

$y = \log_b x$ — y: exponent, b: base, x: argument

ex: 1 a) $\log_4 2 = \frac{1}{2} \longrightarrow 4^{\frac{1}{2}} = 2$

base: 4, argument (result): 2, exponent: $\frac{1}{2}$

ex: 2 b) $3^{-2} = \frac{1}{9} \longrightarrow \log_3\left(\frac{1}{9}\right) = -2$

base: 3, exponent: -2, result (argument): $\frac{1}{9}$

$\log 3x = \square$

$\log_3 x$

switching b/w log + exponent form helps to simplify equation or expressions

ex: 3 a) $\log_4 8 = \overset{?}{x}$

$\log_4 8 = x. \xrightarrow{\text{change to exponent form}} 4^x = 8$

$\log_4 8 = x \xrightarrow{\text{change exponent}} 4^x = 8$

no restriction

$2^{2x} = 2^3$

$\frac{2x}{2} = \frac{3}{2}$

$\boxed{x = \frac{3}{2}}$ ✓

☆ Restrictions (Domain) exist in log equations.

$y = \log_b x$ $x > 0$

no restriction

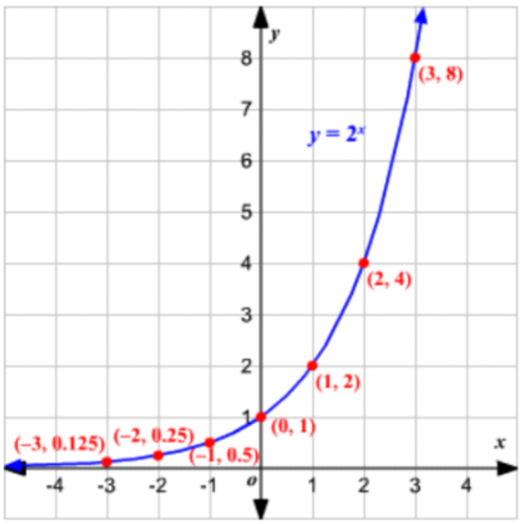
5.2 p.211 1-5 practice.

Graphs and Transformations of Exponential Functions

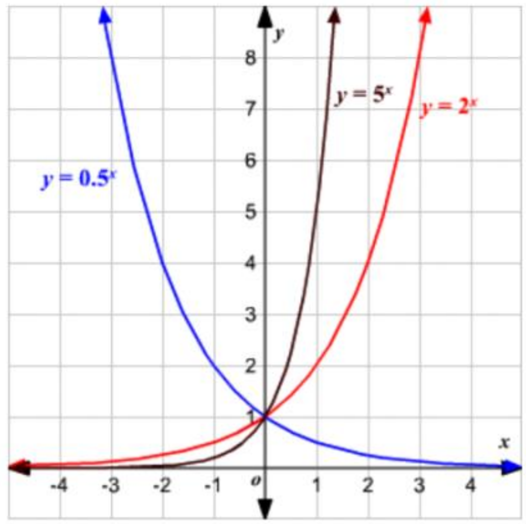
https://www.varsitytutors.com/hotmath/hotmath_help/topics/graphing-exponential-functions

A simple exponential function to graph is $y = 2^x$.

x	-3	-2	-1	0	1	2	3
$y = 2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

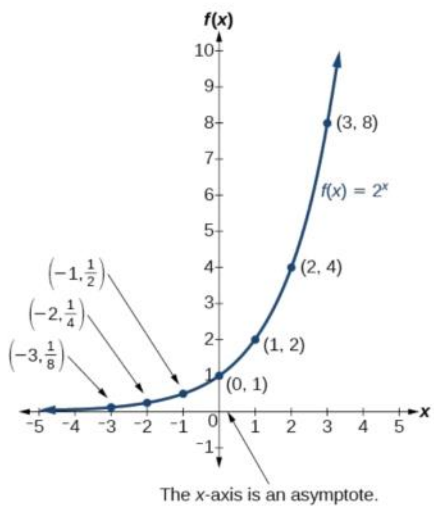


Changing the base changes the shape of the graph.



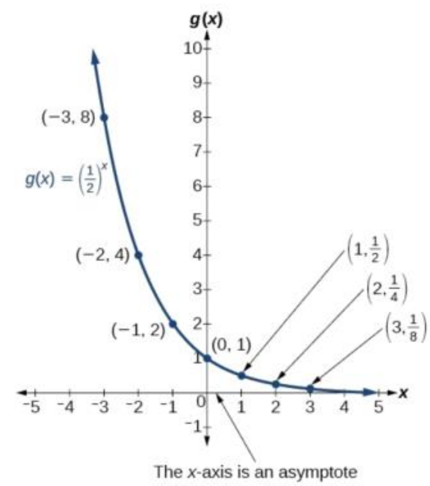
Notice that the graph has the x -axis as an asymptote on the left, and increases very fast on the right.

The graph below shows the exponential growth function $f(x) = 2^x$.



Notice that the graph gets close to the x-axis but never touches it.

The graph below shows the exponential decay function, $g(x) = \left(\frac{1}{2}\right)^x$.



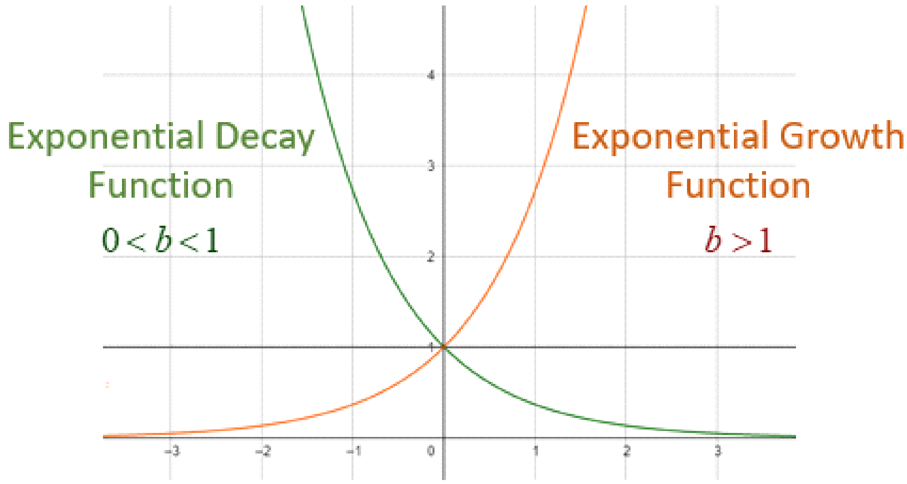
The domain of $g(x) = \left(\frac{1}{2}\right)^x$ is all real numbers, the range is $(0, \infty)$, and the horizontal asymptote is $y = 0$.

Exponential Growth and Decay Functions

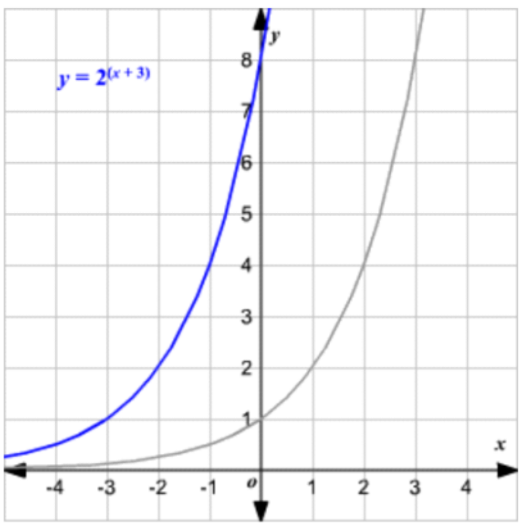
An exponential function f is given by

$$f(x) = b^x$$

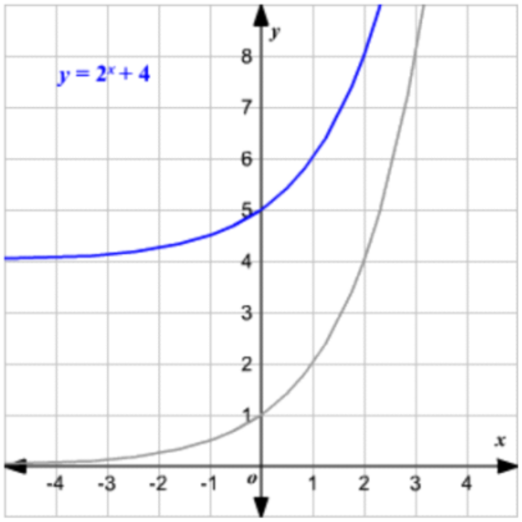
where x is any real number, b > 0, and b ≠ 1.



Replacing x with $x + h$ translates the graph h units to the left.



Replacing y with $y - k$ (which is the same as adding k to the right side) translates the graph k units up.



A GENERAL NOTE: CHARACTERISTICS OF THE GRAPH OF THE PARENT FUNCTION $f(x) = b^x$

An exponential function with the form $f(x) = b^x$, $b > 0$, $b \neq 1$, has these characteristics:

- **one-to-one** function
- horizontal asymptote: $y = 0$
- domain: $(-\infty, \infty)$
- range: $(0, \infty)$
- *x*-intercept: none
- *y*-intercept: $(0, 1)$
- increasing if $b > 1$
- decreasing if $b < 1$

HOW TO: GIVEN AN EXPONENTIAL FUNCTION OF THE FORM $f(x) = b^x$, GRAPH THE FUNCTION

1. Create a table of points.
2. Plot at least 3 point from the table including the *y*-intercept $(0, 1)$.
3. Draw a smooth curve through the points.
4. State the domain, $(-\infty, \infty)$, the range, $(0, \infty)$, and the horizontal asymptote, $y = 0$.

Transformation	Equation	Description
Horizontal stretch	$g(x) = c^{bx}$	Horizontal stretch about the y-axis by a factor of $\frac{1}{\lvert b \rvert}$.
Vertical stretch	$g(x) = a\,c^{x}$	• Vertical stretch about the x-axis by a factor of $\lvert a \rvert$. • Multiplying **y**-coordintates of $\boldsymbol{f(x) = c^x}$ by a.
Reflecting	$g(x) = -c^{x}$	• Reflects the graph of $\boldsymbol{f(x) = c^x}$ about the **x**-axis.
	$g(x) = c^{-x}$	• Reflects the graph of $\boldsymbol{f(x) = c^x}$ about the **y**-axis.
Vertical translation	$g(x) = c^{x} + k$	• Shifts the graph of $f(x) = c^x$ upward k units if k > 0. • Shifts the graph of $\boldsymbol{f(x) = c^x}$ downward k units if k < 0.
Horizontal translation	$g(x) = c^{x-h}$	• Shifts the graph of $\boldsymbol{f(x) = c^x}$ to the right h units if $h > 0$. • Shifts the graph of $\boldsymbol{f(x) = c^x}$ to the left h units if $h < 0$.

Transformations of Exponential Functions			
Transformation	**$f(x)$ Notation**	**Examples**	
Vertical translation	$f(x) + k$	$y = 2^x + 3$	3 units up
		$y = 2^x - 6$	6 units down
Horizontal translation	$f(x - h)$	$y = 2^{x-2}$	2 units right
		$y = 2^{x+1}$	1 unit left
Vertical stretch or compression	$af(x)$	$y = 6(2^x)$	stretch by 6
		$y = \frac{1}{2}(2^x)$	compression by $\frac{1}{2}$
Horizontal stretch or compression	$f\left(\frac{1}{b}x\right)$	$y = 2^{\left(\frac{1}{5}x\right)}$	stretch by 5
		$y = 2^{3x}$	compression by $\frac{1}{3}$
Reflection	$-f(x)$	$y = -2^x$	across x-axis
	$f(-x)$	$y = 2^{-x}$	across y-axis

The basic properties of the graph $f(x) = b^x$ can be stated as follows:

Basic Properties of the Graph $f(x) = b^x,\ b > 0,\ b \neq 0$

1. All graphs go through the point $(0, 1)$, and the graph has no x-intercept.
2. The x-axis is a horizontal asymptote with equation $y = 0$.
3. When $b > 1$, $f(x) = b^x$ is an increasing function.
4. When $0 < b < 1$, $f(x) = b^x$ is a decreasing function.

Practice

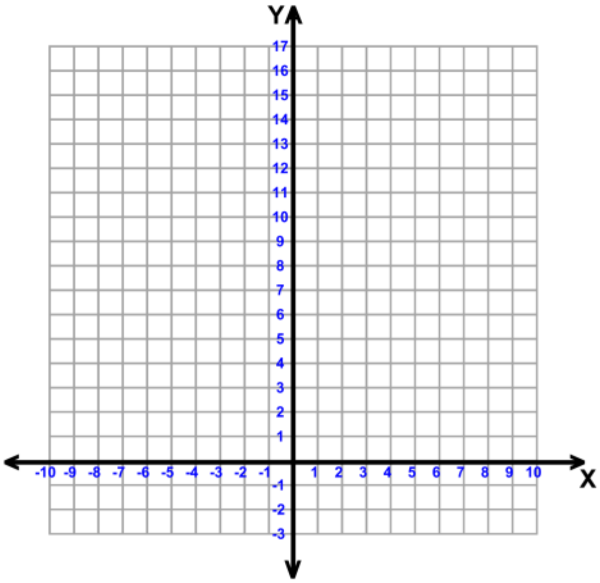
Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

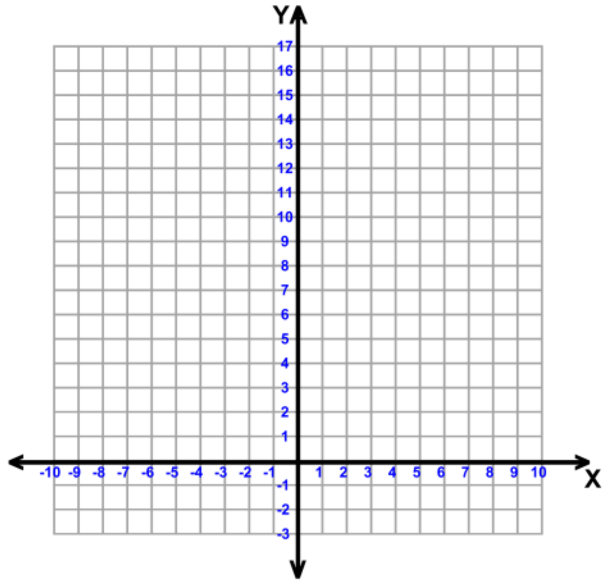
Graphing Exponential Functions

Sketch the graph of each function.

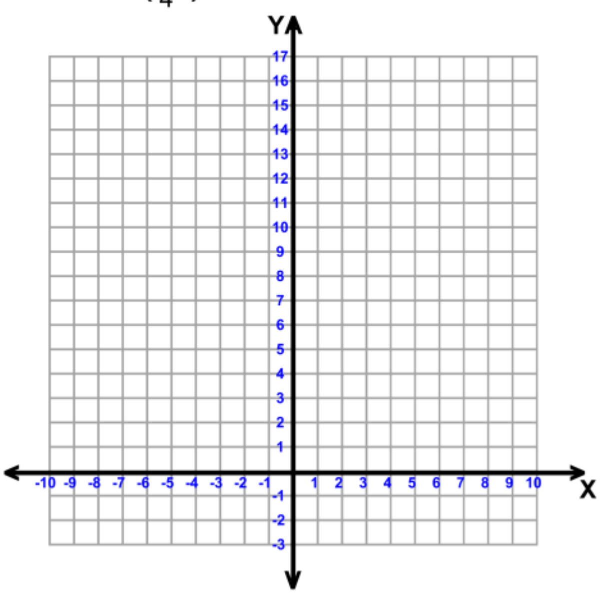
1) $y = 3 \cdot \left(\frac{1}{2}\right)^{x+3} - 3$



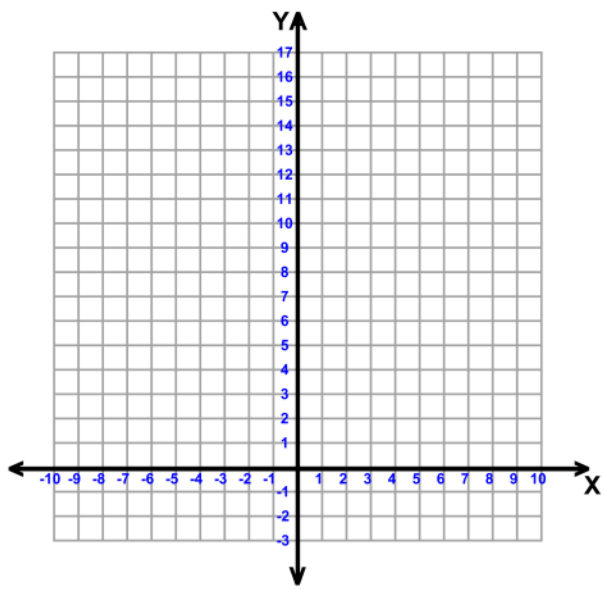
2) $y = 4 \cdot 2^{x+3} - 3$



3) $y = 3 \cdot \left(\frac{1}{4}\right)^{x}$



4) $y = 4 \cdot 3^{x}$



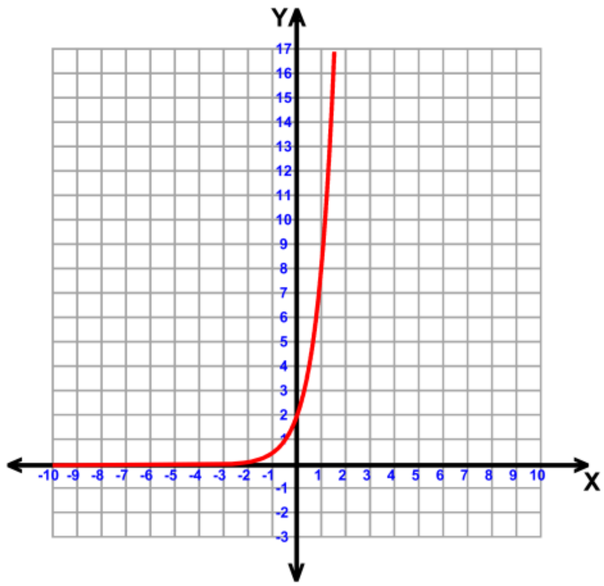
Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

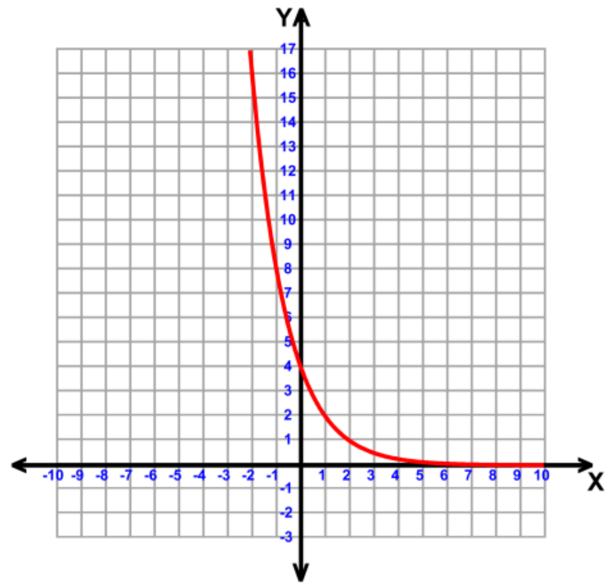
Graphing Exponential Functions

Write an equation for each graph.

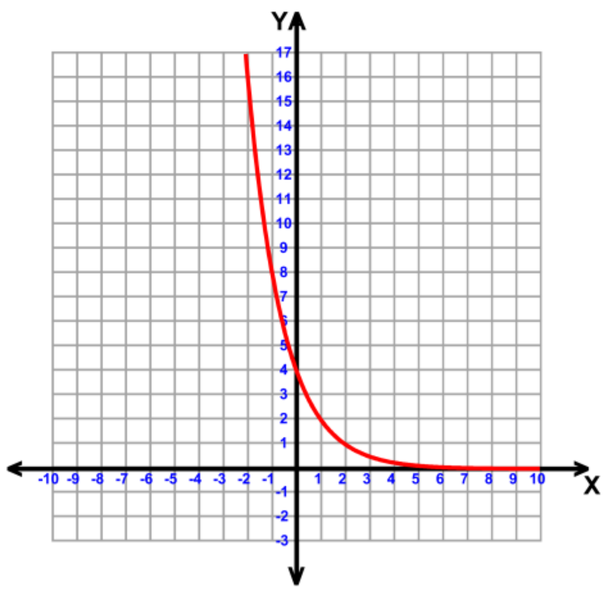
5)



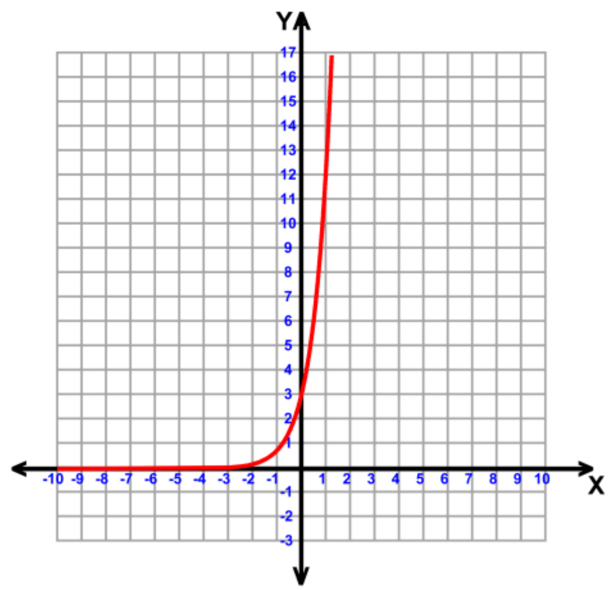
6)



7)



8)



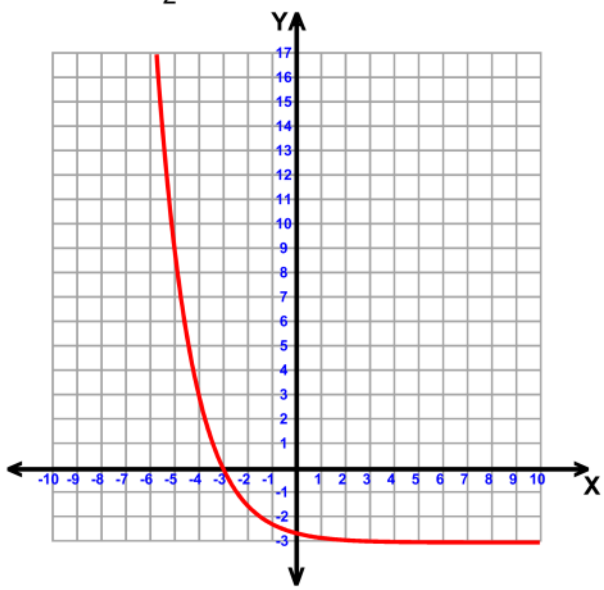
Name : ____________________ Score : __________

Teacher : ____________________ Date : __________

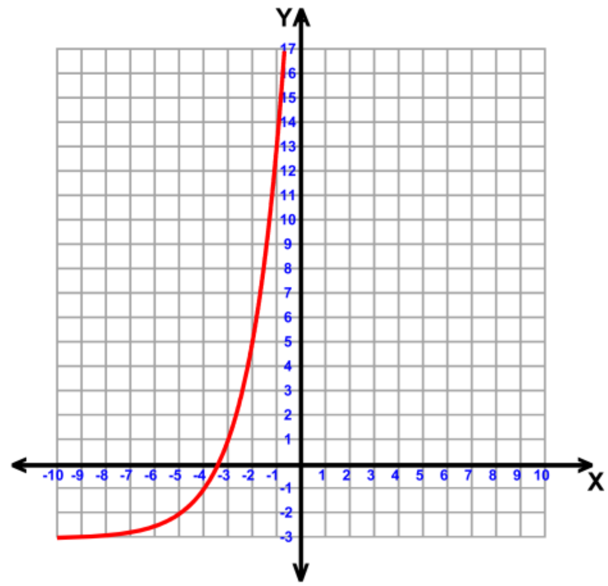
Graphing Exponential Functions

Sketch the graph of each function.

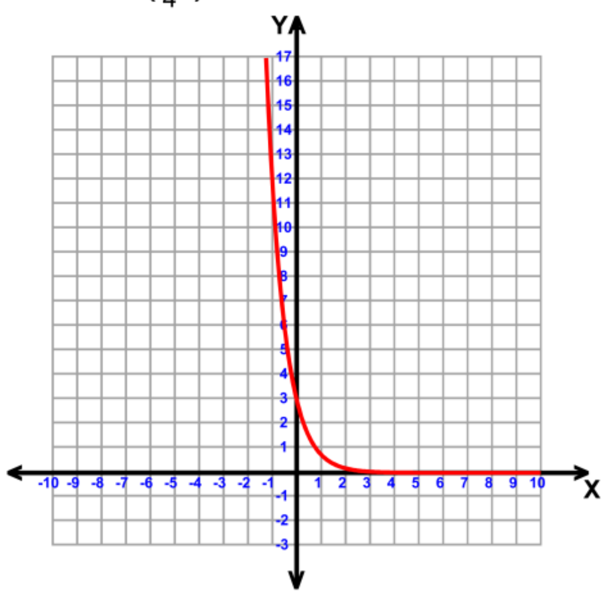
1) $y = 3 \cdot \left(\frac{1}{2}\right)^{x+3} - 3$



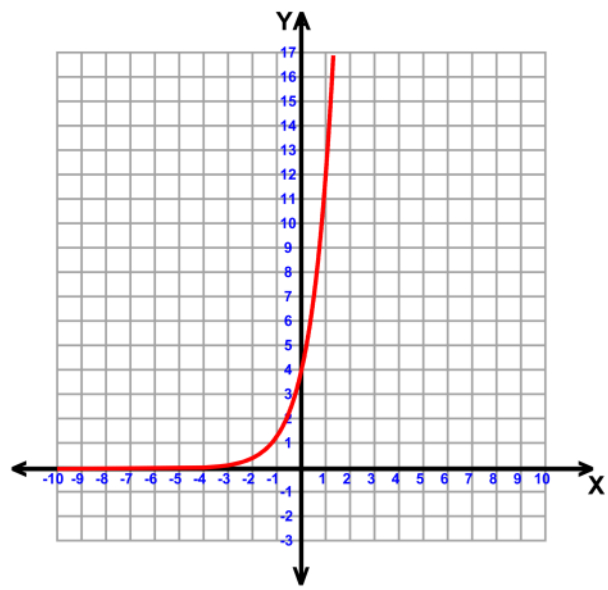
2) $y = 4 \cdot 2^{x+3} - 3$



3) $y = 3 \cdot \left(\frac{1}{4}\right)^{x}$



4) $y = 4 \cdot 3^{x}$



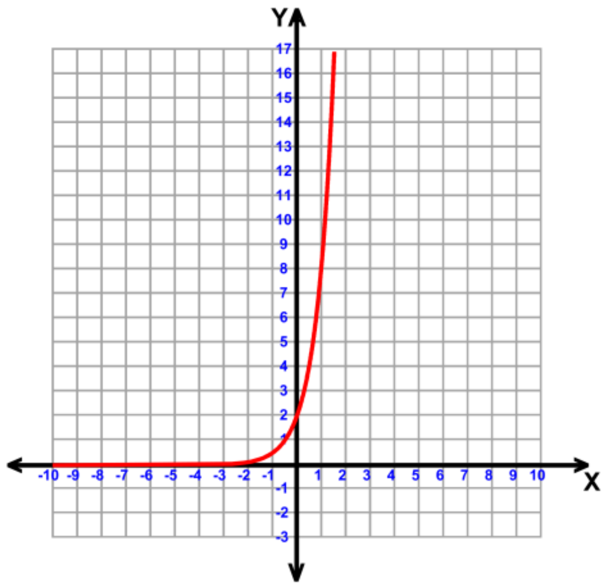
Name : ____________________ Score : __________

Teacher : ____________________ Date : __________

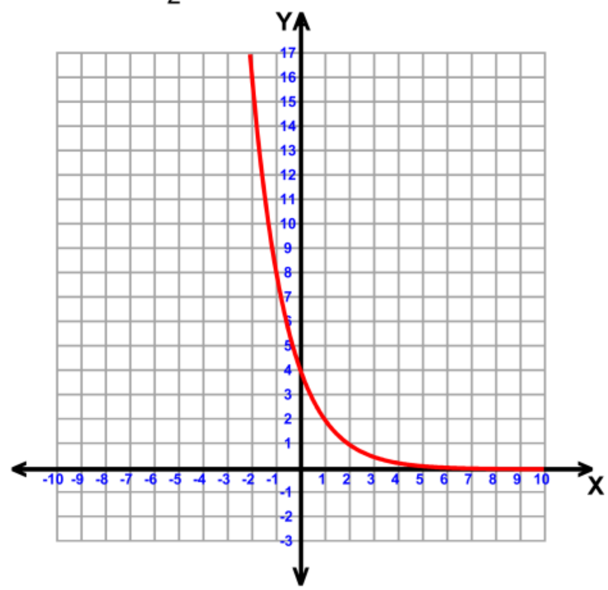
Graphing Exponential Functions

Write an equation for each graph.

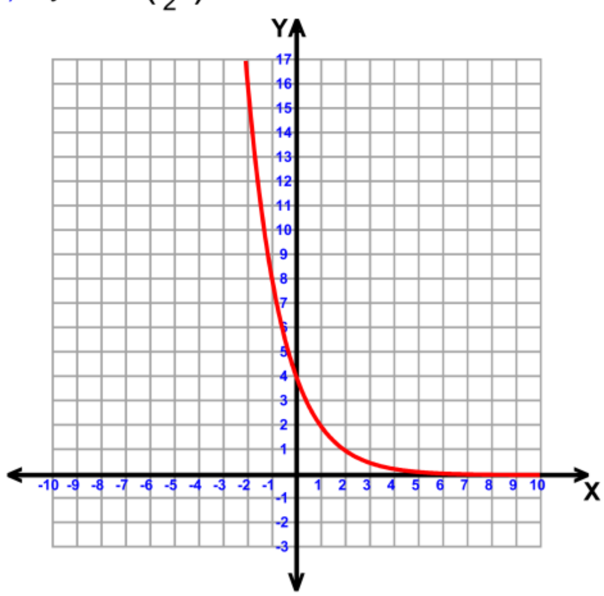
5) $y = 2 \cdot 4^x$



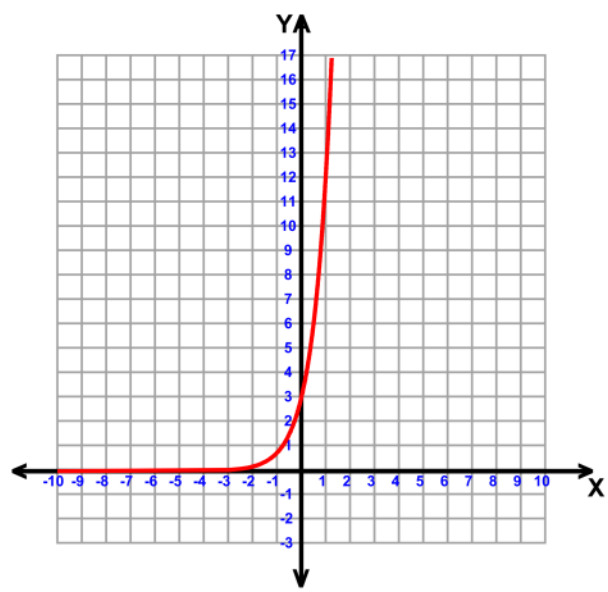
6) $y = 4 \cdot \left(\frac{1}{2}\right)^x$



7) $y = 4 \cdot \left(\frac{1}{2}\right)^x$



8) $y = 3 \cdot 4^x$

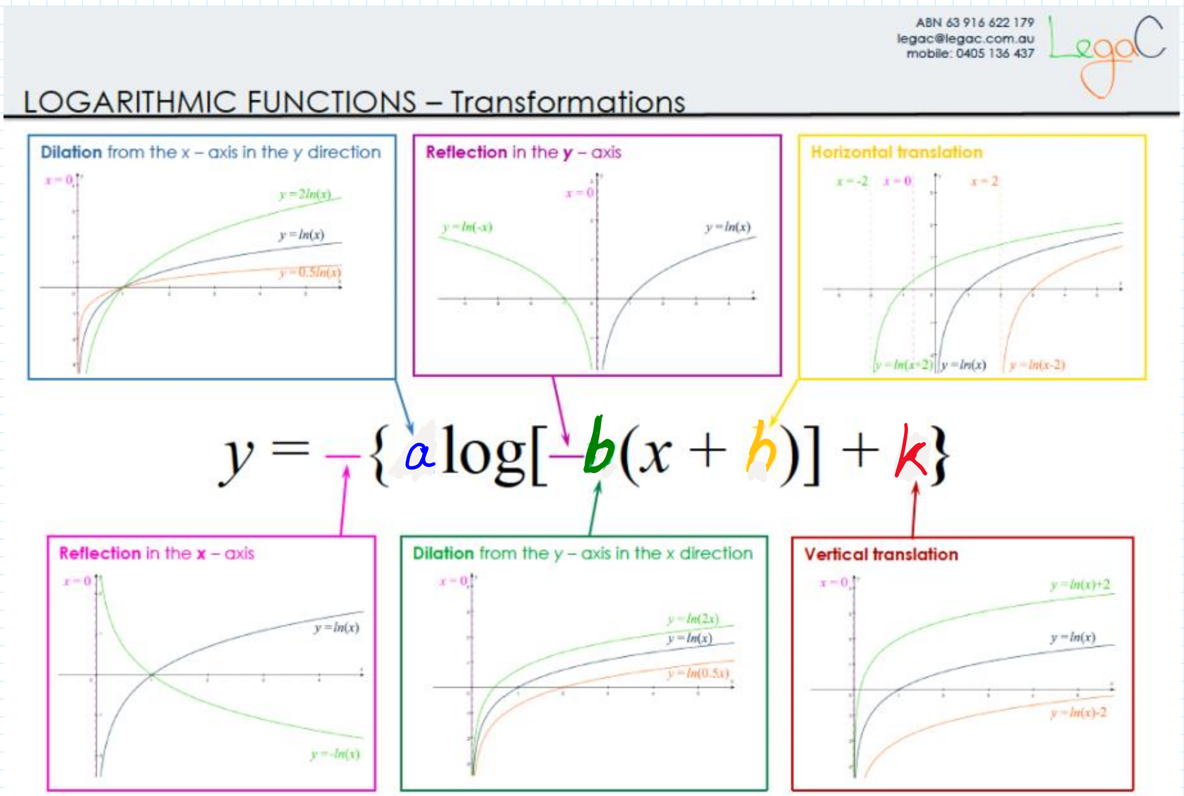


Transformations of Logarithmic Functions

Logarithmic Graphs as Inverses of the Exponential Graphs

Exponential	Logarithmic
$y = b^x,\ b > 0,\ b \neq 1,\ x > 0$	$y = \log_b x,\ b > 0,\ b \neq 1,\ x > 0$
all go through $(0, 1)$	all go through $(1, 0)$
asymptote on x-axis	asymptote on y-axis
$b > 1$ increasing	$b > 1$ increasing
$0 < b < 1$ decreasing	$0 < b < 1$ decreasing
all go through $(1, b)$, $\left(-1, \frac{1}{b}\right)$	all go through $(b, 1)$, $\left(\frac{1}{b}, -1\right)$

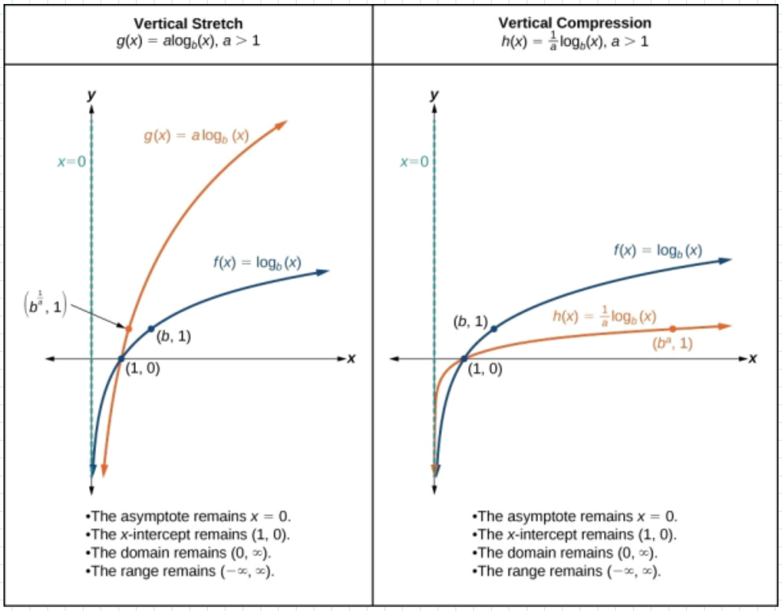
Exponential and logarithmic graphs are inverses of each other.

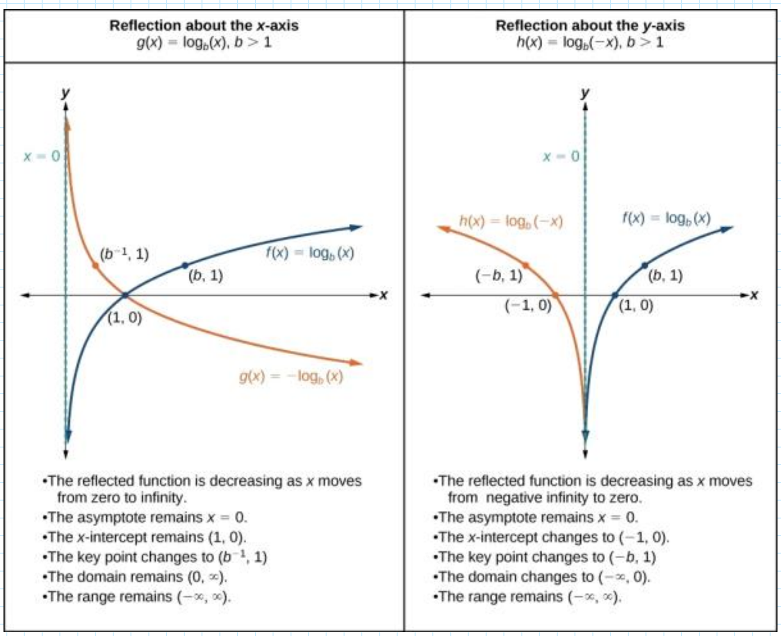


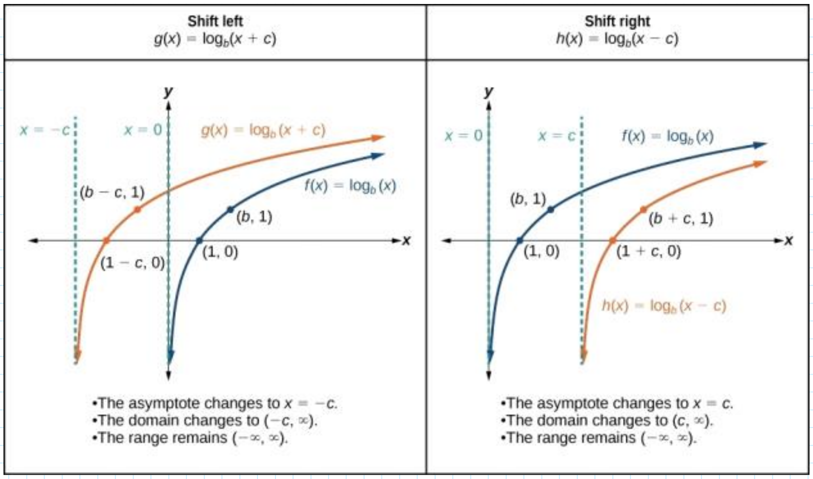
Transformations $y = \log x$

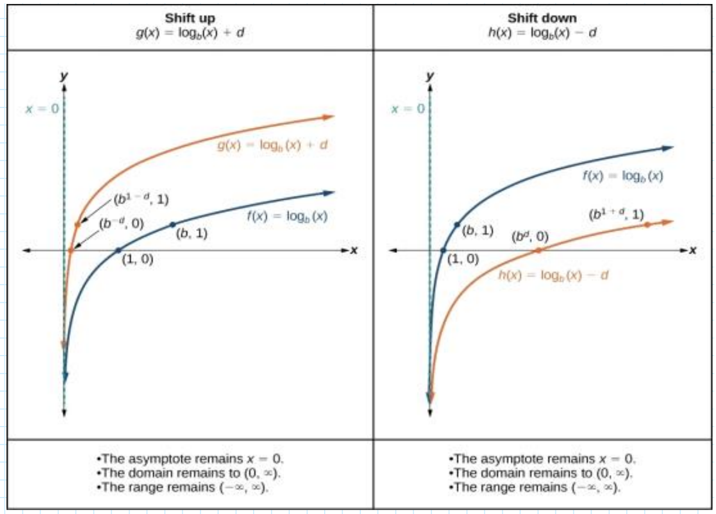
Transformation	f(x) Notation	Examples
Horizontal Translation Graph shifts left or right.	$f(x-h)$	$g(x) = \log(x-3) \rightarrow 3$ units right $g(x) = \log(x+4) \rightarrow 4$ units left
Vertical Translation Graph shifts up or down.	$f(x)+k$	$g(x) = \log x + 4 \rightarrow 4$ units up $g(x) = \log x - 5 \rightarrow 5$ units down
Reflection Graph flips over x-axis.	$-f(x)$	$g(x) = -\log x \rightarrow$ over x-axis
Reflection Graph flips over y-axis.	$f(-x)$	$g(x) = \log(-x) \rightarrow$ over y-axis
Horizontal Shrink Graph shrinks toward y-axis.	$f(ax), a > 1$	$g(x) = \log 2x \rightarrow$ shrink by $\frac{1}{2}$
Horizontal Stretch Graph stretches away from y-axis.	$f(ax),\ 0 < a < 1$	$g(x) = \log\frac{x}{2} \rightarrow$ stretch by 2
Vertical Stretch [redacted] Graph stretches away from x-axis.	$a \cdot f(x),\ a > 1$	$g(x) = 2 \cdot \log x \rightarrow$ stretch by 2

Graph stretches away from y-axis.	$f(ax),\ 0<a<1$	[illegible]
Vertical Stretch [redacted] Graph stretches away from x-axis.	$a\cdot f(x),\ a>1$	$g(x)=2\cdot\log x \rightarrow$ stretch by 2
Vertical Shrink Graph shrinks toward x-axis.	$a\cdot f(x),\ 0<a<1$	$g(x)=\dfrac{1}{2}\log x \rightarrow$ shrink by $\dfrac{1}{2}$






Shift left
g(x) = log_b(x + c)
Shift right
h(x) = log_b(x − c)
y
x = −c
x = 0
g(x) = log_b (x + c)
(b − c, 1)
f(x) = log_b (x)
(b, 1)
(1 − c, 0)
(1, 0)
x
x = 0
x = c
f(x) = log_b (x)
(b, 1)
(b + c, 1)
(1, 0)
(1 + c, 0)
h(x) = log_b (x − c)
•The asymptote changes to x = −c.
•The domain changes to (−c, ∞).
•The range remains (−∞, ∞).
•The asymptote changes to x = c.
•The domain changes to (c, ∞).
•The range remains (−∞, ∞).


Shift up
g(x) = log_b(x) + d
Shift down
h(x) = log_b(x) − d
y
x = 0
g(x) = log_b (x) + d
(b^{1 − d}, 1)
(b^{−d}, 0)
(b, 1)
f(x) = log_b (x)
(1, 0)
x
f(x) = log_b (x)
(b, 1)
(b^{1 + d}, 1)
(b^d, 0)
(1, 0)
h(x) = log_b (x) − d
•The asymptote remains x = 0.
•The domain remains to (0, ∞).
•The range remains (−∞, ∞).
•The asymptote remains x = 0.
•The domain remains to (0, ∞).
•The range remains (−∞, ∞).

Log Graphs Practice

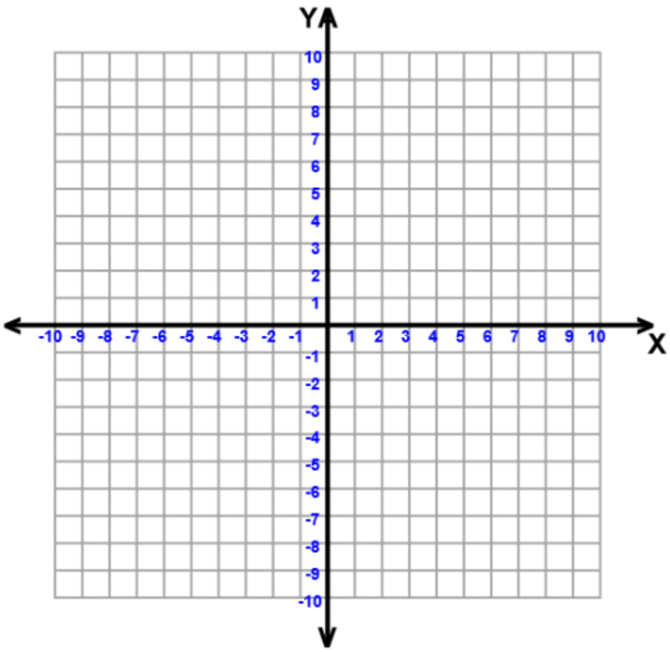
Name : ______________________________ Score : ____________

Teacher : ____________________________ Date : ____________

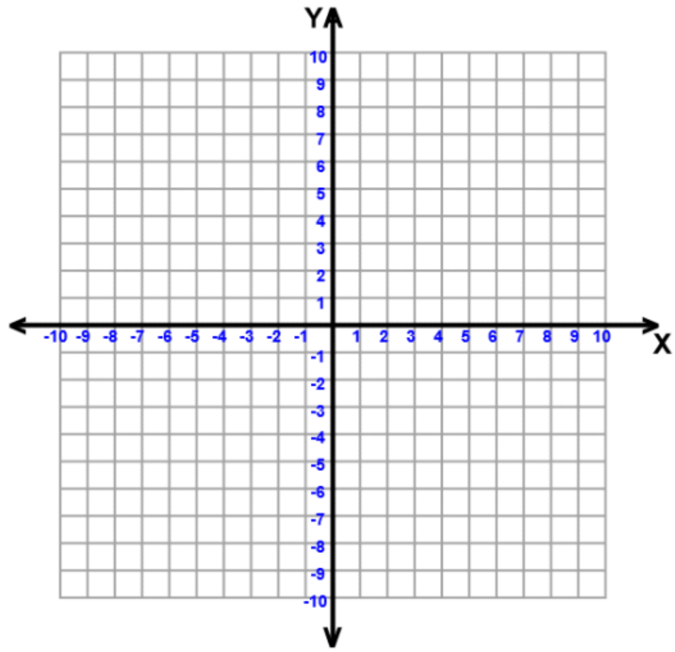
Graphing Logarithms

Give the domain and range of each function, then graph.

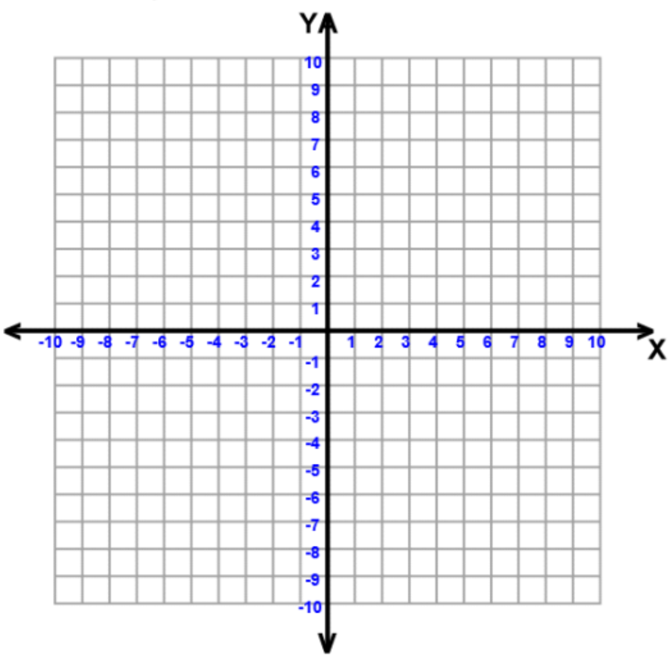
1) $y = \log_9 (x + 4) + 3$



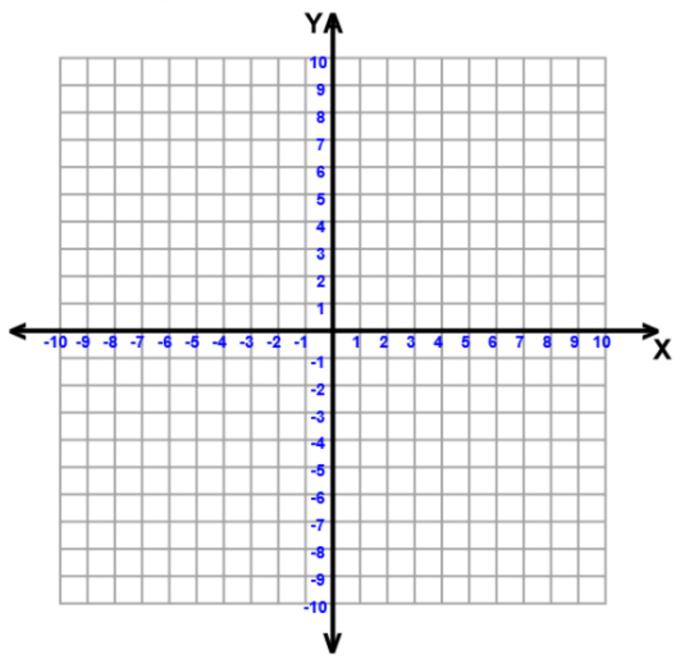
2) $y = \log_5 (x - 2) + 5$



3) $y = \log_7 (x - 4) - 2$



4) $y = \log_4 (x - 2) - 5$



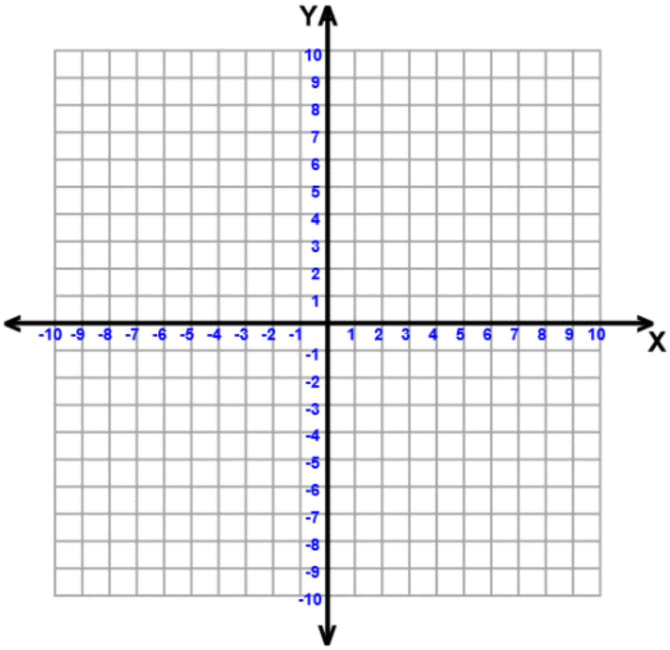
Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

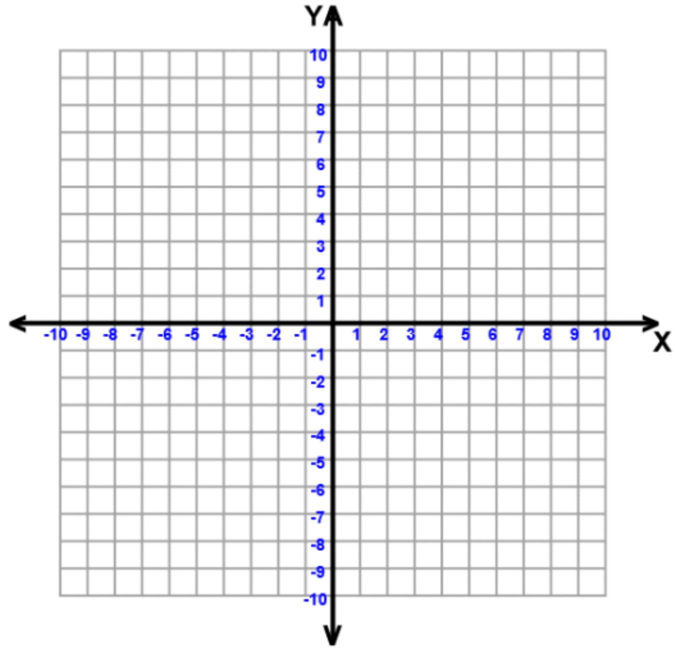
Graphing Logarithms

Give the domain and range of each function, then graph.

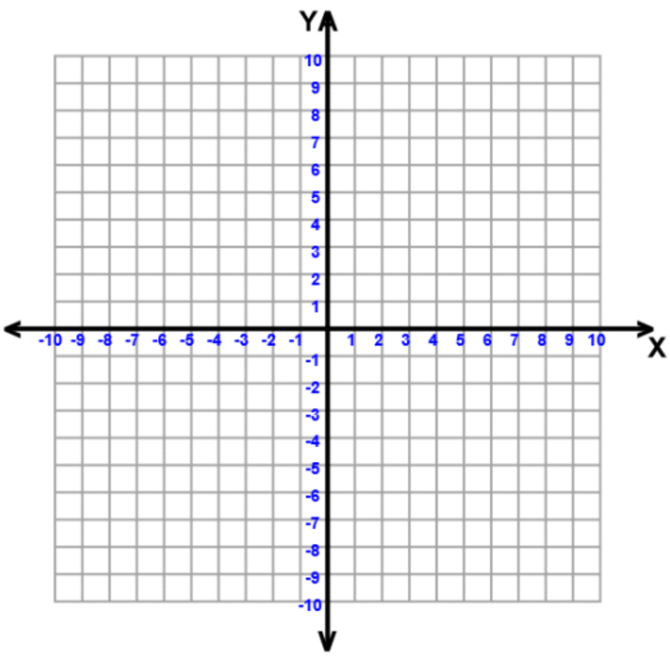
5) $y = \log_5 (4x - 5) - 2$



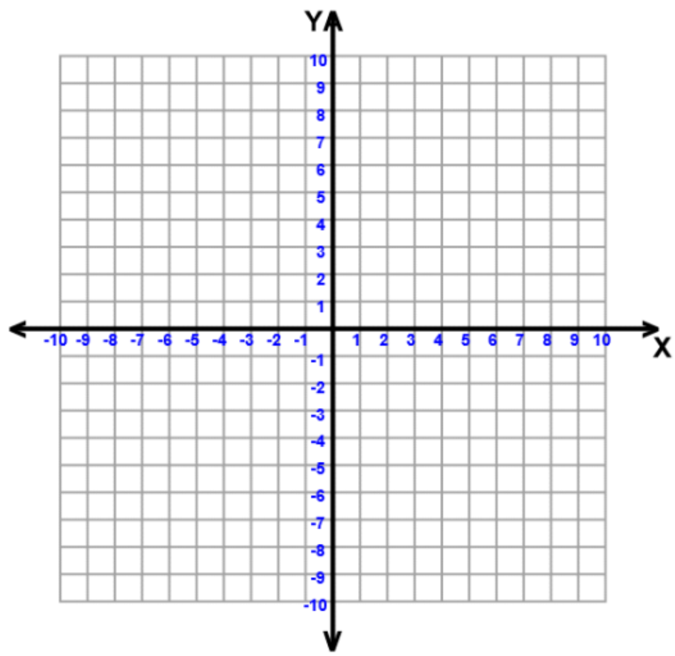
6) $y = \log (4x - 3) - 4$



7) $y = \log_8 (2x + 5) + 3$



8) $y = \log_7 (4x + 4) - 3$



Name : ______________________________ Score : ____________

Teacher : _____________________________ Date : ____________

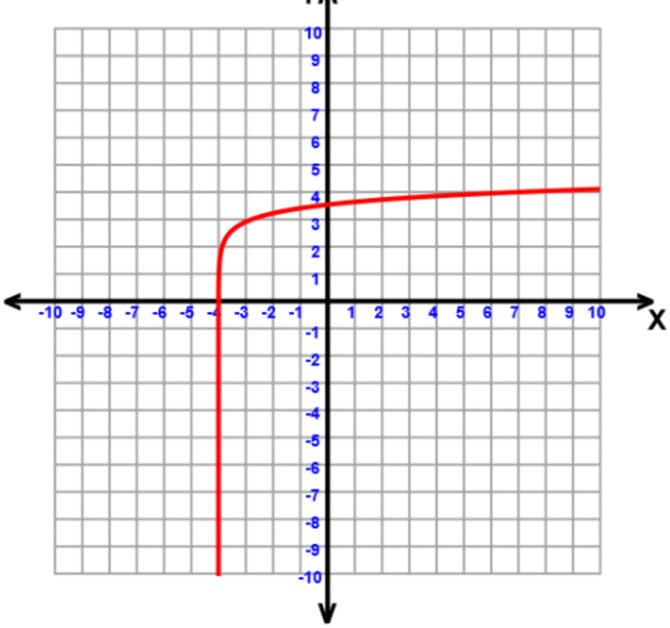
Graphing Logarithms

Give the domain and range of each function, then graph.

1) $y = \log_9 (x + 4) + 3$

Domain: x > -4

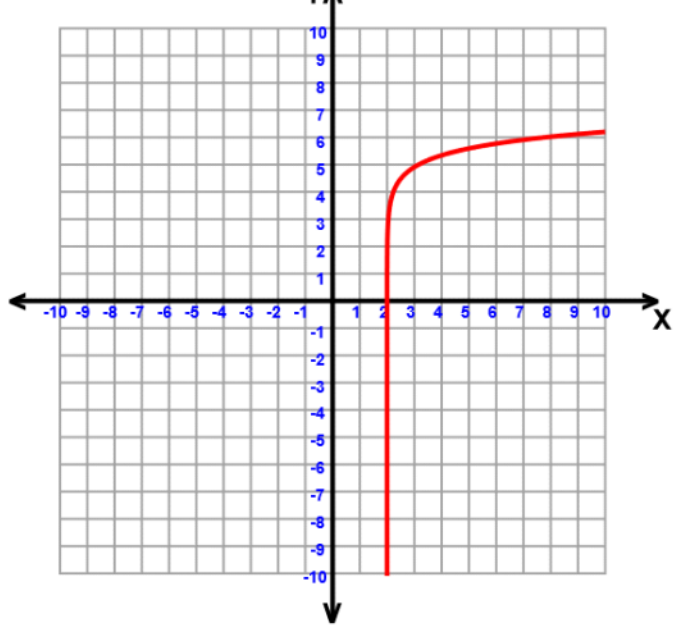
Range: All real numbers



2) $y = \log_5 (x - 2) + 5$

Domain: x > 2

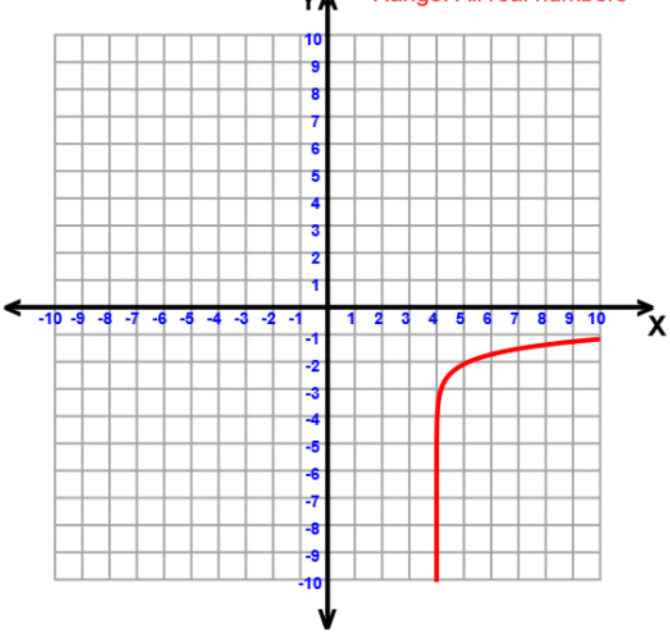
Range: All real numbers



3) $y = \log_7 (x - 4) - 2$

Domain: x > 4

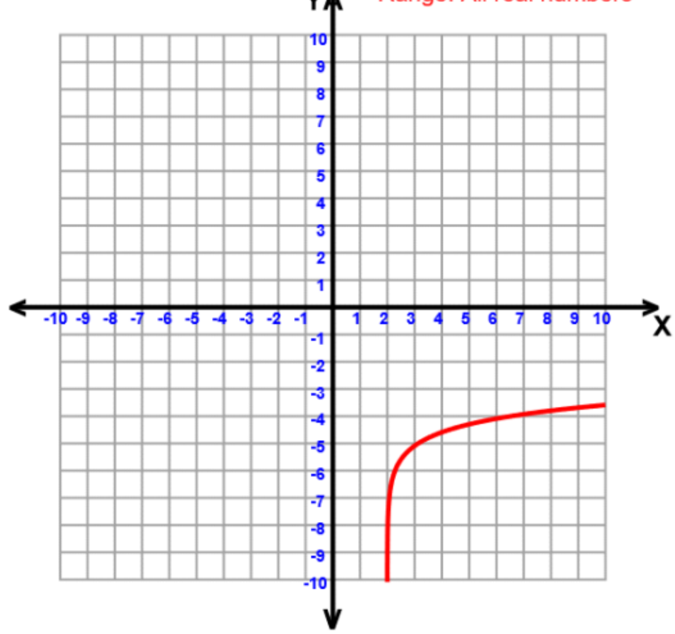
Range: All real numbers



4) $y = \log_4 (x - 2) - 5$

Domain: x > 2

Range: All real numbers



Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

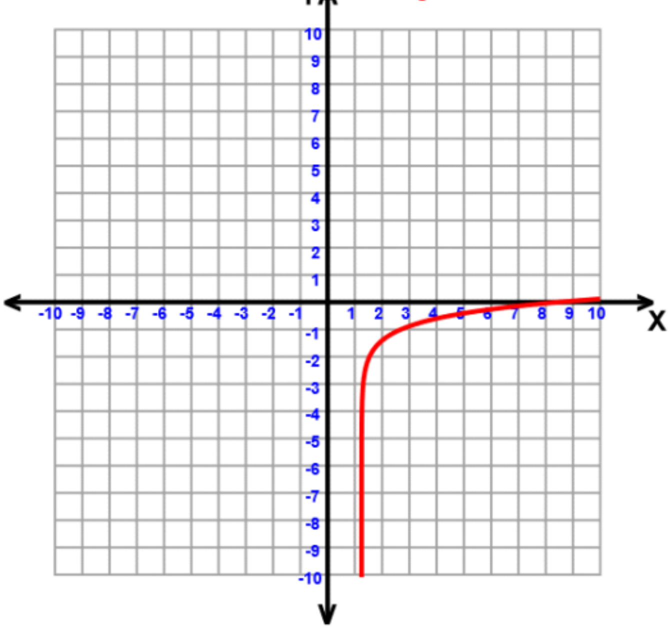
Graphing Logarithms

Give the domain and range of each function, then graph.

5) $y = \log_5 (4x - 5) - 2$

Domain: $x > \frac{5}{4}$

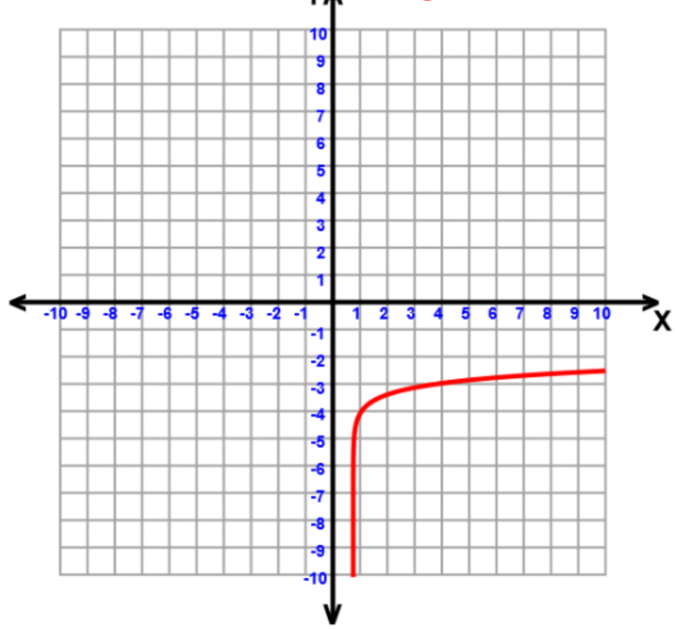
Range: All real numbers



6) $y = \log (4x - 3) - 4$

Domain: $x > \frac{3}{4}$

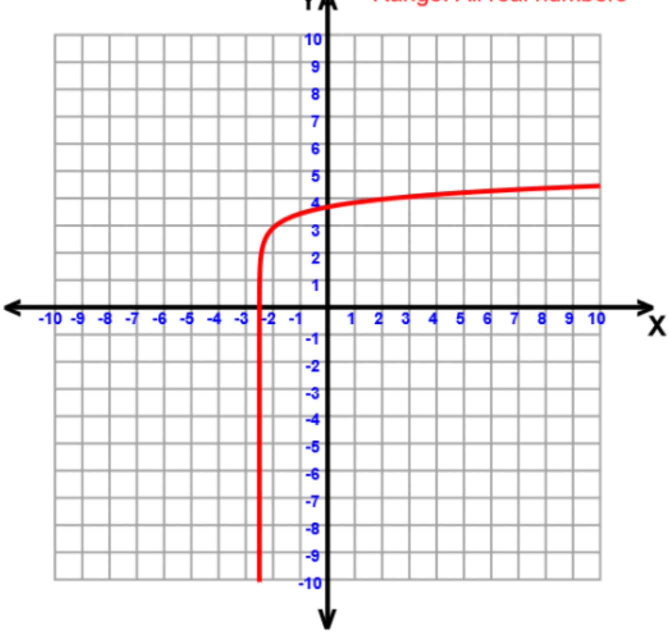
Range: All real numbers



7) $y = \log_8 (2x + 5) + 3$

Domain: $x > \frac{-5}{2}$

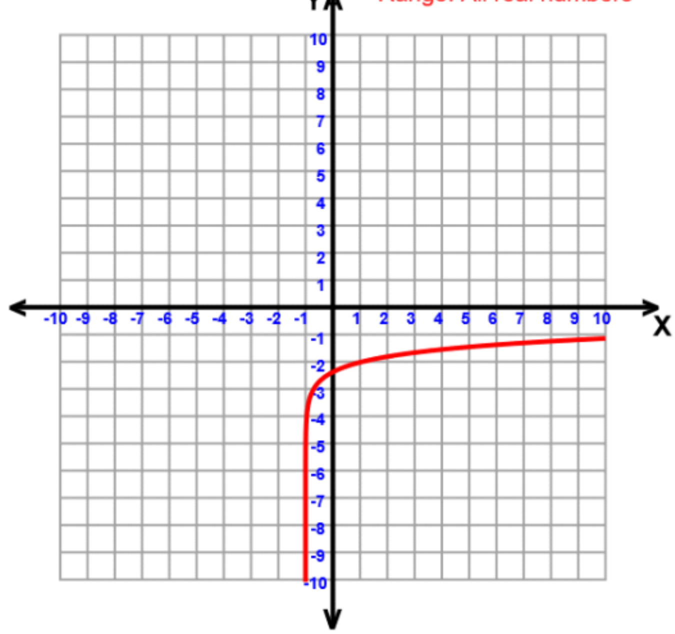
Range: All real numbers



8) $y = \log_7 (4x + 4) - 3$

Domain: $x > -1$

Range: All real numbers



Understanding Logarithms

What is a log?
A logarithm is an exponent.

$$\log_b(a) = c \iff b^c = a$$

Logarithmic Form		Exponential Form
$\log_3 x = 5$	➡	$3^5 = x$

Both forms use the same base.

The logarithm is equal to the exponent.

Changing between log and exponent form:

Convert each to logarithmic form.

a) $2^m = n$ base $\qquad \log_2 n = m$

b) $10^{x-1} = 1000 \rightarrow \log 1000 = x-1$

c) $(x+1) = 3^{z+1} \rightarrow \log_3(x+1) = z+1$

More:
p.263 # 2 + #3

Convert each to exponential form.

a) $\log_2 x = 3$ $\rightarrow$ $x = 2^3$

b) $\log_{x-1} 3 = 4$ $\rightarrow$ $(x-1)^4 = 3$

c) $\log_x (x+2) = 2$ $\rightarrow$ $x^2 = x+2$

Evaluating and solving logarithms by changing to exponential form.
But there is a short-cut in some cases!

Try solving these:

BOOT the BASE

a) $\log_x 27 = 3$ $\rightarrow$ $\sqrt[3]{x^3} = \sqrt[3]{27}$

$\boxed{x = 3}$

☆ Note:
log have restrictions
$y = \log_c x$
$x > 0$
base is also restricted
$c > 0, c \neq 1$

Extra #4-7
p. 264

b) $\log_2 (2x-5) = 4$ $\rightarrow$ $2x-5 = 2^4$

$2x-5 = 16$
$\quad +5$

$\frac{2x}{2} = \frac{21}{2}$

$\boxed{x = \frac{21}{2}}$

☆ Restrictions
$2x-5 > 0$
$x > \frac{5}{2}$

c) $\log_{(x+1)} (2x-1) = 2$

$(x+1)^2 = 2x-1$

$(x+1)(x+1)$

$x^2 + 2x + 1 = 2x - 1$
$\quad -2x \quad +1$

$x^2 + 2 = 0$
$\quad -2$

$\sqrt{x^2} = \sqrt{-2}$

$x = \emptyset$ undefined

$\boxed{x = \text{no solution}}$ (no real roots)

- **Change of Base Law=** $\log_a x = \dfrac{\log x}{\log a}$

$$\log_7(10) = \frac{\log(10)}{\log(7)}$$
$$\approx \frac{1}{0.8451}$$
$$\approx 1.183$$

Try to evaluate these logs:

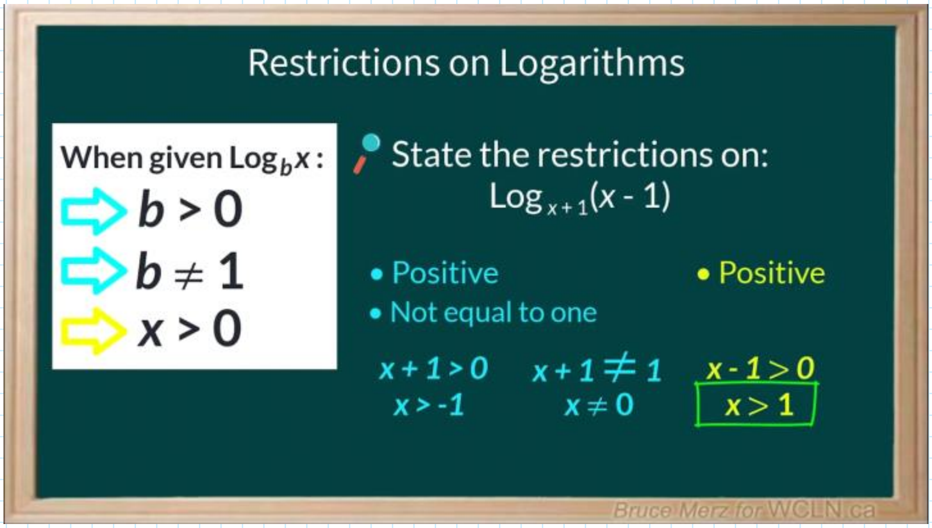
a) $\log_2 64$

b) $\log_4 0.0625$

c) $\log_{\frac{1}{2}} 32$

d) $\log_{1.2} 223.87$

Logarithms have restrictions (non-permissible values = NPVs)



Graphing a Logarithmic Function

Graphing a logarithmic function is the same as graphing the inverse of the exponential function with the same base.

Recall: to graph the inverse, switch the x and y for each coordinate and plot (the graph reflects over the line y=x and the domain and range also inverse)

Ex. Graph the function $y = \log_2 x$

1. First determine the points on the function $y = 2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

Step ① = graph exponential function with the same base as log functron

2. Inverse each coordinate and the asymptote to become $x = 0$

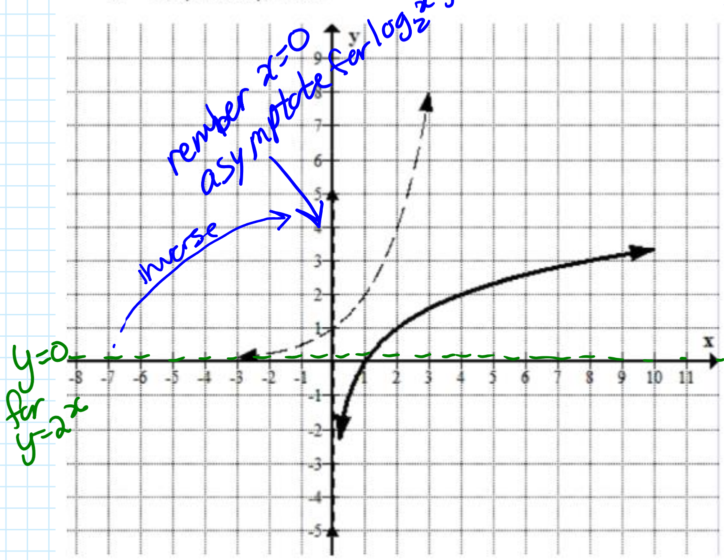
$y=2^x$

x	y
-2	¼
-1	½
0	1
1	2
2	4

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

Step ② = inverse the coordinates from $y=2^x$ to get $y=\log_2 x$

3. Graph the points.



Step ③ Graph the inverse to toget $y=\log_2 x$ graph.

Try graphing the following in the same steps as above:

a) $y = \log_3 x$ (base 3)

b) $y = \log_{\left(\frac{1}{2}\right)} x$ (base $\frac{1}{2}$)

step 1 $y = 2^x$

step 2

① , x

② inverse

step 1

$y = 3^x$

x	y
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

$3^{-2} \to \frac{1}{3^2}$

$3^0 \to 1$

asymptote
$y=0$

step 2

inverse
$\Rightarrow y = \log_3 x$

x	y
1/9	-2
1/3	-1
1	0
3	1
9	2

asymptote
$x=0$

① $y = \frac{1}{2}^x$

x	y
-3	8
-2	4
-1	2
0	1
1	1/2
2	1/4

$\left(\frac{1}{2}\right)^{-3} = 2^3$

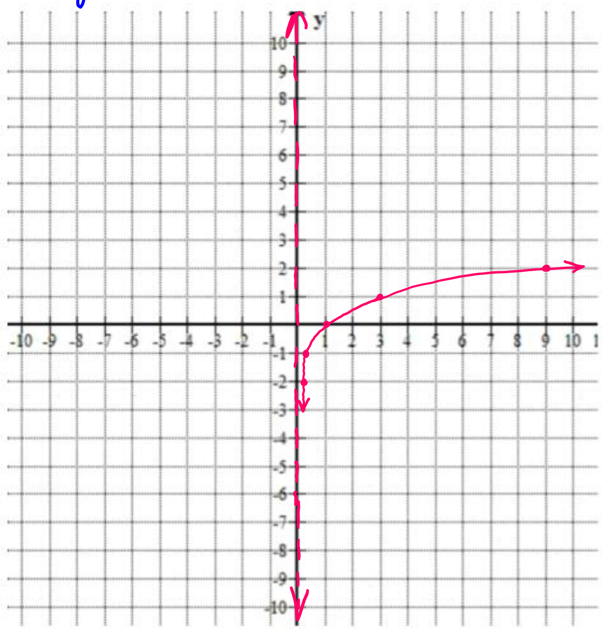
② inverse

$y = \log_{\frac{1}{2}} x$

x	y
8	-3
4	-2
2	-1
1	0
1/2	1
1/4	2

Step 3 ☆ inverse asymptote

③ Graph.

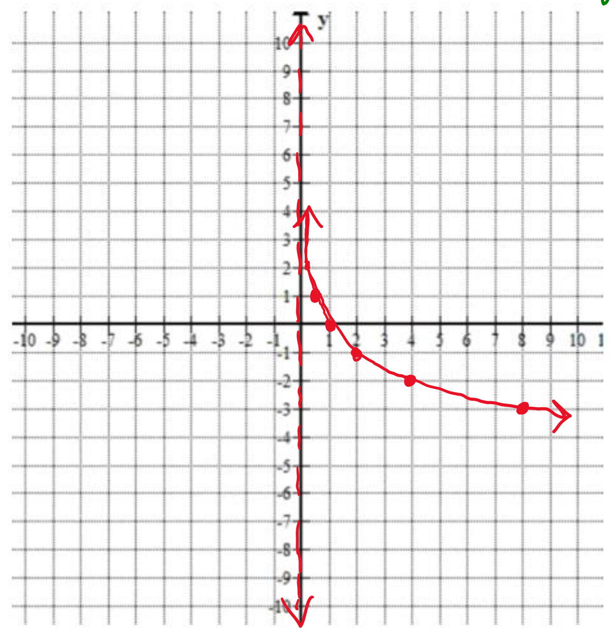


$x=0$

$\{x \mid x>0, x \in \mathbb{R}\}$

$\{y \mid y \in \mathbb{R}\}$

asymptote: $x=0$
(VA)



VA: $x=0$

$\{x \mid x>0, x \in \mathbb{R}\}$

$\{y \mid y \in \mathbb{R}\}$

asymptote $x=0$
VA