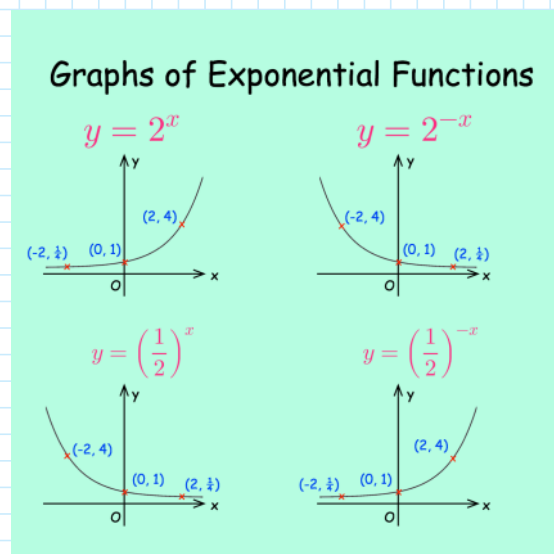


Tuesday, Feb. 20th

Plan For Today:

1. Questions from Chapter 4?
 - * Hand-in Chapter 4 Project
 - * Do Chapter 4 Test
2. Go over any review for Ch3 and Ch4
 - ▶ Unit 2 Exam is on Thursday
 - ▶ Rewrite will be following Tuesday after class
3. Start Chapter 5: Exponents & Logarithms
 - ▶ **5.1: Exponents**
 - ▶ 5.2: Logarithmic Functions and Graphs
 - ▶ 5.3: Properties of Logarithms
 - ▶ 5.4: Exponential and Logarithmic Equations
 - ▶ 5.5: Applications of Exponential and Log Equations
4. Work on Practice Questions from Workbook



Plan Going Forward:

1. Finish going through Chapter 3-4 practice questions and workbook reviews.

▶ **UNIT 2 EXAM ON CH3&4 ON THURSDAY, FEB. 22ND**

- 10 Multiple Choice & 20 marks on the Written
- ~1 hour - please prepare so you are not "learning" while doing the test
- Closed-book - no notes
- Rewrite is following Tuesday after class at 12:30pm
- I will email you this weekend when marks are posted so you can decide on the rewrite
- I will go over the marked exam on Tuesday

2. We will continue in Chapter 5 after the exam on Thursday.

- **CHAPTER 5 PROJECT DUE THURSDAY, MAR. 7TH**
- **CHAPTER 5 TEST ON THURSDAY, MAR. 7TH**

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class. Anurita Dhiman = adhiman@sd35.bc.ca

Tuesday, Feb. 20th In-Class Notes

Exponent Rules

EXPONENT RULES & PRACTICE

1. **PRODUCT RULE:** To multiply when two bases are the same, write the base and ADD the exponents.

$$x^m \cdot x^n = x^{m+n}$$

Examples:

A. $x^3 \cdot x^8 = x^{11}$

B. $2^4 \cdot 2^2 = 2^6$

C. $(x^2y)(x^3y^4) = x^5y^5$

2. **QUOTIENT RULE:** To divide when two bases are the same, write the base and SUBTRACT the exponents.

$$\frac{x^m}{x^n} = x^{m-n}$$

Examples:

A. $\frac{x^5}{x^2} = x^3$

B. $\frac{3^5}{3^3} = 3^2$

C. $\frac{x^2y^5}{xy^3} = xy^2$

3. **ZERO EXPONENT RULE:** Any base (except 0) raised to the zero power is equal to one.

$$x^0 = 1$$

Examples:

A. $y^0 = 1$

B. $6^0 = 1$

C. $(7a^3b^{-1})^0 = 1$

4. **POWER RULE:** To raise a power to another power, write the base and MULTIPLY the exponents.

$$(x^m)^n = x^{m \cdot n}$$

Examples:

A. $(x^3)^2 = x^6$

B. $(3^2)^4 = 3^8$

C. $(z^5)^2 = z^{10}$

5. **EXPANDED POWER RULE:**

$$(xy)^m = x^m y^m$$

$$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$$

$$(x+y)^2 \neq x^2 + y^2$$

$$= (x+y)(x+y)$$

$$= x^2 + 2xy + y^2$$

Examples:

A. $(2a)^3 = 2^3 a^3 = 8a^3$

B. $(6x^3)^2 = 6^2 (x^3)^2 = 36x^6$

C. $\left(\frac{x^2}{y}\right)^4 = \frac{(x^2)^4}{y^4} = \frac{x^8}{y^4}$

D. $\left(\frac{2x}{3y^2}\right)^3 = \frac{(2x)^3}{(3y^2)^3} = \frac{2^3 x^3}{3^3 (y^2)^3} = \frac{8x^3}{27y^6}$

6. **NEGATIVE EXPONENTS:** If a factor in the numerator or denominator is moved across the fraction bar, the sign of the exponent is changed.

$$x^{-m} = \frac{1}{x^m} \quad \frac{1}{x^{-m}} = x^m \quad \left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$$

Examples:

A. $x^{-3} = \frac{1}{x^3}$

B. $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

C. $-4x^5y^{-2} = \frac{-4x^5}{y^2}$

D. $\left(\frac{x^2}{y}\right)^{-3} = \left(\frac{y}{x^2}\right)^3 = \frac{y^3}{x^6}$

E. $(3x^{-2}y)(-2xy^{-3}) = -6x^{-1}y^{-2} = \frac{-6}{xy^2}$

F. $\frac{a^{-2}b^3}{c^{-4}d^{-1}} = \frac{b^3c^4d}{a^2}$

G. $(-2x^2y^{-4})^{-2} = \left(\frac{-2x^2}{y^4}\right)^{-2} = \left(\frac{y^4}{-2x^2}\right)^2 = \frac{y^8}{4x^4}$

CAUTION: $-x \neq \frac{1}{x}$ For example: $-3 \neq \frac{1}{3}$

REMEMBER: An exponent applies to only the factor it is directly next to *unless* parentheses enclose other factors.

Examples:

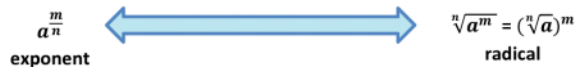
A. $(-3)^2 = (-3)(-3) = 9$

B. $-3^2 = -9$

An *exponential expression* with a fractional exponent can be expressed as a *radical* where the denominator is the index of the root, and the numerator remains as the exponent.

$$a^{\frac{m}{n}} \longleftrightarrow \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

exponent radical



Example 1: Write $125^{\frac{1}{3}}$ as a radical expression.

$$125^{\frac{1}{3}} = \sqrt[3]{125^1} = \sqrt[3]{125} \rightarrow = 5$$

EXPONENTS PRACTICE

Simplify:

- | | | |
|-------------------------|---|--|
| 1. $3 \cdot 4^3$ | 15. $\frac{x^5y^6}{xy^2}$ | 27. $\frac{x^{-1}}{x^{-8}}$ |
| 2. $4x^3 \cdot 2x^3$ | 16. $\frac{x^2y^5}{xy^4}$ | 28. $\frac{52x^6}{13x^{-7}}$ |
| 3. $x^5 \cdot x^3$ | 17. $\left(\frac{4x^5y}{16xy^4}\right)^3$ | 29. $f^{-3}(f^2)(f^{-3})$ |
| 4. $2x^3 \cdot 2x^2$ | 18. $\left(\frac{5x^3y}{20xy^5}\right)^4$ | 30. $\frac{x^{-4}}{x^{-9}}$ |
| 5. $\frac{6^5}{6^3}$ | 19. y^{-7} | 31. $\frac{24x^6}{12x^{-8}}$ |
| 6. $\frac{x^4}{x^7}$ | 20. 7^{-2} | 32. $\frac{3x^2y^{-3}}{12x^6y^3}$ |
| 7. 8^0 | 21. $\frac{1}{x^{-5}}$ | 33. $(2x^3y^{-3})^{-2}$ |
| 8. $-(9x)^0$ | 22. $\frac{1}{2^{-4}}$ | 34. $\frac{2x^4y^{-4}}{8x^7y^3}$ |
| 9. $(y^4)^3$ | 23. $x^5 \cdot x^{-1}$ | 35. $(4x^4y^{-4})^3$ |
| 10. $(x^2y)^4$ | 24. x^{-6} | 36. $5x^2y(2x^4y^{-3})$ |
| 11. $\frac{6x^7}{2x^4}$ | 25. $x^9 \cdot x^{-7}$ | 37. $\left(\frac{-7a^2b^3c^0}{3a^3b^4c^3}\right)^{-4}$ |
| 12. $\frac{8x^5}{4x^2}$ | 26. $(j^{-13})(j^4)(j^6)$ | 38. $\left(\frac{-2a^3b^2c^0}{3a^2b^3c^7}\right)^{-2}$ |
| 13. $(2cd^4)^2(cd)^5$ | | |
| 14. $(2fg^4)^4(fg)^6$ | | |

EXPONENTS PRACTICE ANSWERS

- | | | |
|----------------------|-----------------------------|-----------------------------------|
| 1. 192 | 16. xy | 29. $\frac{1}{f^4}$ |
| 2. $8x^6$ | 17. $\frac{x^{12}}{64y^9}$ | 30. x^5 |
| 3. x^8 | 18. $\frac{x^8}{256y^{16}}$ | 31. $2x^{14}$ |
| 4. $4x^5$ | 19. $\frac{1}{y^7}$ | 32. $\frac{1}{4x^4y^6}$ |
| 5. 36 | 20. $\frac{1}{49}$ | 33. $\frac{y^6}{4x^6}$ |
| 6. $\frac{1}{x^3}$ | 21. x^5 | 34. $\frac{1}{4x^3y^7}$ |
| 7. 1 | 22. 16 | 35. $\frac{64x^{12}}{y^{12}}$ |
| 8. -1 | 23. x^4 | 36. $\frac{10x^6}{y^2}$ |
| 9. y^{12} | 24. $\frac{1}{x^6}$ | 37. $\frac{81a^4b^4c^{12}}{2401}$ |
| 10. x^8y^4 | 25. x^2 | 38. $\frac{9b^2c^{14}}{4a^2}$ |
| 11. $3x^3$ | 26. $\frac{1}{j^3}$ | |
| 12. $2x^3$ | 27. x^7 | |
| 13. $4c^7d^{13}$ | 28. $4x^{13}$ | |
| 14. $16f^{10}g^{22}$ | | |
| 15. x^4y^4 | | |

Review Exponent Rules

Recall some exponent rules:

<https://demonstrations.wolfram.com/LawsOfExponents/>

One as Exponent	Any number raised to the power of 1 is that number
$x^1 = x$	$6^1 = 6$

ZERO EXPONENTS

Any number or any letter raised to the zero exponent is always one.

$$x^0 = 1$$

NEGATIVE EXPONENT

$$x^{-a} = \frac{1}{x^a}$$

$$\frac{1}{x^{-a}} = x^a$$

EXPONENT RULES EXPLAINED! MULTIPLYING EXPONENTS

$$3^2 \circ 3^3 \quad | \quad x^a \cdot x^b = x^{a+b}$$

$$3 \circ 3 \quad | \quad 3 \circ 3 \circ 3$$

EXPONENT RULES EXPLAINED!! DIVIDING EXPONENTS

$$\frac{5^7}{5^4} = \frac{\cancel{5} \cdot \cancel{5} \cdot \cancel{5} \cdot \cancel{5} \cdot 5 \cdot 5 \cdot 5}{\cancel{5} \cdot \cancel{5} \cdot \cancel{5} \cdot \cancel{5}} \quad | \quad \frac{x^a}{x^b} = x^{a-b}$$

EXPONENT RULES EXPLAINED!! POWER TO POWER

$$(4^2)^5 \quad | \quad (x^a)^b = x^{ab}$$

$$4^2 \circ 4^2 \circ 4^2 \circ 4^2 \circ 4^2$$

ALGEBRA BASICS

WHAT IS THE FRACTIONAL EXPONENTS RULE?

$$\sqrt{49^1} = 49^{\frac{1}{2}} \quad | \quad \text{RULE} \quad \sqrt[b]{x^a} = x^{\frac{a}{b}}$$

$$\sqrt[3]{27^1} = 27^{\frac{1}{3}}$$

EXPONENT RULES EXPLAINED!! PRODUCT TO A POWER

$$(2xy^3)^4 \quad | \quad 2^4 x^4 y^{3 \cdot 4} = 16x^4y^{12}$$

NOT $(2 + x + y^3)^4 \neq 2^4 + x^4 + y^{12}$

Raising a Quotient to a Power

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$

Example: $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

NOT $(2 + x + y^3)^4 \neq 2^4 + x^4 + y^{12}$

Example: $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

Summary:

Exponent Rules

The Product Rule for Exponents (Multiplying Like Bases With Exponents)

When you multiply like bases you add your exponents.

$$x^n \cdot x^m = x^{n+m}$$

$$2^3 \cdot 2^5 = 2^{3+5} = 2^8$$

$$w^2 \cdot w^3 = w^5$$

Quotient Rule for Exponents (Dividing Like Bases With Exponents)

When you divide like bases you subtract their exponents.

$$a^m \div a^n = a^{m-n}$$

$$7^5 \div 7^2 = 7^{5-2} = 7^3$$

$$2^2 \div 2^5 = 2^{2-5} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

Power of a Power Rule for Exponents (Base Raised to Two Exponents)

When you raise a base to two exponents, you multiply those exponents together.

$$(a^m)^n = a^{m \times n}$$

$$(a^5)^2 = a^{5 \times 2} = a^{10}$$

$$(2^2)^{-3} = 2^{2 \times -3} = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$$

Power of a Product Rule for Exponents (A Product Raised to an Exponent)

When you have a **PRODUCT** (not a sum or difference) raised to an exponent, you can simplify by raising each base in the product to that exponent.

$$(ab)^m = a^m b^m \quad (2x^2)^3 = 2^3 x^6 = 8x^6 \quad (2x^2)^{-3} = 2^{-3} x^{-6} = \frac{1}{2^3 x^6} = \frac{1}{8x^6}$$

Power of a Quotient (A Quotient Raised to an Exponent)

When you have a **QUOTIENT** (not a sum or difference) raised to an exponent, you raise each base in the numerator and denominator of the quotient to that exponent.

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad \left(\frac{x^5}{y^2}\right)^3 = \frac{x^{15}}{y^6} \quad \left(\frac{2^3}{4}\right)^2 = \frac{2^6}{4^2} = \frac{64}{16} = 4$$

Zero Exponents

Any base raised to an exponent of 0 has a value of 1.

$$a^0 = 1 \quad 100^0 = 1 \quad 5xyz^0 = 5xy \quad (5xyz)^0 = 1$$

Negative Exponents

A base raised to a negative exponent has the same value as the reciprocal of the base to the positive of the exponent.

$$a^{-4} = \left(\frac{1}{a}\right)^4 = \frac{1}{a^4} \quad 10^{-4} = \left(\frac{1}{10}\right)^4 = \frac{1}{10^4} \quad \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$$

Rule	Example	Notes
$a^n \cdot a^m = a^{n+m}$	$2^2 \cdot 2^3 = 2^{2+3} = 2^5$	The bases, which are the a's in this case, must be the same.
$\frac{a^n}{a^m} = a^{n-m}$	$\frac{2^5}{2^3} = 2^{5-3} = 2^2 = 4$	The bases, which are the a's in this case, must be the same.
$a^n \cdot b^n = (a \cdot b)^n$	$2^2 \cdot 3^2 = (2 \cdot 3)^2 = (6)^2 = 36$	The exponents, which are the n's in this case, must be the same.
$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$	$\frac{2^2}{3^2} = \left(\frac{2}{3}\right)^2 = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$	The exponents, the n's in this case, must be the same.
$(b^n)^m = b^{n \cdot m}$	$(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$	Alternatively, we can use the first rule: $(2^3)^2 = 2^3 \cdot 2^3 = 2^{3+3} = 2^6$
$a^{-m} = \frac{1}{a^m}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$	In order to solve, we must change negative exponents into positives with this method.
$\sqrt[m]{a^n} = a^{n/m}$	$\sqrt[3]{2^2} = 2^{2/3}$	In some situations, we must change radicals into exponents with this method.
$a^0 = 1$	$\left(\frac{1}{2}\right)^0 = 1, 3^0 = 1, \dots$	Any number raised to the power of zero is equal to 1.
$a^1 = a$	$\left(\frac{1}{4}\right)^1 = \frac{1}{4}, 6^1 = 6, \dots$	Any number raised to the power of 1 is equal to itself.

Lesson 1 Assignment
Review The Laws of Exponents

1.) Simplify the following and write answers with positive exponents only.

a) $-3x^{-2}y^5$ b) $\frac{24a^2}{3a^5}$ c) $\frac{16a^{-3}}{48a^{-7}b^2}$ d) $(3x^{-3}y^2)(-4x^{-2}y^{-7})$ e) $(-2m^4n^{-3})^{-3}$

f) $\frac{-m^{-3}(n^{-2})^3}{n^5(m^3)^2}$ g) $\left(\frac{2a^{-3}b^2}{3a^4b^{-1}}\right)^{-2}$ h) $5x^{-4}y^3 \times (15x^{-2}y^3)^{-1}$ i) $\frac{(15a^{-2})(18a^{-5})}{30a^{-8}}$ j) $\frac{7a^{-4}bc^3}{(2a^{-2}b^4c^{-1})^3}$

2.) Find the exact value of the following.

a) 7^{-3} b) $32^{\frac{2}{5}}$ c) $\left(\frac{3}{7}\right)^{-2}$ d) $\left(\frac{1}{6}\right)^{-3}$

Change the base of an exponential expression or equation:

3.) Evaluate the expressions for, $x = -1$, $y = 2$, $z = 3$.

a) $(2x^{-1}y^{-3}z)(3x^2y^3z^{-1})$ b) $\frac{(2x^2y^4z^{-2})^3}{4x^4y^{10}z^{-4}}$

4.) Convert the following to the indicated base.

a) 25^{-3x} to base 5. b) 64^{a+4} to base 4. c) $\frac{1}{216^{3x}}$ to base 6.

d) $\left(\frac{1}{32}\right)^{a+1}$ to base 2. e) $\left(\frac{81}{49}\right)^{2a}$ to base $\frac{3}{7}$.

ANSWERS

1a. $\frac{-3y^5}{x^2}$ 1b. $\frac{8}{a^3}$ 1c. $\frac{a^4}{3b^2}$ 1d. $\frac{-12}{x^5y^5}$ 1e. $-\frac{n^9}{8m^{12}}$ 1f. $\frac{-1}{m^9n^{11}}$ 1g. $\frac{9a^{14}}{4b^6}$ 1h. $\frac{1}{3x^2}$

1i. $9a$ 1j. $\frac{7a^2c^6}{8b^{11}}$

2a. $\frac{1}{343}$ 2b. 4 2c. $\frac{49}{9}$ 2d. 216

3a. -6 3b. $\frac{8}{9}$

4a. 5^{-6x} 4b. 4^{3a+12} 4c. 6^{-9x} 4d. 2^{-5a-5} 4e. $\frac{3^{8a}}{7^{4a}}$

Solving Exponents

RECALL:

Rules of Exponents or Laws of Exponents	
Multiplication Rule	$a^x \times a^y = a^{x+y}$
Division Rule	$a^x \div a^y = a^{x-y}$
Power of a Power Rule	$(a^x)^y = a^{xy}$
Power of a Product Rule	$(ab)^x = a^x b^x$
Power of a Fraction Rule	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
Zero Exponent	$a^0 = 1$
Negative Exponent	$a^{-x} = \frac{1}{a^x}$
Fractional Exponent	$a^{\frac{x}{y}} = \sqrt[y]{a^x}$

Solving when the base is a variable x :

Example 1 Solve the following equation for x .

$$x^{\frac{3}{2}} = 125$$

Example 2 Solve the following equation for x .

$$2x^{\frac{3}{4}} = 54$$

Example 3 Solve the following equation for x .

$$(2x + 1)^{\frac{2}{3}} = 4$$

When the variable is in the exponent:

- **convert each base to the same base**

Exponential Equations with Same Base

$$a^x = a^y \text{ If and only if } x = y$$

Examples:

$$\begin{aligned} 2^x &= 64 \\ \Rightarrow 2^x &= 2^6 \\ \Rightarrow x &= 6 \end{aligned}$$

$$\begin{aligned} 3^{2x} &= 27^{x-4} \\ \Rightarrow 3^{2x} &= (3^3)^{x-4} \\ \Rightarrow 2x &= 3(x-4) \\ \Rightarrow 2x &= 3x-12 \\ \Rightarrow -x &= -12 \\ \Rightarrow x &= 12 \end{aligned}$$

<https://slideplayer.com/slide/6897597/>

Solve by Equating Exponents

Solve $4^{3x} = 8^{x+1}$ Original Problem

$$(2^2)^{3x} = (2^3)^{x+1} \quad \text{Rewrite with base 2}$$

$$2^{6x} = 2^{3x+3} \quad \text{Simplify exponents}$$

$$6x = 3x + 3 \quad \text{Equate exponents}$$

$$x = 1 \quad \text{Simplify}$$

Goal 1: Solving Exponential Functions

Solving Exponential Equations:

- If possible, express both sides as powers of the same base

$$\begin{aligned} 27(3^{x+1}) &= 9^{2x-7} \\ 3^3(3^{x+1}) &= (3^2)^{2x-7} \\ 3^{3+x+1} &= 3^{2(2x-7)} \\ 3^{x+4} &= 3^{4x-14} \end{aligned}$$

- Equate the exponents
- Solve for variable

$$x + 4 = 4x - 14$$

$$4 = 3x - 14$$

$$18 = 3x$$

$$x = 6$$

Solve: $\left(\frac{2}{3}\right)^{x+6} = \left(\frac{8}{27}\right)^{3x}$

1. Find a common base

$$\frac{8}{27} = \left(\frac{2}{3}\right)^3$$

$$\left(\frac{2}{3}\right)^{x+6} = \left[\left(\frac{2}{3}\right)^3\right]^{3x}$$

$$\left(\frac{2}{3}\right)^{x+6} = \left(\frac{2}{3}\right)^{9x}$$

2. Equate the exponents

$$x + 6 = 9x$$

$$x = \frac{3}{4}$$

Example 1 Simplify $\frac{4^{6x+1}}{8^{4x+2}}$

Example 2 Solve $9^{2x-3} = 27^{1-x}$

Example 1 Solve the following equation for x .

$$2^{3x-1} = 16$$

Example 2 Solve the following equation for x .

$$27^{2x-1} = 9^{x+2}$$

Example 3 Solve the following equation for x .

$$\frac{1}{343^{x-1}} = 49^{2x-1}$$

APPLICATIONS

EXPONENTIAL GROWTH formula

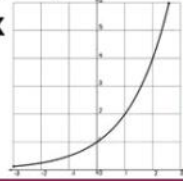
$$y = a(1 + r)^t$$

y = final amount
 a = initial amount
 r = rate (as a decimal)
 t = # of time periods

EXPONENTIAL GROWTH graph

$$y = a(b)^x$$

$a > 0$
 $b > 1$



EXPONENTIAL DECAY formula

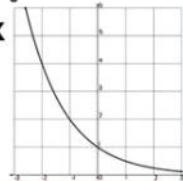
$$y = a(1 - r)^t$$

y = final amount
 a = initial amount
 r = rate (as a decimal)
 t = # of time periods

EXPONENTIAL DECAY graph

$$y = a(b)^x$$

$a > 0$
 $0 < b < 1$



COMPOUND INTEREST formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = final amount
 P = principal (starting amount)
 r = interest rate (as a decimal)
 n = # of times compounded per year
 t = time (in years)

Annually: $n = 1$
Semi-Annually: $n = 2$
Quarterly: $n = 4$
Monthly: $n = 12$
Daily: $n = 365$



EXPONENTIAL GROWTH & DECAY

Exponential growth and decay can be modeled using the formula: $A = A_0(b)^{\frac{t}{n}}$

A = final amount

A_0 = initial amount

b = base which is the factor of change (growth or decay factor)

t = time elapsed

n = interval of time for growth or decay

Compound Interest: $A = P(1 + i)^{tn}$ OR $A = P\left(1 + \frac{i}{n}\right)^{tn}$

P = Principal (initial amount)

i = Interest rate divided by the number of times compounded per year

n = number of compounding periods per year

Richter Scales for Earthquakes:

$$I = I_0(10)^{R_{\text{high}} - R_{\text{low}}}$$

I = the intensity between the two Richter scale magnitudes

R = difference in Richter scale magnitudes where 1 unit represents a 10-fold increase or decrease in magnitude.

Decibel Scale:

$$I = I_0(10)^{\frac{Db_{\text{high}} - Db_{\text{low}}}{10}}$$

I = the intensity of sound between the two decibel levels

Db = difference in decibel scale levels where 10 units represent a 10-fold increase or decrease in decibel level.

pH Scale:

$$I = I_0(10)^{pH_{\text{high}} - pH_{\text{low}}}$$

I = the level of acidity or basicity between the two pH values

pH = difference in values on pH scale where 1 unit represents a 10-fold increase or decrease in pH level (translates to change in acidity of a solution; either more or less acidic or basic).

The growth and decay formula is another variation of $f(x) = A(a^x)$.

Growth and Decay Formulas

$$A = A_0(x)^{\frac{t}{T}}$$

A : final amount

A_0 : initial amount

x : growth or decay value*

t : total time remaining

T : time of growth or decay by factor of x **

*for half-life questions, use $x = \frac{1}{2}$

for increasing by 10%, use $1 + .1 = 1.1$

for decreasing by 10%, use $1 - .1 = 0.9$

**for half-life questions, T = half-life

$$A = A_0e^{kt}$$

A : final amount

A_0 : initial amount

e : constant ≈ 2.71828

k : proportional constant

t : time

Example 7

The half-life of plutonium-239 is about 25 000 years. How much of a given sample will remain after 2000 years?

Example 8

The number of fruit flies increases by 25% every 3 days. If the population was 2000 fruit flies after 25 days, how many were there initially?

Solving Extra Practice

Exponential Equations Not Requiring Logarithms

Solve each equation.

1) $4^{2x+3} = 1$

2) $5^{3-2x} = 5^{-x}$

3) $3^{1-2x} = 243$

4) $3^{2a} = 3^{-a}$

5) $4^{3x-2} = 1$

6) $4^{2p} = 4^{-2p-1}$

7) $6^{-2a} = 6^{2-3a}$

8) $2^{2x+2} = 2^{3x}$

9) $6^{3m} \cdot 6^{-m} = 6^{-2m}$

10) $\frac{2^x}{2^x} = 2^{-2x}$

11) $10^{-3x} \cdot 10^x = \frac{1}{10}$

12) $3^{-2x+1} \cdot 3^{-2x-3} = 3^{-x}$

$$13) 4^{-2x} \cdot 4^x = 64$$

$$14) 6^{-2x} \cdot 6^{-x} = \frac{1}{216}$$

$$15) 2^x \cdot \frac{1}{32} = 32$$

$$16) 2^{-3p} \cdot 2^{2p} = 2^{2p}$$

$$17) 64 \cdot 16^{-3x} = 16^{3x-2}$$

$$18) \frac{81^{3n+2}}{243^{-n}} = 3^4$$

$$19) 81 \cdot 9^{-2b-2} = 27$$

$$20) 9^{-3x} \cdot 9^x = 27$$

$$21) \left(\frac{1}{6}\right)^{3x+2} \cdot 216^{3x} = \frac{1}{216}$$

$$22) 243^{k+2} \cdot 9^{2k-1} = 9$$

$$23) 16^r \cdot 64^{3-3r} = 64$$

$$24) 16^{2p-3} \cdot 4^{-2p} = 2^4$$

Exponential Equations Not Requiring Logarithms

Solve each equation.

1) $4^{2x+3} = 1$

$$\left\{-\frac{3}{2}\right\}$$

2) $5^{3-2x} = 5^{-x}$

$$\{3\}$$

3) $3^{1-2x} = 243$

$$\{-2\}$$

4) $3^{2a} = 3^{-a}$

$$\{0\}$$

5) $4^{3x-2} = 1$

$$\left\{\frac{2}{3}\right\}$$

6) $4^{2p} = 4^{-2p-1}$

$$\left\{-\frac{1}{4}\right\}$$

7) $6^{-2a} = 6^{2-3a}$

$$\{2\}$$

8) $2^{2x+2} = 2^{3x}$

$$\{2\}$$

9) $6^{3m} \cdot 6^{-m} = 6^{-2m}$

$$\{0\}$$

10) $\frac{2^x}{2^x} = 2^{-2x}$

$$\{0\}$$

11) $10^{-3x} \cdot 10^x = \frac{1}{10}$

$$\left\{\frac{1}{2}\right\}$$

12) $3^{-2x+1} \cdot 3^{-2x-3} = 3^{-x}$

$$\left\{-\frac{2}{3}\right\}$$

$$13) 4^{-2x} \cdot 4^x = 64$$
$$\{-3\}$$

$$14) 6^{-2x} \cdot 6^{-x} = \frac{1}{216}$$
$$\{1\}$$

$$15) 2^x \cdot \frac{1}{32} = 32$$
$$\{10\}$$

$$16) 2^{-3p} \cdot 2^{2p} = 2^{2p}$$
$$\{0\}$$

$$17) 64 \cdot 16^{-3x} = 16^{3x-2}$$
$$\left\{\frac{7}{12}\right\}$$

$$18) \frac{81^{3n+2}}{243^{-n}} = 3^4$$
$$\left\{-\frac{4}{17}\right\}$$

$$19) 81 \cdot 9^{-2b-2} = 27$$
$$\left\{-\frac{3}{4}\right\}$$

$$20) 9^{-3x} \cdot 9^x = 27$$
$$\left\{-\frac{3}{4}\right\}$$

$$21) \left(\frac{1}{6}\right)^{3x+2} \cdot 216^{3x} = \frac{1}{216}$$
$$\left\{-\frac{1}{6}\right\}$$

$$22) 243^{k+2} \cdot 9^{2k-1} = 9$$
$$\left\{-\frac{2}{3}\right\}$$

$$23) 16^r \cdot 64^{3-3r} = 64$$
$$\left\{\frac{6}{7}\right\}$$

$$24) 16^{2p-3} \cdot 4^{-2p} = 2^4$$
$$\{4\}$$

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