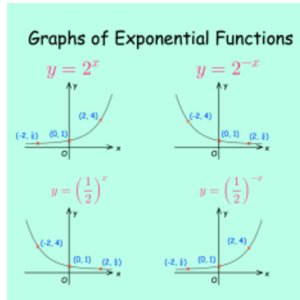
Thursday, Feb. 22nd

Plan For Today:

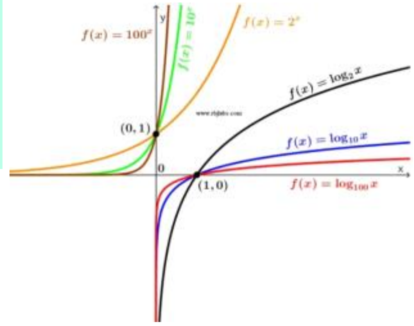
1. Questions from Chapter 3 or 4?
   - Do Unit 2 Exam
   - I will send an email on the weekend when they are marked
2. Start Chapter 5: Exponents & Logarithms
   - **5.1: Exponents**
   - **5.2: Logarithmic Functions and Graphs**
   - 5.3: Properties of Logarithms
   - 5.4: Exponential and Logarithmic Equations
   - 5.5: Applications of Exponential and Log Equations
3. Work on Practice Questions from Workbook



Argument

$$b^x = a \Leftrightarrow \log_b a = x$$

base



Plan Going Forward:

1. If you're planning to rewrite the U2 exam, continue reviewing until Tuesday.

   - **UNIT 2 EXAM REWRITE ON TUESDAY, FEB. 27TH**
     - 12 Multiple Choice & 18 marks on the Written
     - ~1.5 hour - please prepare so you are not "learning" while doing the test
     - Closed-book - no notes
     - Rewrite is following Tuesday after class at 12:30pm
     - I will email you this weekend when marks are posted so you can decide on the rewrite
     - I will go over the marked exam on Tuesday

2. We will continue in Chapter 5 on Tuesday.

   - **CHECK-IN QUIZ ON 5.1-5.2 ON TUESDAY, FEB. 27TH**
   - **CHAPTER 5 PROJECT (PART A&B) DUE THURSDAY, MAR. 7TH**
     - PART A IS IN DESMOS: http://tinyurl.com/PC12-Feb2024-Ch5PartA
     - PART B IS ON HANDOUT
   - **CHAPTER 5 TEST ON THURSDAY, MAR. 7TH**

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.
Anurita Dhiman = adhiman@sd35.bc.ca

Thursday, Feb. 22nd In-Class Notes

REVIEW FROM 5.1

Example 1 Simplify $\frac{4^{6x+1}}{8^{4x+2}}$

$4 = 2^2$

$8 = 2^3$

$$\frac{(2^2)^{6x+1}}{(2^3)^{4x+2}} = \frac{2^{12x+2}}{2^{12x+6}}$$

$\div \rightarrow -$

$$= 2^{12x+2-(12x+6)}$$

$$= 2^{12x+2-12x-6}$$

$$= 2^{-4}$$

$- \rightarrow$ reciprocal.

$$= \frac{1}{2^4}$$

$$= \boxed{\frac{1}{16}}$$

Example 2 Solve $9^{2x-3} = 27^{1-x}$

Example 7 The half-life of plutonium-239 is about 25 000 years. How much of a given sample will remain after 2000 years?

Example 8 The number of fruit flies increases by 25% every 3 days. If the population was 2000 fruit flies after 25 days, how many were there initially?

Intro to Exponential Functions

An exponential function is written in the form: $y = C^x$, where C is the base of the function.

To review exponent rules, complete all of the practice questions on the Exponent Rules handout.

Try these:

What is the base of each of the following functions?

a) $y = 3^x$

b) $y = \frac{1}{4}^x$

c) $y = 3\left(\frac{1}{2}\right)^{-x} + 1$

d) $y = -2(5)^{x-1} - 3$

To graph a basic exponential function in the form of $y = C^x$, start with a general set of x-values, substitute them into the function and determine the y-values. Do this in a table of values and graph the points.

Ex. Graph the function $y = 2^x$ you will have to keep in mind the fraction rules.

1. First determine the points on the function $y = 2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

$y = 2^x$

$y = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$y = 2^{-1} = \frac{1}{2^1} = \frac{1}{2}$

$y = 2^0 = 1$

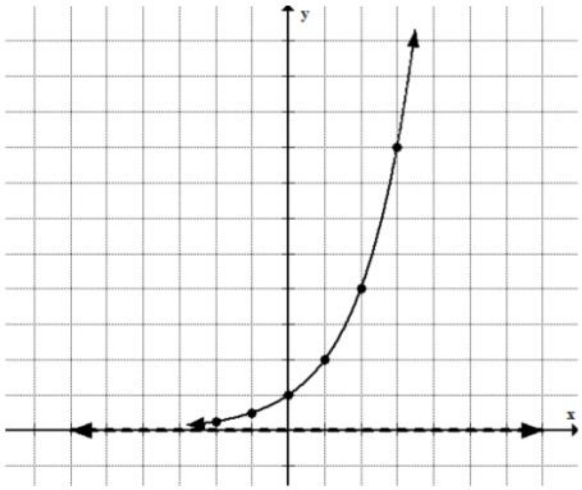
$y = 2^1 = 2$

$y = 2^2 = 4$

$y = 2^3 = 8$

Notice you will never be able to make y = 0 no matter how low x becomes. y will just get closer and closer to zero. The means there's an asymptote at y=0

2. Graph the points.

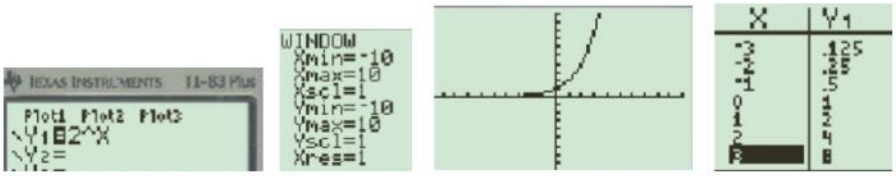


3. Determine the domain and range:

$\{x \mid x \in \mathbb{R}\}$

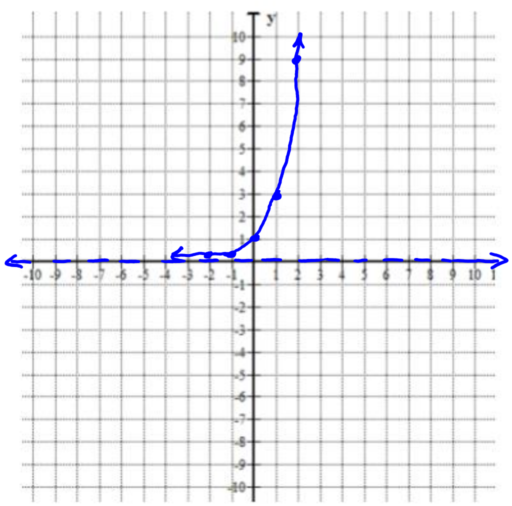
$\{y \mid y \geq 0, y \in \mathbb{R}\}$

To graph it in your calculator use the ^ to raise the base to x.



Try graphing the following in the same steps as above AND determine the domain and range:

a) $y = 3^x$



x	y
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

output → $y = 3^x$ ← input

$y = 3^{-2} \to \frac{1}{3^2} = \frac{1}{9}$

$y = 3^{-1} \to \frac{1}{3}$

$y = 3^0 \Rightarrow 1$

$y = 3^1 \Rightarrow 3$

$y = 3^2 \Rightarrow 9$

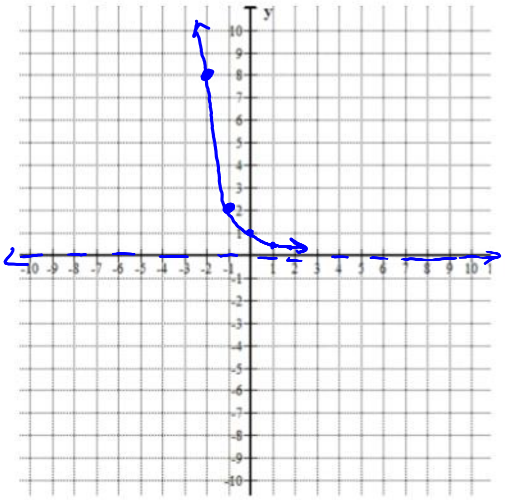
b) $y = \left(\frac{1}{2}\right)^x$

decreasing or decay

x	y
-3	8

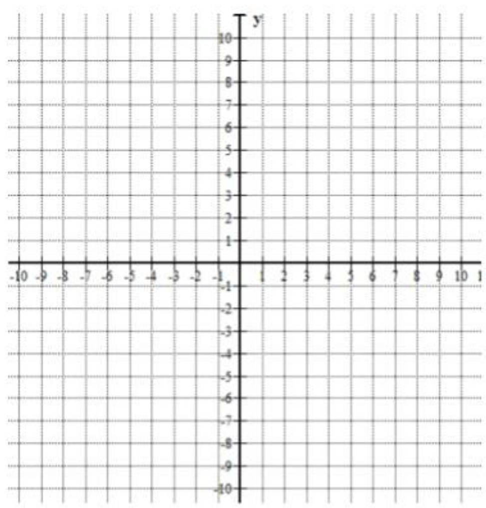
$y = \left(\frac{1}{2}\right)^x$

$= \left(\frac{1}{2}\right)^{-3} \rightarrow 2^3 = 8$



increasing or growth

a) c) $y = 4^x$



Example 3 Sketch the graph.

a) $f(x) = \left(\frac{1}{2}\right)^x - 1$ down 1 **b)** $g(x) = -3^{x-1} + 1$ up 1

right

ref in x-axis.

Base $y =$

Intro to Exponential Functions

An exponential function is written in the form: $y = C^x$, where C is the base of the function.

To review exponent rules, complete all of the practice questions on the Exponent Rules handout.

Try these:

What is the base of each of the following functions?

a) $y = 3^x$ base = 3

b) $y = \frac{1}{4}^x$ base = $\frac{1}{4}$

c) $y = 3\left(\frac{1}{2}\right)^{-x} + 1$ base = $\frac{1}{2}$

d) $y = -2(5)^{x-1} - 3$ base = 5

To graph a basic exponential function in the form of $y = C^x$, start with a general set of x-values, substitute them into the function and determine the y-values. Do this in a table of values and graph the points.

Ex. Graph the function $y = 2^x$ you will have to keep in mind the ~~fraction~~ exponent rules.

1. First determine the points on the function $y = 2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

$y = 2^x$

$y = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$y = 2^{-1} = \frac{1}{2^1} = \frac{1}{2}$

$y = 2^0 = 1$

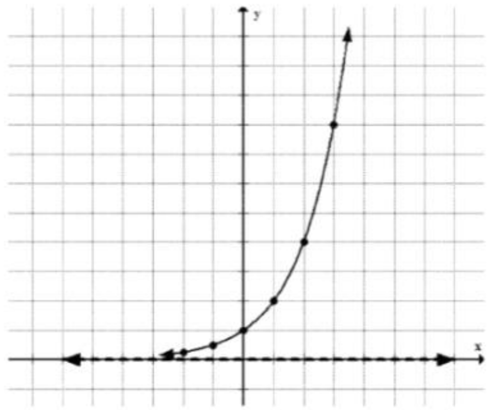
$y = 2^1 = 2$

$y = 2^2 = 4$

$y = 2^3 = 8$

Notice you will never be able to make y = 0 no matter how low x becomes. y will just get closer and closer to zero. The means there's an asymptote at y=0

2. Graph the points.

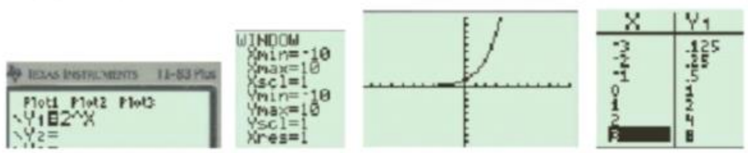


3. Determine the domain and range:

$\{x \mid x \in \mathbb{R}\}$

$\{y \mid y > 0, y \in \mathbb{R}\}$

To graph it in your calculator use the ^ to raise the base to x.



Try graphing the following in the same steps as above AND determine the domain and range:

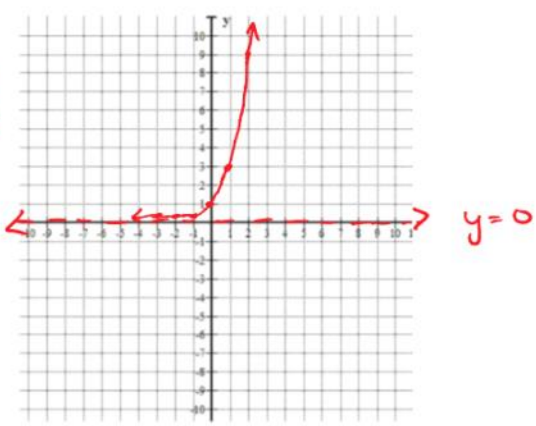
a) $y = 3^x$

x	y
-3	1/27
-2	1/9
-1	1/3
0	1
1	3
2	9
3	27

$3^{-3} = \frac{1}{3^3}$

$3^{-2} = \frac{1}{3^2}$

$3^{-1} = \frac{1}{3}$

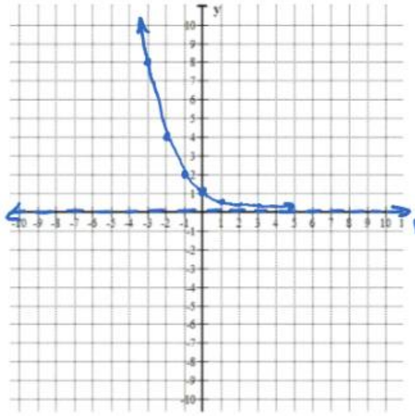


$\{x \mid x \in \mathbb{R}\}$

$\{y \mid y > 0, y \in \mathbb{R}\}$

b) $y = \left(\frac{1}{2}\right)^x$

x	y	
-3	8	$\left(\frac{1}{2}\right)^{-3} = 2^3$
-2	4	$\left(\frac{1}{2}\right)^{-2} = 2^2$
-1	2	$\left(\frac{1}{2}\right)^{-1} = 2^1$
0	1	$\left(\frac{1}{2}\right)^0$
1	$\frac{1}{2}$	$\left(\frac{1}{2}\right)^1$
2	$\frac{1}{4}$	$\left(\frac{1}{2}\right)^2$
3	$\frac{1}{8}$	$\left(\frac{1}{2}\right)^3$



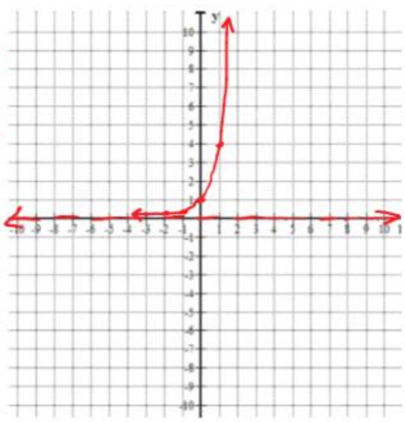
asymptote at $y=0$ (equation of asymptote)

$\{x \mid x \in \mathbb{R}\}$

$\{y \mid y > 0, y \in \mathbb{R}\}$.

a) c) $y = 4^x$

x	y	
-3	$\frac{1}{64}$	$(4)^{-3} = \frac{1}{4^3}$
-2	$\frac{1}{16}$	$4^{-2} = \frac{1}{4^2}$
-1	$\frac{1}{4}$	$4^{-1} = \frac{1}{4}$
0	1	4^0
1	4	4^1
2	16	4^2
3	64	4^3



$y=0$ ← equation of the asymptote

$\{x \mid x \in \mathbb{R}\}$

$\{y \mid y > 0, y \in \mathbb{R}\}$

Solving Exponents

RECALL:

Rules of Exponents or Laws of Exponents	
Multiplication Rule	$a^x \times a^y = a^{x+y}$
Division Rule	$a^x \div a^y = a^{x-y}$
Power of a Power Rule	$(a^x)^y = a^{xy}$
Power of a Product Rule	$(ab)^x = a^x b^x$
Power of a Fraction Rule	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
Zero Exponent	$a^0 = 1$
Negative Exponent	$a^{-x} = \frac{1}{a^x}$
Fractional Exponent	$a^{\frac{x}{y}} = \sqrt[y]{a^x}$

Solving when the base is a variable x:

Example 1 Solve the following equation for x .

$$x^{\frac{3}{2}} = 125$$

Example 2 Solve the following equation for x .

$$2x^{\frac{3}{4}} = 54$$

Example 3 Solve the following equation for x.

$$(2x+1)^{\frac{2}{3}} = 4$$

When the variable is in the exponent:

- **convert each base to the same base**

Exponential Equations with Same Base

$a^x = a^y$ If and only if $x = y$

Examples:

$2^x = 64$
$\Rightarrow 2^x = 2^6$
$\Rightarrow x = 6$

$3^{2x} = 27^{x-4}$
$\Rightarrow 3^{2x} = \left(3^3\right)^{x-4}$
$\Rightarrow 2x = 3(x-4)$
$\Rightarrow 2x = 3x - 12$
$\Rightarrow -x = -12$
$\Rightarrow x = 12$

https://slideplayer.com/slide/6897597/

Solve by Equating Exponents

Solve $4^{3x} = 8^{x+1}$	Original Problem
$\left(2^2\right)^{3x} = \left(2^3\right)^{x+1}$	Rewrite with base 2
$2^{6x} = 2^{3x+3}$	Simplify exponents
$6x = 3x + 3$	Equate exponents
$x = 1$	Simplify

Goal 1: Solving Exponential Functions

Solving Exponential Equations:

- If possible, express both sides as powers of the same base

$27(3^{x+1}) = 9^{2x-7}$
$3^3(3^{x+1}) = (3^2)^{2x-7}$
$3^{3+x+1} = 3^{2(2x-7)}$
$3^{x+4} = 3^{4x-14}$

- Equate the exponents

$x + 4 = 4x - 14$

- Solve for variable

$4 = 3x - 14$
$18 = 3x$
$\boxed{x = 6}$

Solve: $\left(\frac{2}{3}\right)^{x+6} = \left(\frac{8}{27}\right)^{3x}$

1. Find a common base

$\frac{8}{27} = \left(\frac{2}{3}\right)^3$

$\left(\frac{2}{3}\right)^{x+6} = \left[\left(\frac{2}{3}\right)^3\right]^{3x}$

$\left(\frac{2}{3}\right)^{x+6} = \left(\frac{2}{3}\right)^{9x}$

2. Equate the exponents

$x + 6 = 9x$

$x = \frac{3}{4}$

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Example 1 Solve the following equation for x.

$2^{3x-1} = 16$

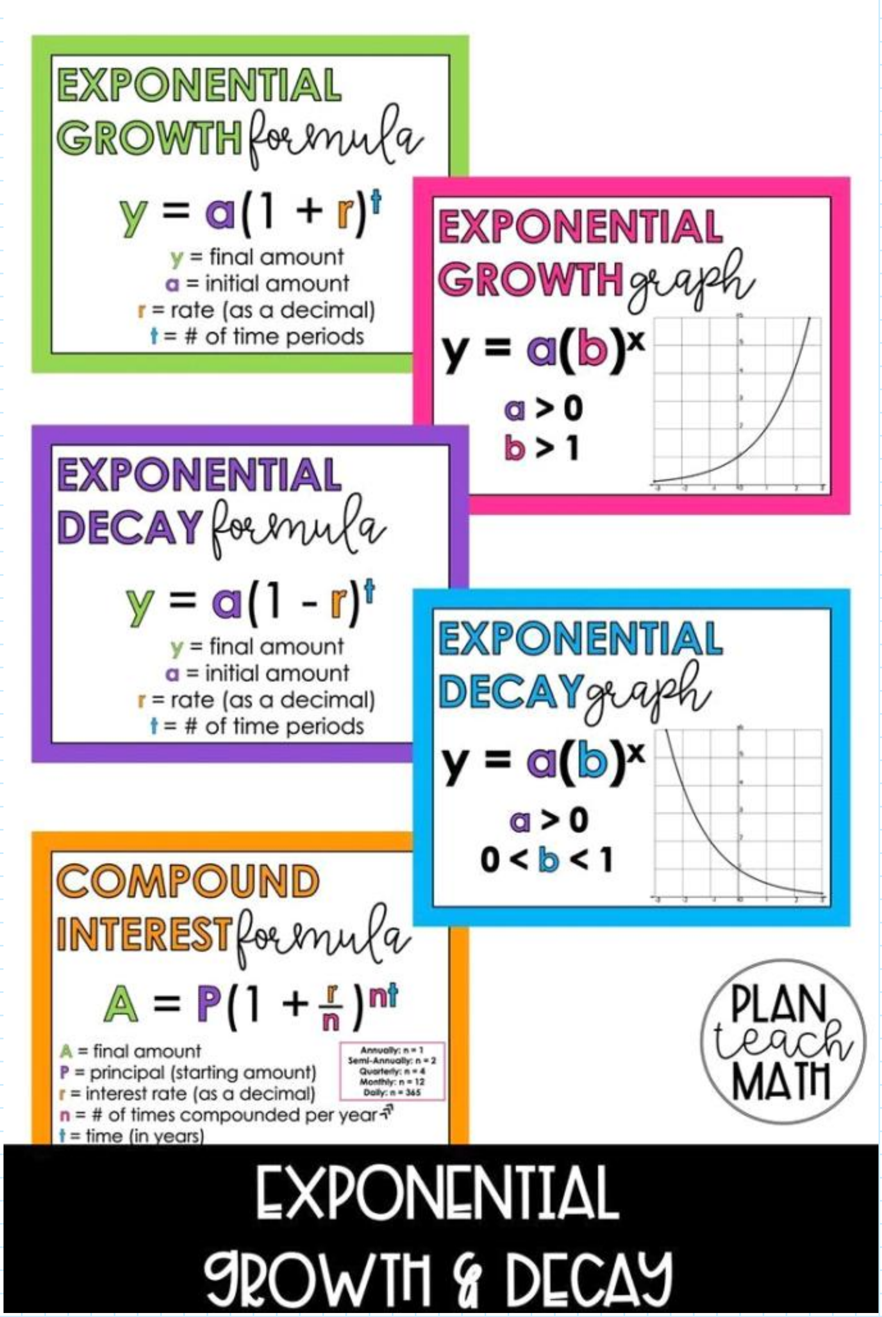
Example 2 Solve the following equation for x.

$27^{2x-1} = 9^{x+2}$

Example 3 Solve the following equation for x.

$\frac{1}{343^{x-1}} = 49^{2x-1}$

APPLICATIONS



Exponential growth and decay can be modeled using the formula: $A = A_o(b)^{\frac{t}{n}}$

A = final amount
A_o = initial amount
b = base which is the factor of change (growth or decay factor)
t = time elapsed
n = interval of time for growth or decay

Compound Interest: $A = P(1+i)^{tn}$ OR $A = P\left(1+\frac{r}{n}\right)^{tn}$
P = Principal (initial amount)
i = Interest rate divided by the number of times compounded per year
n = number of compounding periods per year

Richter Scales for Earthquakes:

$$I = I_o(10)^{R_{high} - R_{low}}$$

I = the intensity between the two Richter scale magnitudes
R = difference in Richter scale magnitudes where 1 unit represents a 10-fold increase or decrease in magnitude.

Decibel Scale:

$$I = I_o(10)^{\frac{Db_{high} - Db_{low}}{10}}$$

I = the intensity of sound between the two decibel levels
Db = difference in decibel scale levels where 10 units represent a 10-fold increase or decrease in decibel level.

pH Scale:

$$I = I_o(10)^{pH_{high} - pH_{low}}$$

I = the level of acidity or basicity between the two pH values
pH = difference in values on pH scale where 1 unit represents a 10-fold increase or decrease in pH level (translates to change in acidity of a solution; either more or less acidic or basic).

The growth and decay formula is another variation of $f(x) = A(a^x)$.

Growth and Decay Formulas

$A = A_0(x)^{\frac{t}{T}}$	$A = A_0e^{kt}$
A: final amount	A: final amount
A_0: initial amount	A_0: initial amount
x: growth or decay value*	e: constant ≈ 2.71828
t: total time remaining	k: proportional constant
T: time of growth or decay by factor of x**	t: time

*for half-life questions, use $x = \frac{1}{2}$
for increasing by 10%, use $1 + .1 = 1.1$
for decreasing by 10%, use $1 - .1 = 0.9$
**for half-life questions, T = half-life

Solving Extra Practice

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Exponential Equations Not Requiring Logarithms

Date________________ Period____

Solve each equation.

1) $4^{2x+3} = 1$

2) $5^{3-2x} = 5^{-x}$

3) $3^{1-2x} = 243$

4) $3^{2a} = 3^{-a}$

5) $4^{3x-2} = 1$

6) $4^{2p} = 4^{-2p-1}$

7) $6^{-2a} = 6^{2-3a}$

8) $2^{2x+2} = 2^{3x}$

9) $6^{3m} \cdot 6^{-m} = 6^{-2m}$

10) $\frac{2^{x}}{2^{x}} = 2^{-2x}$

11) $10^{-3x} \cdot 10^{x} = \frac{1}{10}$

12) $3^{-2x+1} \cdot 3^{-2x-3} = 3^{-x}$

13) $4^{-2x} \cdot 4^{x} = 64$

14) $6^{-2x} \cdot 6^{-x} = \frac{1}{216}$

15) $2^{x} \cdot \frac{1}{32} = 32$

16) $2^{-3p} \cdot 2^{2p} = 2^{2p}$

17) $64 \cdot 16^{-3x} = 16^{3x - 2}$

18) $\frac{81^{3n + 2}}{243^{-n}} = 3^{4}$

19) $81 \cdot 9^{-2b - 2} = 27$

20) $9^{-3x} \cdot 9^{x} = 27$

21) $\left(\frac{1}{6}\right)^{3x + 2} \cdot 216^{3x} = \frac{1}{216}$

22) $243^{k + 2} \cdot 9^{2k - 1} = 9$

23) $16^{r} \cdot 64^{3 - 3r} = 64$

24) $16^{2p - 3} \cdot 4^{-2p} = 2^{4}$

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Exponential Equations Not Requiring Logarithms

Date________________ Period____

Solve each equation.

1) $4^{2x+3} = 1$

$\left\{-\frac{3}{2}\right\}$

2) $5^{3-2x} = 5^{-x}$

$\{3\}$

3) $3^{1-2x} = 243$

$\{-2\}$

4) $3^{2a} = 3^{-a}$

$\{0\}$

5) $4^{3x-2} = 1$

$\left\{\frac{2}{3}\right\}$

6) $4^{2p} = 4^{-2p-1}$

$\left\{-\frac{1}{4}\right\}$

7) $6^{-2a} = 6^{2-3a}$

$\{2\}$

8) $2^{2x+2} = 2^{3x}$

$\{2\}$

9) $6^{3m} \cdot 6^{-m} = 6^{-2m}$

$\{0\}$

10) $\frac{2^{x}}{2^{x}} = 2^{-2x}$

$\{0\}$

11) $10^{-3x} \cdot 10^{x} = \frac{1}{10}$

$\left\{\frac{1}{2}\right\}$

12) $3^{-2x+1} \cdot 3^{-2x-3} = 3^{-x}$

$\left\{-\frac{2}{3}\right\}$

13) $4^{-2x} \cdot 4^{x} = 64$

$\{-3\}$

14) $6^{-2x} \cdot 6^{-x} = \frac{1}{216}$

$\{1\}$

15) $2^{x} \cdot \frac{1}{32} = 32$

$\{10\}$

16) $2^{-3p} \cdot 2^{2p} = 2^{2p}$

$\{0\}$

17) $64 \cdot 16^{-3x} = 16^{3x-2}$

$\left\{\frac{7}{12}\right\}$

18) $\frac{81^{3n+2}}{243^{-n}} = 3^{4}$

$\left\{-\frac{4}{17}\right\}$

19) $81 \cdot 9^{-2b-2} = 27$

$\left\{-\frac{3}{4}\right\}$

20) $9^{-3x} \cdot 9^{x} = 27$

$\left\{-\frac{3}{4}\right\}$

21) $\left(\frac{1}{6}\right)^{3x+2} \cdot 216^{3x} = \frac{1}{216}$

$\left\{-\frac{1}{6}\right\}$

22) $243^{k+2} \cdot 9^{2k-1} = 9$

$\left\{-\frac{2}{3}\right\}$

23) $16^{r} \cdot 64^{3-3r} = 64$

$\left\{\frac{6}{7}\right\}$

24) $16^{2p-3} \cdot 4^{-2p} = 2^{4}$

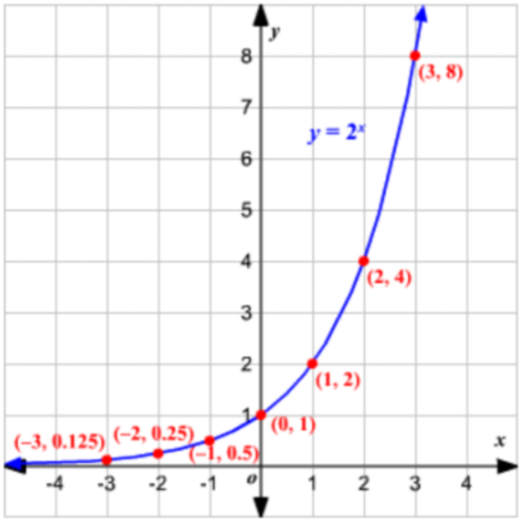
$\{4\}$

Graphs and Transformations of Exponential Functions

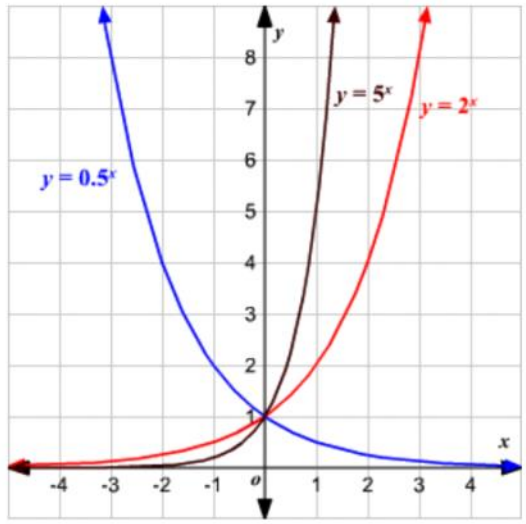
https://www.varsitytutors.com/hotmath/hotmath_help/topics/graphing-exponential-functions

A simple exponential function to graph is $y = 2^x$.

x	-3	-2	-1	0	1	2	3
$y = 2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

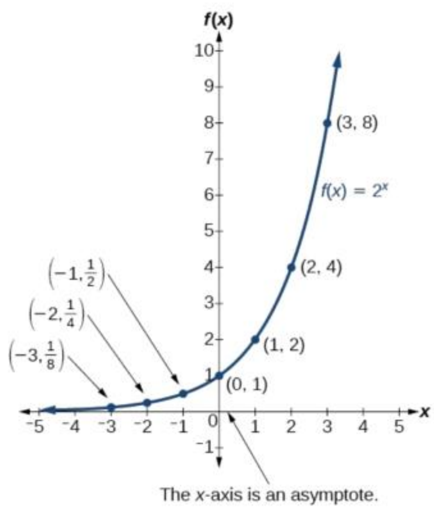


Changing the base changes the shape of the graph.



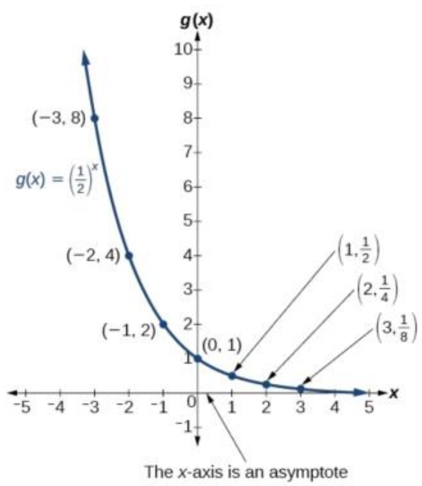
Notice that the graph has the x -axis as an asymptote on the left, and increases very fast on the right.

The graph below shows the exponential growth function $f(x) = 2^x$.



Notice that the graph gets close to the x-axis but never touches it.

The graph below shows the exponential decay function, $g(x) = \left(\frac{1}{2}\right)^x$.



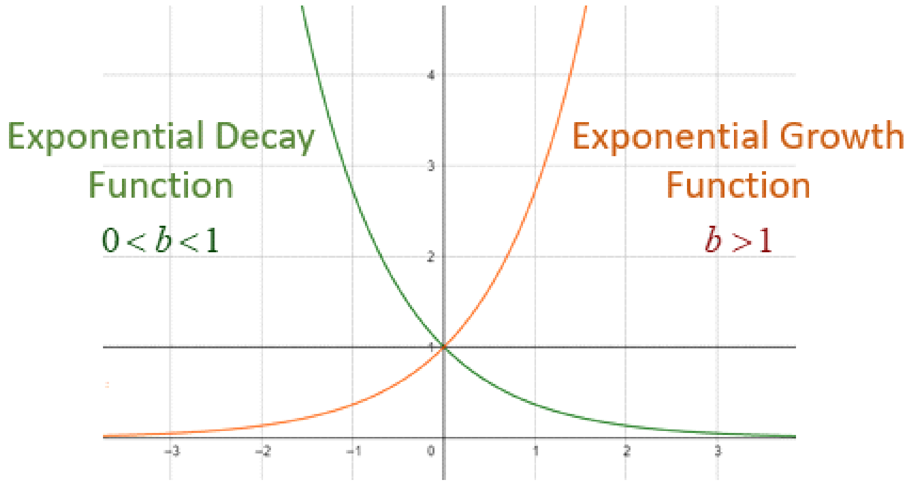
The domain of $g(x) = \left(\frac{1}{2}\right)^x$ is all real numbers, the range is $(0, \infty)$, and the horizontal asymptote is $y = 0$.

Exponential Growth and Decay Functions

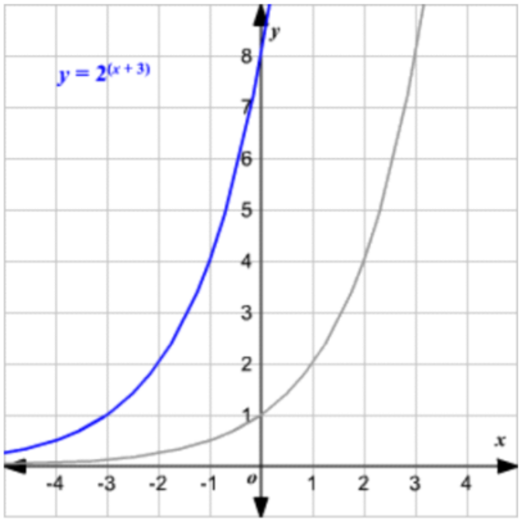
An exponential function f is given by

$$f(x) = b^x$$

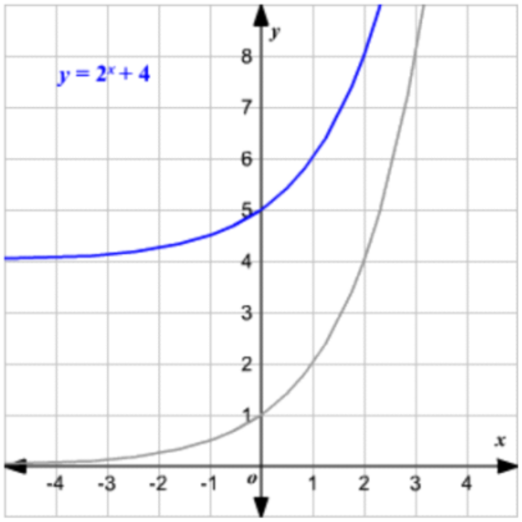
where x is any real number, b > 0, and b ≠ 1.



Replacing x with $x + h$ translates the graph h units to the left.



Replacing y with $y - k$ (which is the same as adding k to the right side) translates the graph k units up.



A GENERAL NOTE: CHARACTERISTICS OF THE GRAPH OF THE PARENT FUNCTION $f(x) = b^x$

An exponential function with the form $f(x) = b^x$, $b > 0$, $b \neq 1$, has these characteristics:

- **one-to-one** function
- horizontal asymptote: $y = 0$
- domain: $(-\infty, \infty)$
- range: $(0, \infty)$
- *x*-intercept: none
- *y*-intercept: $(0, 1)$
- increasing if $b > 1$
- decreasing if $b < 1$

HOW TO: GIVEN AN EXPONENTIAL FUNCTION OF THE FORM $f(x) = b^x$, GRAPH THE FUNCTION

1. Create a table of points.
2. Plot at least 3 point from the table including the *y*-intercept $(0, 1)$.
3. Draw a smooth curve through the points.
4. State the domain, $(-\infty, \infty)$, the range, $(0, \infty)$, and the horizontal asymptote, $y = 0$.

Transformation	Equation	Description
Horizontal stretch	$g(x) = c^{bx}$	Horizontal stretch about the y-axis by a factor of $\frac{1}{\lvert b \rvert}$.
Vertical stretch	$g(x) = a\,c^x$	• Vertical stretch about the x-axis by a factor of $\lvert a \rvert$. • Multiplying **y**-coordintates of $f(x) = c^x$ by a.
Reflecting	$g(x) = -c^x$	• Reflects the graph of $f(x) = c^x$ about the **x**-axis.
	$g(x) = c^{-x}$	• Reflects the graph of $f(x) = c^x$ about the **y**-axis.
Vertical translation	$g(x) = c^x + k$	• Shifts the graph of $f(x) = c^x$ upward k units if $k > 0$. • Shifts the graph of $f(x) = c^x$ downward k units if $k < 0$.
Horizontal translation	$g(x) = c^{x-h}$	• Shifts the graph of $f(x) = c^x$ to the right h units if $h > 0$. • Shifts the graph of $f(x) = c^x$ to the left h units if $h < 0$.

Transformations of Exponential Functions

Transformation	$f(x)$ Notation	Examples	
Vertical translation	$f(x) + k$	$y = 2^x + 3$	3 units up
		$y = 2^x - 6$	6 units down
Horizontal translation	$f(x - h)$	$y = 2^{x-2}$	2 units right
		$y = 2^{x+1}$	1 unit left
Vertical stretch or compression	$af(x)$	$y = 6(2^x)$	stretch by 6
		$y = \frac{1}{2}(2^x)$	compression by $\frac{1}{2}$
Horizontal stretch or compression	$f\left(\frac{1}{b}x\right)$	$y = 2^{\left(\frac{1}{5}x\right)}$	stretch by 5
		$y = 2^{3x}$	compression by $\frac{1}{3}$
Reflection	$-f(x)$	$y = -2^x$	across x-axis
	$f(-x)$	$y = 2^{-x}$	across y-axis

The basic properties of the graph $f(x) = b^x$ can be stated as follows:

Basic Properties of the Graph $f(x) = b^x$, $b > 0$, $b \neq 0$

1. All graphs go through the point $(0, 1)$, and the graph has no x-intercept.
2. The x-axis is a horizontal asymptote with equation $y = 0$.
3. When $b > 1$, $f(x) = b^x$ is an increasing function.
4. When $0 < b < 1$, $f(x) = b^x$ is a decreasing function.

Practice

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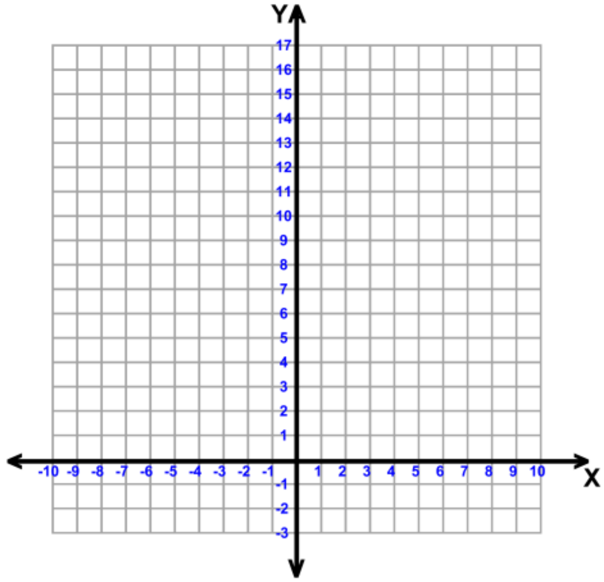
Teacher : ____________________________ Date : ___________

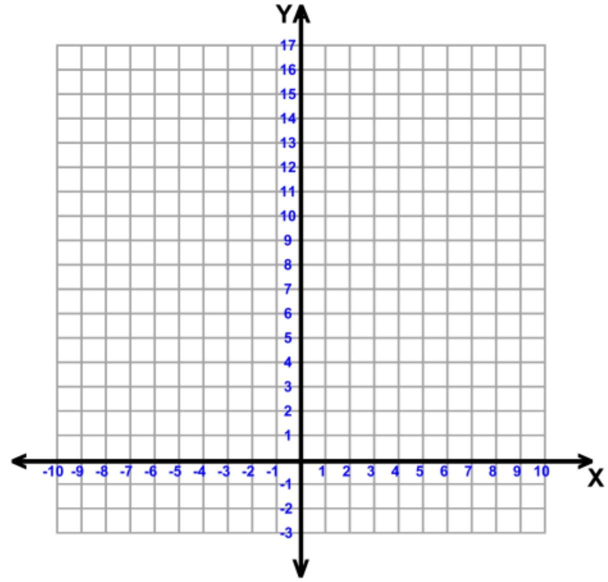
Graphing Exponential Functions

Sketch the graph of each function.

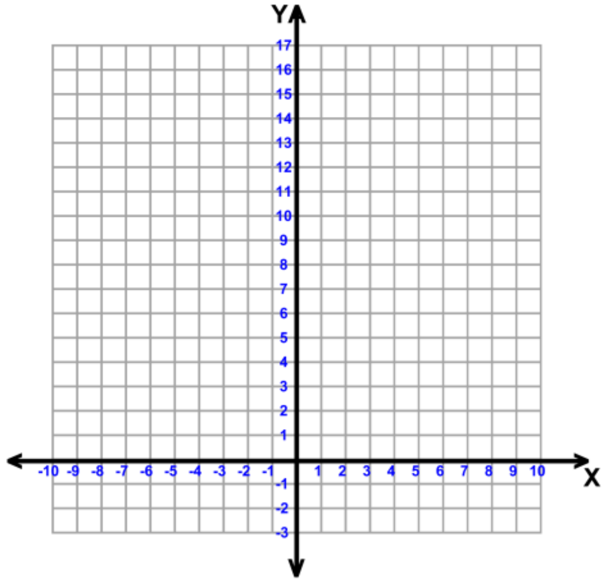
1) $y = 3 \cdot \left(\frac{1}{2}\right)^{x+3} - 3$



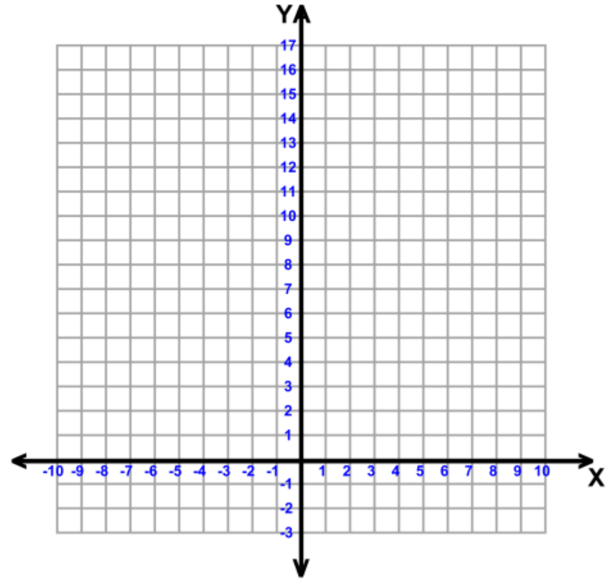
2) $y = 4 \cdot 2^{x+3} - 3$



3) $y = 3 \cdot \left(\frac{1}{4}\right)^{x}$



4) $y = 4 \cdot 3^{x}$



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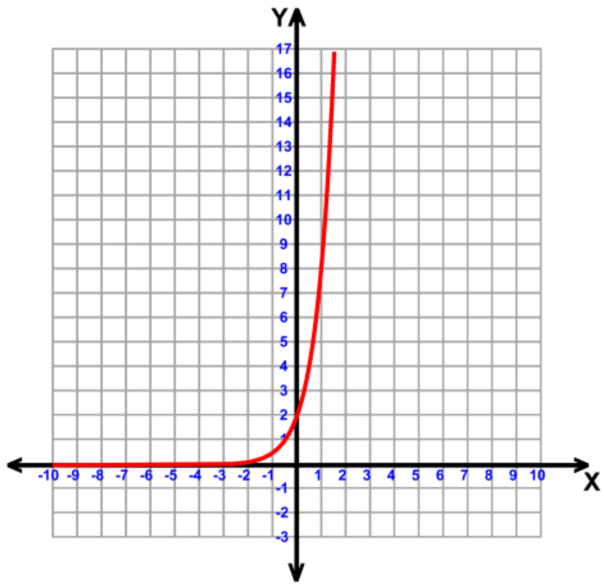
Graphing Exponential Functions

Write an equation for each graph.

5)



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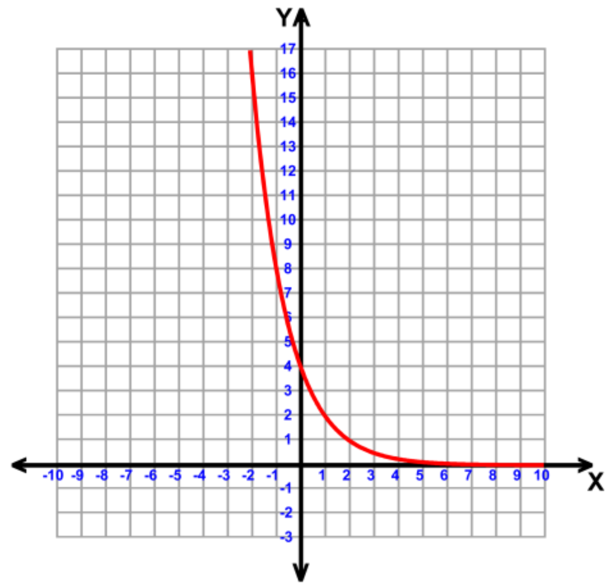

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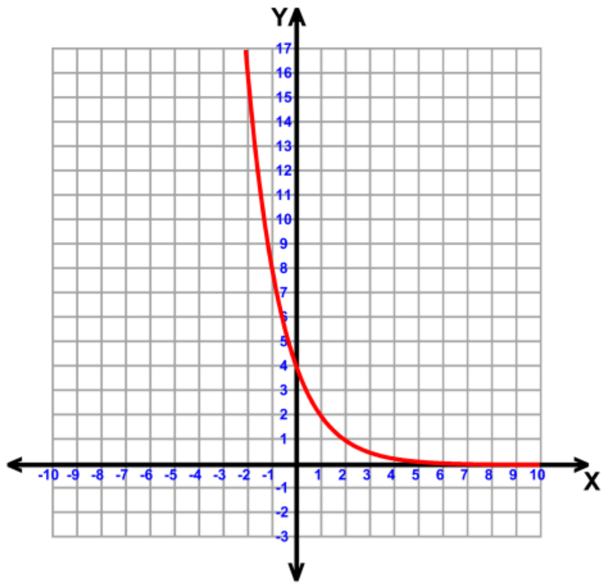

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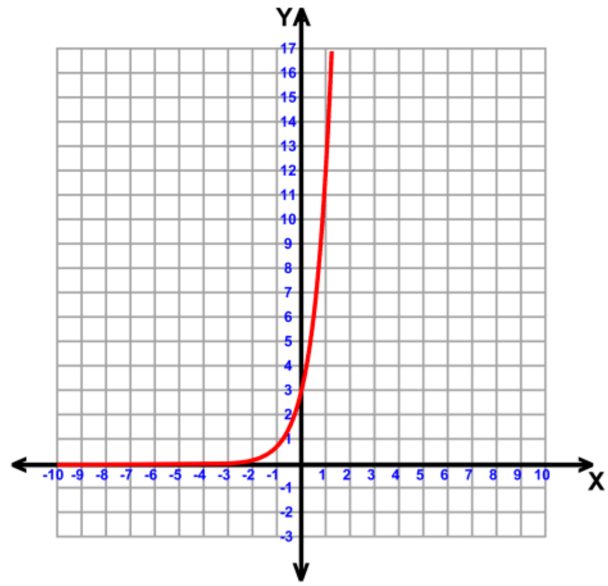

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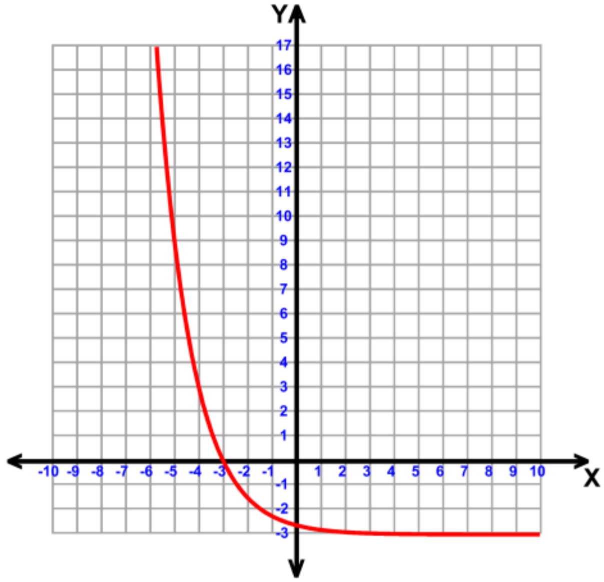
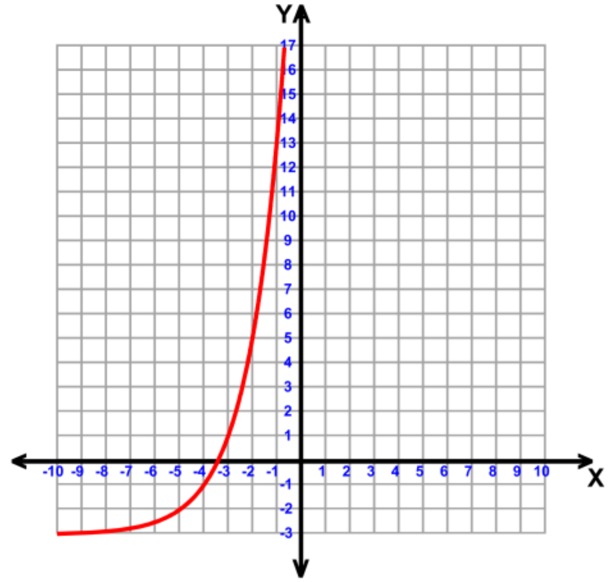
Graphing Exponential Functions

Sketch the graph of each function.

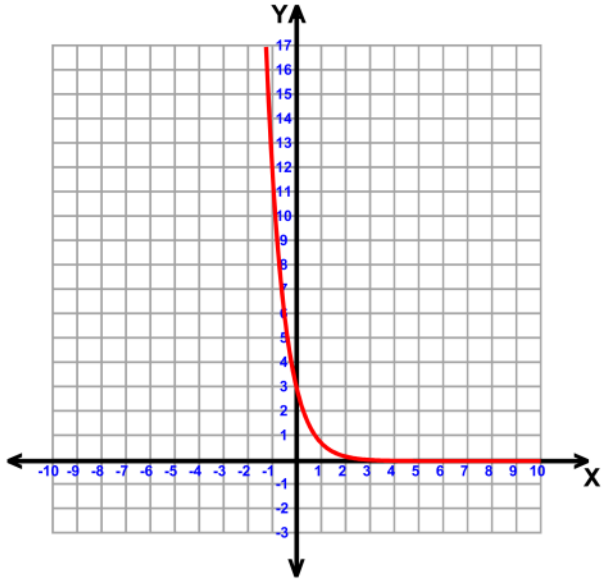
1) $y = 3 \cdot \left(\frac{1}{2}\right)^{x+3} - 3$



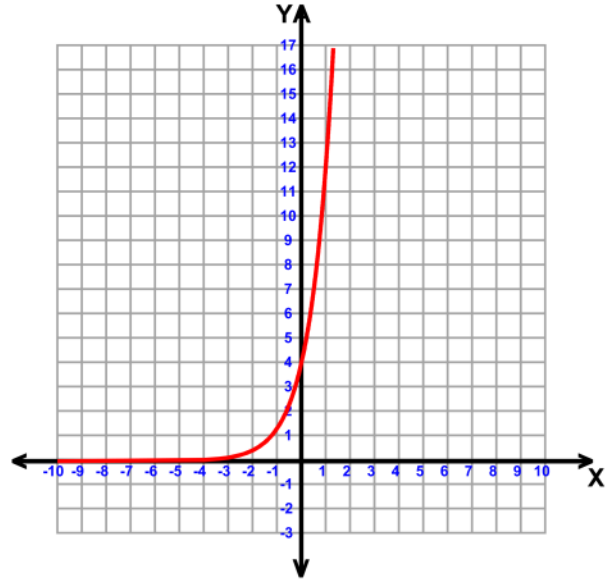
2) $y = 4 \cdot 2^{x+3} - 3$



3) $y = 3 \cdot \left(\frac{1}{4}\right)^{x}$



4) $y = 4 \cdot 3^{x}$



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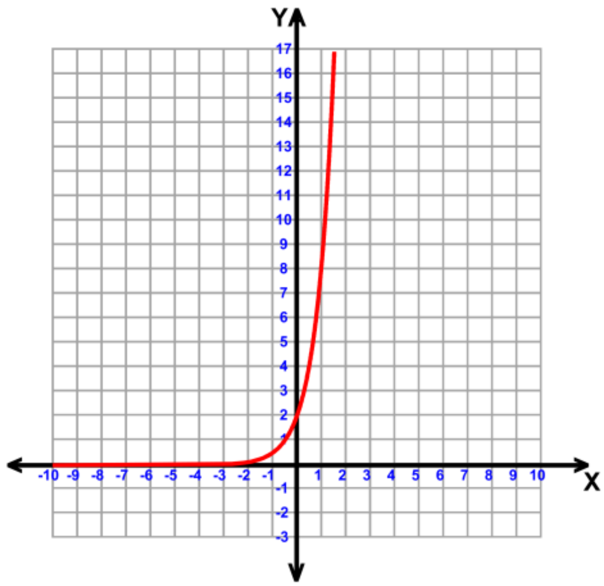
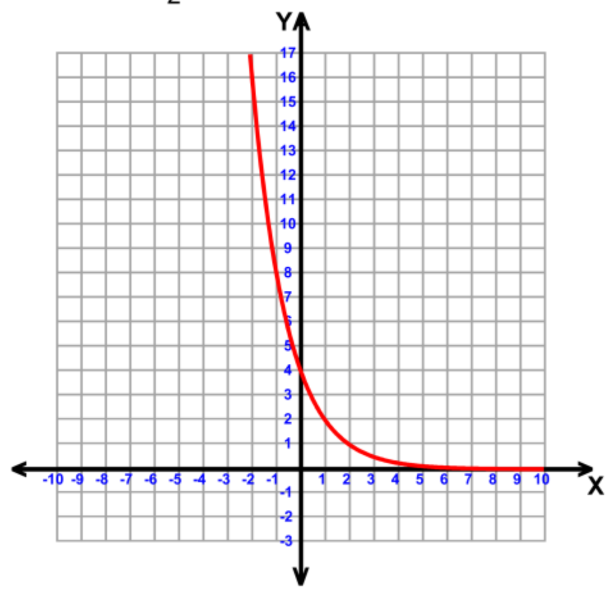
Graphing Exponential Functions

Write an equation for each graph.

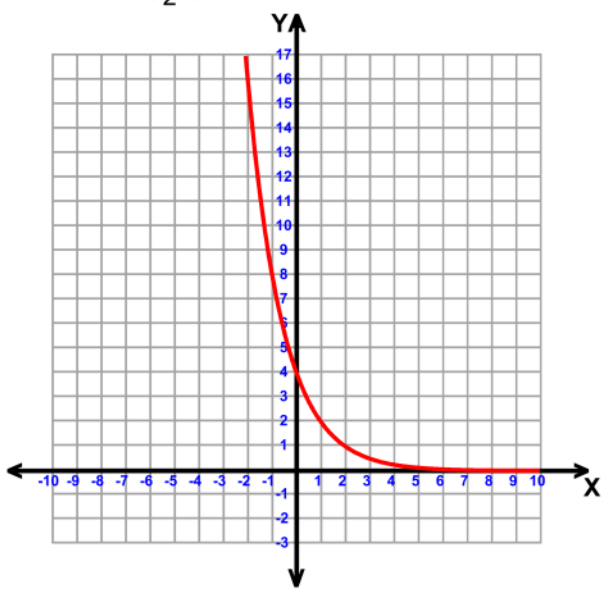
5) $y = 2 \cdot 4^x$



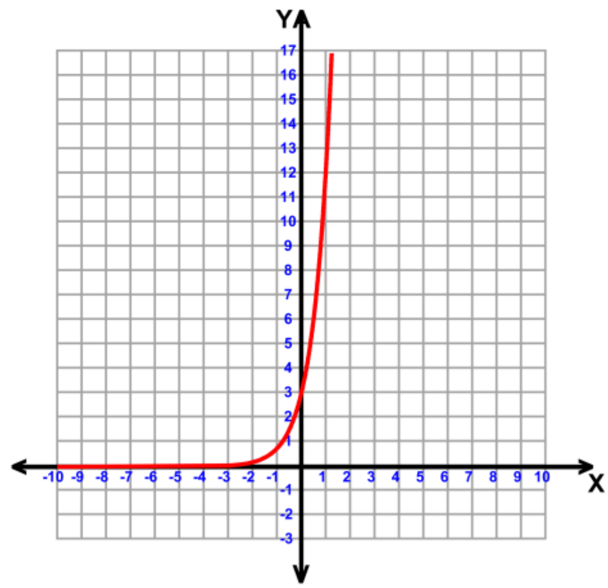
6) $y = 4 \cdot \left(\frac{1}{2}\right)^x$



7) $y = 4 \cdot \left(\frac{1}{2}\right)^x$



8) $y = 3 \cdot 4^x$



Understanding Logarithms

What is a log?
A logarithm is an exponent.

$$\log_b(a) = c \iff b^c = a$$

Logarithmic Form		Exponential Form
$\log_3 x = 5$	➡	$3^5 = x$

Both forms use the same base.

The logarithm is equal to the exponent.

Changing between log and exponent form:

Convert each to logarithmic form.

a) $2^m = n$ (base) $\log_2 n = m$

b) $10^{x-1} = 1000$ → $\log 1000 = x-1$

c) $(x+1) = 3^{z+1}$ → $\log_3 (x+1) = z+1$

More:
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Convert each to exponential form.

a) $\log_2 x = 3$ → $x = 2^3$

b) $\log_{x-1} 3 = 4$ → $(x-1)^4 = 3$

c) $\log_x (x+2) = 2$ → $x^2 = x+2$

Evaluating and solving logarithms by changing to exponential form.

But there is a short-cut in some cases!

Try solving these:

BOOT the BASE

a) $\log_x 27 = 3$ → $\sqrt[3]{x^3} = \sqrt[3]{27}$

$x = 3$

b) $\log_2 (2x-5) = 4$ → $2x-5 = 2^4$

$2x-5 = 16$

+5

$2x = 21$

$x = \frac{21}{2}$

c) $\log_{(x+1)} (2x-1) = 2$

Note:

log have restrictions

$y = \log_c x$

$x > 0$

base is also restricted

$c > 0,\ c \neq 1$

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Restrictions

$2x-5 > 0$

$x > \frac{5}{2}$

$(x+1)^2 = 2x-1$

$(x+1)(x+1)$

$x^2 + 2x + 1 = 2x - 1$

$-2x \quad +1$

$x^2 + 2 = 0$

-2

$\sqrt{x^2} = \sqrt{-2}$

$x = \emptyset$ undefined

$x =$ no solution (no real roots)

- **Change of Base Law=** $\log_a x = \dfrac{\log x}{\log a}$

$$\log_7(10) = \frac{\log(10)}{\log(7)}$$

$$\approx \frac{1}{0.8451}$$

$$\approx 1.183$$

Try to evaluate these logs:

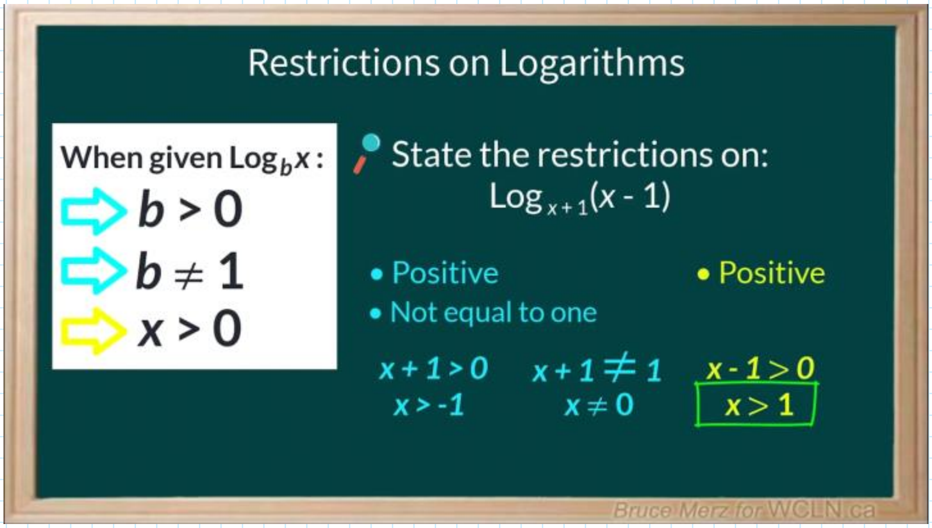
a) $\log_2 64$

b) $\log_4 0.0625$

c) $\log_{\frac{1}{2}} 32$

d) $\log_{1.2} 223.87$

Logarithms have restrictions (non-permissible values = NPVs)



Graphing a Logarithmic Function

Graphing a logarithmic function is the same as graphing the inverse of the exponential function with the same base.

Recall: to graph the inverse, switch the x and y for each coordinate and plot (the graph reflects over the line y=x and the domain and range also inverse)

Ex. Graph the function $y = \log_2 x$

1. First determine the points on the function $y = 2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

Step① = graph exponential function with the same base as log functron

2. Inverse each coordinate and the asymptote to become $x = 0$

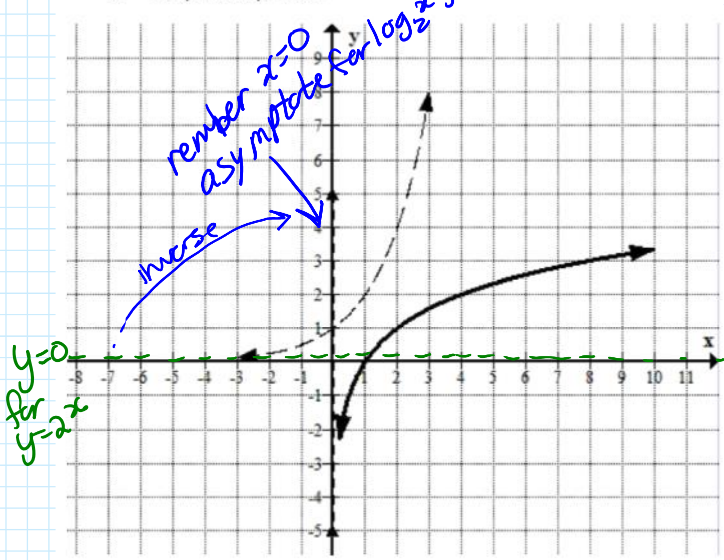
$y=2^x$

x	y
-2	¼
-1	½
0	1
1	2
2	4

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

Step② = inverse the coordinates from $y=2^x$ to get $y=\log_2 x$

3. Graph the points.



Step③ Graph the inverse to toget $y=\log_2 x$ graph.

Try graphing the following in the same steps as above:

a) $y = \log_3 x$ (base 3)

b) $y = \log_{\left(\frac{1}{2}\right)} x$ (base $\frac{1}{2}$)

step 1 $y=3^x$

step 2

① , x

② inverse

Step 1

$y = 3^x$

x	y
-2	1/9
-1	1/3
0	1
1	3
2	9

$3^{-2} \to \frac{1}{3^2}$

$3^0 \to 1$

asymptote $y = 0$

Step 2

inverse $\Rightarrow y = \log_3 x$

x	y
1/9	-2
1/3	-1
1	0
3	1
9	2

asymptote $x = 0$

① $y = \frac{1}{2}^x$

x	y
-3	8
-2	4
-1	2
0	1
1	1/2
2	1/4

$\left(\frac{1}{2}\right)^{-3} = 2^3$

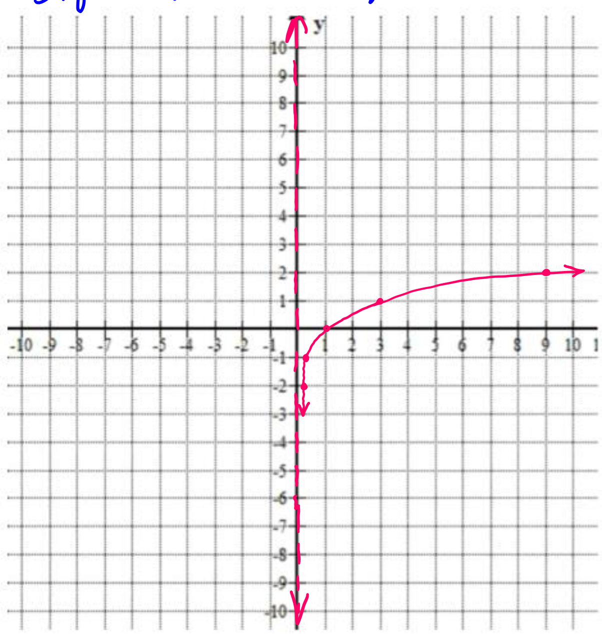
② inverse

$y = \log_{\frac{1}{2}} x$

x	y
8	-3
4	-2
2	-1
1	0
1/2	1
1/4	2

Step 3 ☆ inverse asymptote

③ Graph.

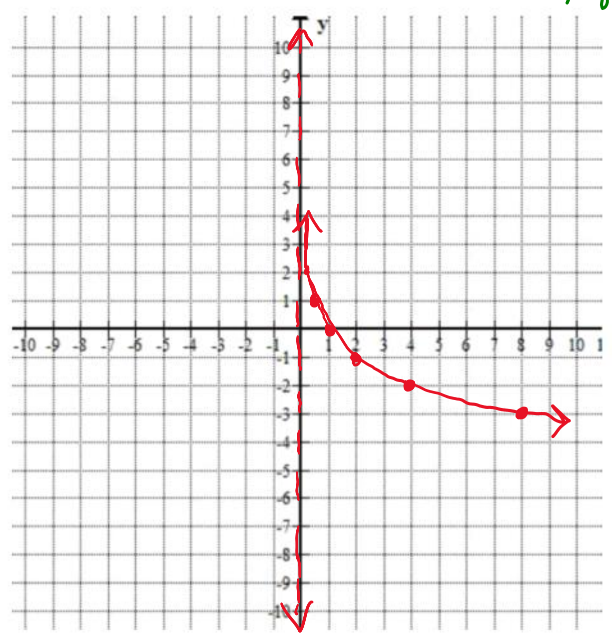


$x = 0$

$\{x \mid x > 0, x \in R\}$

$\{y \mid y \in R\}$

asymptote: $x = 0$ (VA)



VA : $x = 0$

$\{x \mid x > 0, x \in R\}$

$\{y \mid y \in R\}$

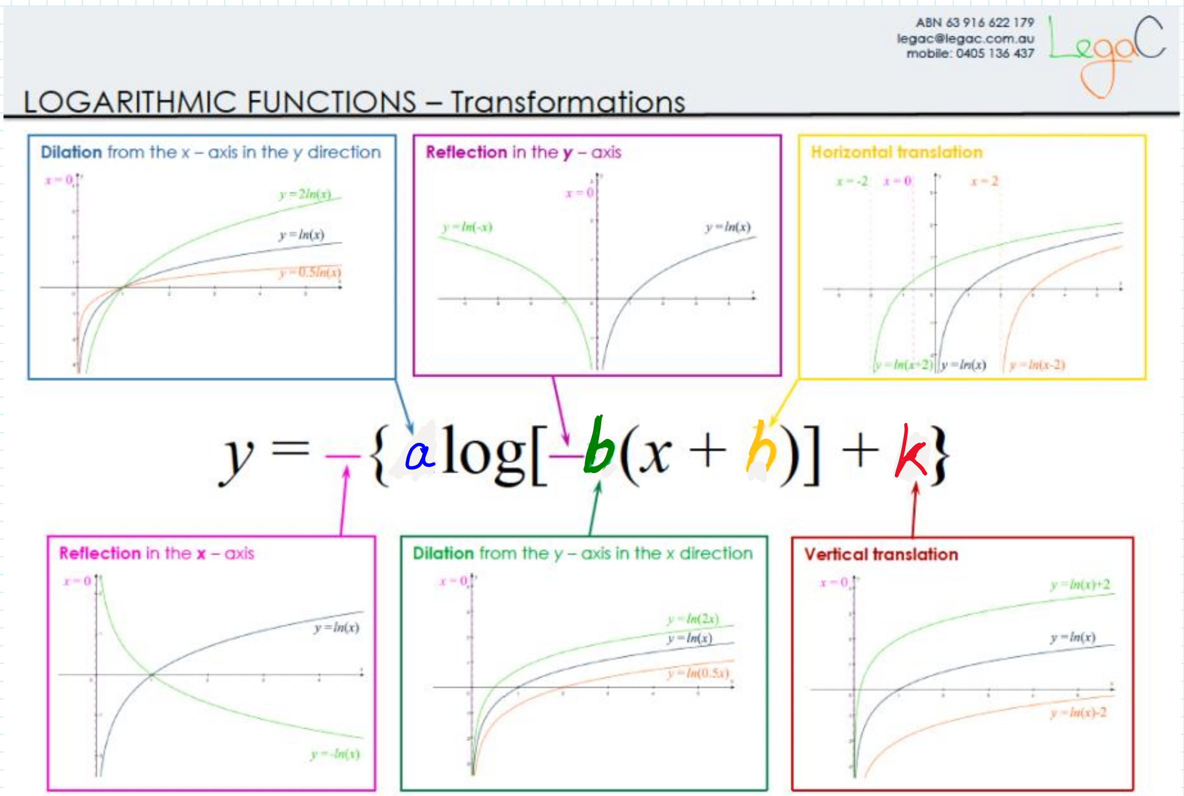
asymptote VA $x = 0$

Transformations of Logarithmic Functions

Logarithmic Graphs as Inverses of the Exponential Graphs

Exponential	Logarithmic
$y = b^x,\ b > 0,\ b \neq 1,\ x > 0$	$y = \log_b x,\ b > 0,\ b \neq 1,\ x > 0$
all go through $(0, 1)$	all go through $(1, 0)$
asymptote on x-axis	asymptote on y-axis
$b > 1$ increasing	$b > 1$ increasing
$0 < b < 1$ decreasing	$0 < b < 1$ decreasing
all go through $(1, b)$, $\left(-1, \frac{1}{b}\right)$	all go through $(b, 1)$, $\left(\frac{1}{b}, -1\right)$

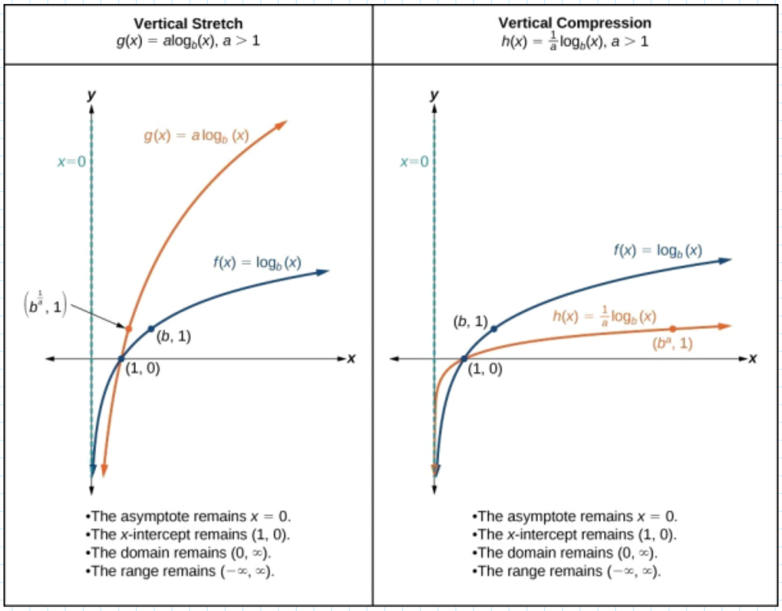
Exponential and logarithmic graphs are inverses of each other.

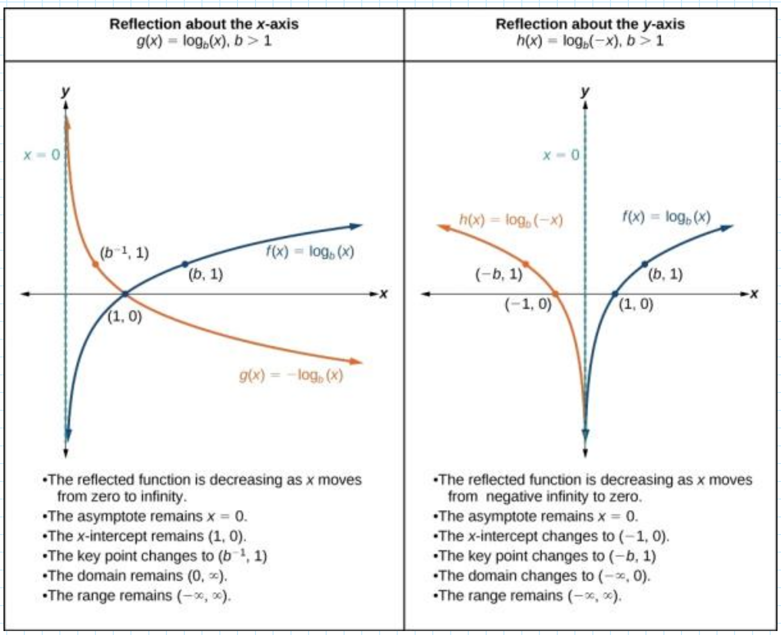


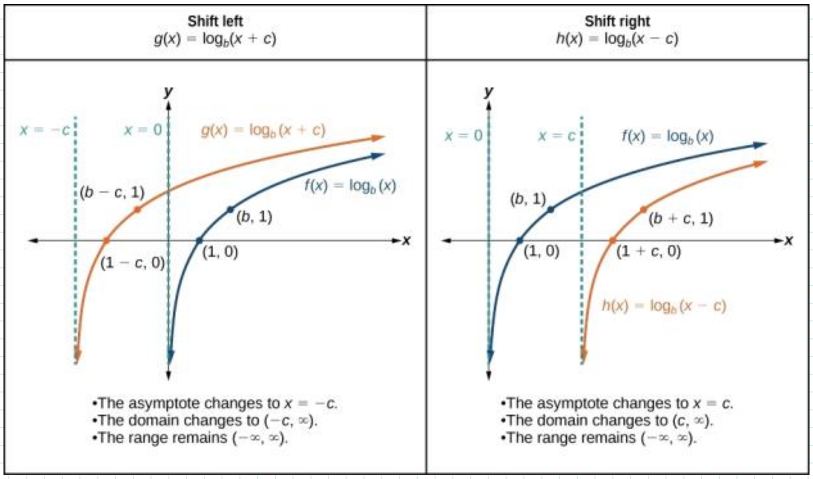
Transformations $y = \log x$

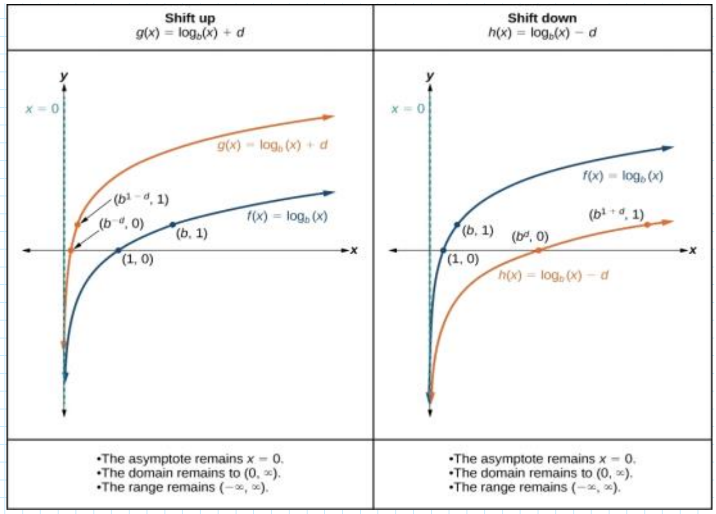
Transformation	f(x) Notation	Examples
Horizontal Translation Graph shifts left or right.	$f(x-h)$	$g(x) = \log(x-3) \to 3$ units right $g(x) = \log(x+4) \to 4$ units left
Vertical Translation Graph shifts up or down.	$f(x)+k$	$g(x) = \log x + 4 \to 4$ units up $g(x) = \log x - 5 \to 5$ units down
Reflection Graph flips over x-axis.	$-f(x)$	$g(x) = -\log x \to$ over x-axis
Reflection Graph flips over y-axis.	$f(-x)$	$g(x) = \log(-x) \to$ over y-axis
Horizontal Shrink Graph shrinks toward y-axis.	$f(ax), a > 1$	$g(x) = \log 2x \to$ shrink by $\frac{1}{2}$
Horizontal Stretch Graph stretches away from y-axis.	$f(ax),\ 0 < a < 1$	$g(x) = \log\frac{x}{2} \to$ stretch by 2
Vertical Stretch [redacted] Graph stretches away from x-axis.	$a \cdot f(x),\ a > 1$	$g(x) = 2 \cdot \log x \to$ stretch by 2

Graph stretches away from y-axis.	$f(ax),\ 0<a<1$	[illegible]
Vertical Stretch [illegible] Graph stretches away from x-axis.	$a\cdot f(x),\ a>1$	$g(x)=2\cdot\log x \rightarrow$ stretch by 2
Vertical Shrink Graph shrinks toward x-axis.	$a\cdot f(x),\ 0<a<1$	$g(x)=\frac{1}{2}\log x \rightarrow$ shrink by $\frac{1}{2}$






Shift left
$g(x) = \log_b(x + c)$
$x = -c$
$x = 0$
$g(x) = \log_b(x + c)$
$f(x) = \log_b(x)$
$(b - c, 1)$
$(b, 1)$
$(1 - c, 0)$
$(1, 0)$
•The asymptote changes to $x = -c$.
•The domain changes to $(-c, \infty)$.
•The range remains $(-\infty, \infty)$.
Shift right
$h(x) = \log_b(x - c)$
$x = 0$
$x = c$
$f(x) = \log_b(x)$
$(b, 1)$
$(b + c, 1)$
$(1, 0)$
$(1 + c, 0)$
$h(x) = \log_b(x - c)$
•The asymptote changes to $x = c$.
•The domain changes to (c, ∞).
•The range remains $(-\infty, \infty)$.


Shift up
$g(x) = \log_b(x) + d$
$x = 0$
$g(x) = \log_b(x) + d$
$(b^{1-d}, 1)$
$(b^{-d}, 0)$
$f(x) = \log_b(x)$
$(b, 1)$
$(1, 0)$
•The asymptote remains $x = 0$.
•The domain remains to $(0, \infty)$.
•The range remains $(-\infty, \infty)$.
Shift down
$h(x) = \log_b(x) - d$
$x = 0$
$f(x) = \log_b(x)$
$(b, 1)$
$(b^{1+d}, 1)$
$(b^d, 0)$
$(1, 0)$
$h(x) = \log_b(x) - d$
•The asymptote remains $x = 0$.
•The domain remains to $(0, \infty)$.
•The range remains $(-\infty, \infty)$.