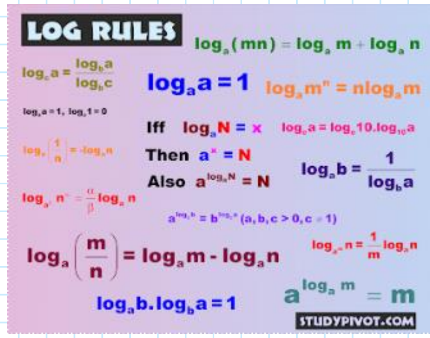
Thursday, Feb. 29th

Plan For Today:

1. Questions from Chapter 3 or 4?
   - Do 5.2 Check-in Quiz
2. Start Chapter 5: Exponents & Logarithms
   - 5.1: Exponents
   - 5.2: Logarithmic Functions and Graphs
   - **5.3: Properties of Logarithms**
   - ~~**5.4: Exponential and Logarithmic Equations**~~
   - 5.5: Applications of Exponential and Log Equations
3. Work on Practice Questions from Workbook



Plan Going Forward:

1. Finish going through 5.2-5.3 and chapter practice questions in workbook and start working on review handout. #1-8

- **CHECK-IN QUIZ ON 5.3- ON TUESDAY, MAR. 5TH**

2. We will finish in Chapter 5 on Tuesday.

- **CHAPTER 5 PROJECT (PART A&B) DUE THURSDAY, MAR. 7TH**
  - PART A IS IN DESMOS: http://tinyurl.com/PC12-Feb2024-Ch5PartA
  - PART B IS ON HANDOUT
- **UNIT 3 EXAM ON THURSDAY, MAR. 7TH**

- **UNIT 3 EXAM ON CH5 ON THURSDAY, MAR. 7TH**
  - 12 Multiple Choice & 18 marks on the Written
  - ~1.5 hour - please prepare so you are not "learning" while doing the test
  - Closed-book - no notes
  - Rewrite is following Tuesday after class at 12:30pm
  - I will email you on the weekend when marks are posted so you can decide on the rewrite
  - I will go over the marked exam on Tuesday

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.
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Thursday, Feb. 29th In-Class Notes

Feb. 29th, 2024 **Name:** KEY **TOTAL =** _____ **/ 6 marks**

Check-in Quiz Section 5.2:
Graphing Logarithmic Equations & Log-Exponent Conversions

Complete the following questions SHOWING ALL WORK and steps where applicable.

1. Describe the transformations or and graph the function $y = 3\log_2(x-2)+1$

3 marks

a) Describe transformations:

base $y = \log_2 x$

$y = 2^x$ ↓ inverse

VE of 3
2 right
1 up.

b) Graph:

$y=2^x$

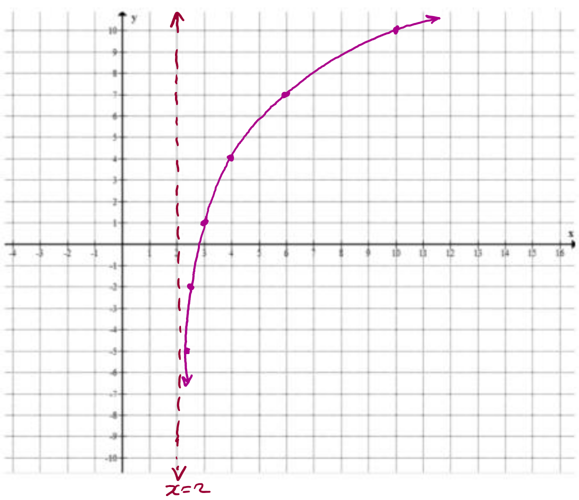
x	y
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8

$y = \log_2 x$

x	y
1/4	-2
1/2	-1
1	0
2	1
4	2
8	3

x	3y
1/4	-6
1/2	-3
1	0
2	3
4	6
8	9

x+2	3y+1
2 1/4	-5
2 1/2	-2
3	1
4	4
6	7
10	10



b) What is the domain and range of this function?

$\{x \mid x > 2, x \in \mathbb{R}\}$.

$\{y \mid y \in \mathbb{R}\}$.

asymptote

$x = 2$

~~VA~~=2

$a = b^c \leftrightarrow \log_b a = c$

Boot the Base.

0.5 each = 1 mark

2. a) Convert the following to exponential form:

$\log_3(m-4) = k$

$3^k = m-4$

b) Convert the following to logarithmic form:

$(a-b)^{2x} = 45$ (Base)

$\log_{(a-b)} 45 = 2x$

3. Determine the value of the following (evaluate):

1 mark each = 2 marks

common base.

a) $\log_2\left(\frac{1}{64}\right)$

$\log_2(2)^{-6}$

$= \boxed{-6}$

b) $\log_5(25^6)$

$\log_5(5^2)^6$

$\log_5 5^{12} = \boxed{12}$

5.3 Log Laws.

Product Law → $\log_b AB = \log_b A + \log_b B$

multiplying changes to adding

ex: $\log 4 + \log 6 = \log(4 \times 6)$ (arguments multiply.)

$= \boxed{\log 24}$

adding ∴ multiply arguments.

Quotient Law → $\log_b \frac{A}{B} = \log_b A - \log_b B$

dividing changes to subtraction

edited ex. $\log_3 12 - \log_3 4 = \log_3\left(\frac{12}{4}\right)$ (divide arguments) edited ex. $\log_3 12 - \log_3 4 = \log_3\left(\frac{12}{4}\right)$

(subtraction ∴ divide arguments.) (divide arguments) (base must be included)

$= \log_3 3$

$= \boxed{1}$ (simplify fully.)

practice p. 221 #1 : a) $\log 6$

$\log(2\times 3) = \boxed{\log 2 + \log 3}$

b) $\log 12$

$\log(4\cdot 3)$

$\log(2\cdot 2\cdot 3)$

$\log 2 + \log 2 + \log 3$.

$\boxed{2\log 2 + \log 3}$

c) $\log 72$

$\log(9\cdot 8)$

$\log(3^2 \cdot 2^3)$

$\log 3^2 + \log 2^3$ (power law)

$\boxed{2\log 3 + 3\log 2}$

d) $\log_{10} 3200$

$\log(32\cdot 100)$

$\log(2^5 \cdot 100)$

$\log 2^5 + \log_{10} 100$.

$5\log 2 + \log 10^2$

$\boxed{5\log 2 + 2}$

Power Law $\log_b A^n \leftrightarrow n\log_b A$

exponent can move to coefficient & vise versa.

ex: $\log_2 8 = \log_2 2^3$

$= 3\log_2 2$

$= 3$

change to common base.

★ $\log_2 2^3 = 3$

IF there is a root (radical) in argument, use power law to simplify.

$\log_b \sqrt[n]{A} \rightarrow \log_b A^{\frac{1}{n}}$

$= \frac{1}{n}\log_b A$ OR $\frac{\log_b A}{n}$

$\log_b \sqrt[n]{A^m} = \log_b A^{\frac{m}{n}}$

$\log_b \sqrt[n]{A}^{m} = \log_b A^{\frac{m}{n}}$

$= \frac{m}{n} \log_b A$

Change of Base Rule.

used to change to common base (usually base 10)

$\log_b a = \frac{\log_x a}{\log_x b}$ or $\frac{\log a}{\log b}$ good for calculator.

(argument = top; base = bottom)

ex: $\log_2 7 \rightarrow \frac{\log 7}{\log 2}$ $\quad \log 7 \div \log 2$

$= 2.807$

ex: $\log_7 2 \rightarrow \frac{\log 2}{\log 7} = 0.356$

Ex 6
p. 217.

a) $\log_7 \sqrt[4]{7} = \log_7 7^{\frac{1}{4}}$

$= \boxed{\frac{1}{4}}$

b) $\log_5 5^6 - \log_5 5^2$

$6-2$

$= \boxed{4}$

IF $\log_5 6^6 - \log_5 4^2$
$\downarrow$
$\div$

$\log_5\left(\frac{6^6}{4^2}\right)$

c) $\log_{\frac{1}{4}}\left(\frac{16^2}{2^{-3}}\right)$

$\frac{16^2}{2^{-3}} \Rightarrow \frac{(2^4)^2}{2^{-3}}$

$= \frac{2^8}{2^{-3}}$ $\quad 8-(-3)$

$= 2^{11}$

$\log_{\frac{1}{4}}(2^{11})$

change of base rule

$\frac{\log_2 2^{11}}{\log_2 \frac{1}{4}} = \frac{\log_2 2^{11}}{\log_2 2^{-2}}$

$\frac{1}{4} = 4^{-1} \rightarrow 2^{-2}$

$= \frac{11}{-2}$

$= \boxed{-\frac{11}{2}}$

Practice # 1, 2, 5

#1

j) $\log_{12} 0.108$

$\log_{12} \frac{108}{1000}$

quotient law

$= \log_{12} 108 - \log_{12} 1000$

change of base rule.

$\frac{\log 108}{\log 12} - \frac{\log 1000}{\log 12}$

common denominator

$\frac{\log 108 - \log 1000}{\log 12}$.

$\log 10^3 = 3$

$\frac{\log(4\cdot 27) - 3}{\log(4\cdot 3)}$

product law

$\frac{\log 4 + \log 27 - 3}{\log 4 + \log 3}$

base 2+3

$\frac{\log 2^2 + \log 3^3 - 3}{\log 2^2 + \log 3}$. power law

$= \boxed{\frac{2\log 2 + 3\log 3 - 3}{2\log 2 + \log 3}}$

2

g) $(\log_4 8)(\log_{16} 32)$ → short cut.

$\log_{2^2} 2^3$

$\frac{3}{2} \log_2 2$

$\frac{3}{2}$

change of base rule.

$\frac{\log_2 8}{\log_2 4} \cdot \frac{\log_2 32}{\log_2 16}$

$\frac{\log_2 2^3}{\log_2 2^2}$

$= \frac{3}{2} \cdot$ $= \boxed{\frac{15}{8}}$

3) b) p.223

$x^{\log_x 20 - \log_x 4}$ ① quotient law

$\frac{20}{4} = 5$

$= x^{\log_x 5}$

$= \boxed{5}$

Practice with laws

Ex:7 p. 218

a) $\log_5 3x^4 y^{-2} \rightarrow \log_5 3 + \log_5 x^4 + \log_5 y^{-2}$ power law

multiplying → adding

product law

$\boxed{\log_5 3 + 4\log_5 x - 2\log_5 y}$

b) $\log \frac{\sqrt{2x-5}}{3} = \log \sqrt{2x-5} - \log 3$

radical → $\frac{1}{2}$ exponent

$\div \rightarrow -$ quotient law

$\log (2x-5)^{\frac{1}{2}} - \log 3$

$\boxed{\frac{1}{2}\log(2x-5) - \log 3}$

$\sqrt[n]{x} = x^{\frac{1}{n}}$

$\sqrt{x} = x^{\frac{1}{2}}$

$\sqrt[3]{x} = x^{\frac{1}{3}}$

Ex: 8

a) $\frac{1}{3}\log x + 2\log(x-1)$ condense.

power law: $\log x^{\frac{1}{3}} + \log(x-1)^2$

product law: $\log\left(x^{\frac{1}{3}}(x-1)^2\right)$

$\boxed{\log\left(\sqrt[3]{x}\,(x-1)^2\right)}$

$(x-1)^2$

$= (x-1)(x-1)$

$x^2 - 2x + 1$

practice #3 + 4.

Applications: Growth + Decay formula

p.202

$A = A_0 (x)^{\frac{t}{T}}$

if $x > 1$ = growth

$0 < x <$ = decay

A = final A = initial

0 < x < 1 = decay →

A = final amount A_0 = initial amount   x = base = growth/decay factor

t = amount of time elapsed

T = time of growth/decay factor

* $A = P\left(1+\frac{r}{n}\right)^{nt}$

compound interest formula
p. 201

p202 ex: half-life = 25000 yr $x = \frac{1}{2}$ or 0.5

T = 25000 yr.

t = 2000 yr have passed.

How much remains? A?

assume 100% initial amount.

↓

$A_0 = 1$

$A = 1(0.5)^{\frac{2000}{25000}}$

$A = 0.9461$

= 94.61% remains

Ex8. increases 25% every 3 days.   decrease 25%

$(1+0.25)$ ↓ $1-r$

$x = 1.25$ T = 3 days. $1-0.25$

$x = 0.75$

$A = 2000$ $A_0 = ?$

t = 25 days

$$\frac{2000}{1.25^{25/3}} = \frac{A_0(1.25)^{25/3}}{1.25^{25/3}}$$

$$A_0 = \frac{2000}{1.25^{25/3}}$$

$$A_0 = 311.49$$

= 311 flies

Log Graphs Practice

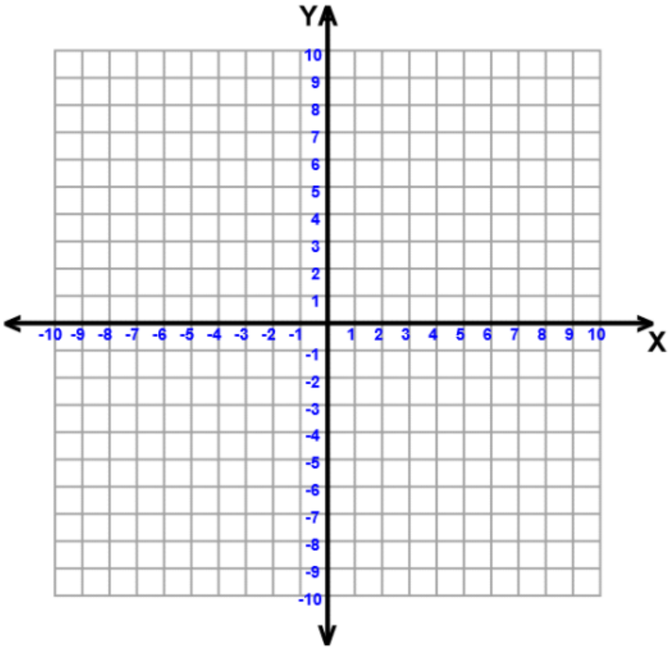
Name : ______________________ Score : ____________

Teacher : ______________________ Date : ____________

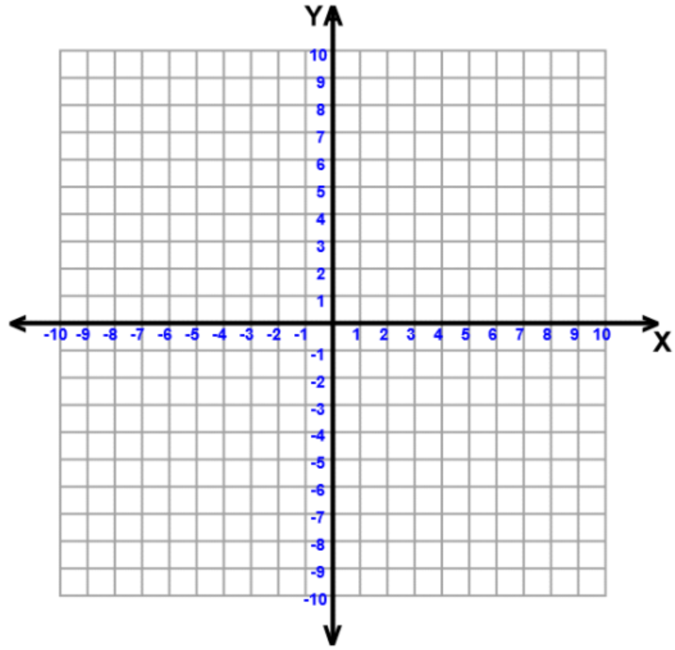
Graphing Logarithms

Give the domain and range of each function, then graph.

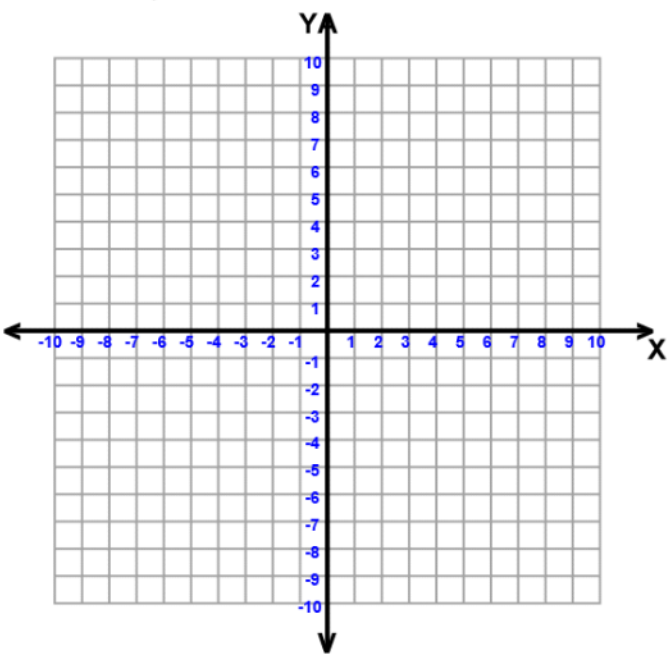
1) $y = \log_9 (x + 4) + 3$



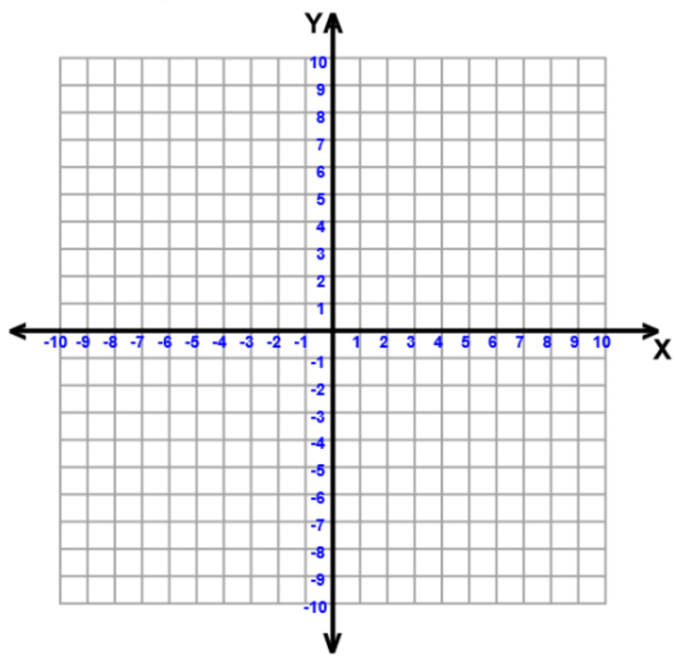
2) $y = \log_5 (x - 2) + 5$



3) $y = \log_7 (x - 4) - 2$



4) $y = \log_4 (x - 2) - 5$



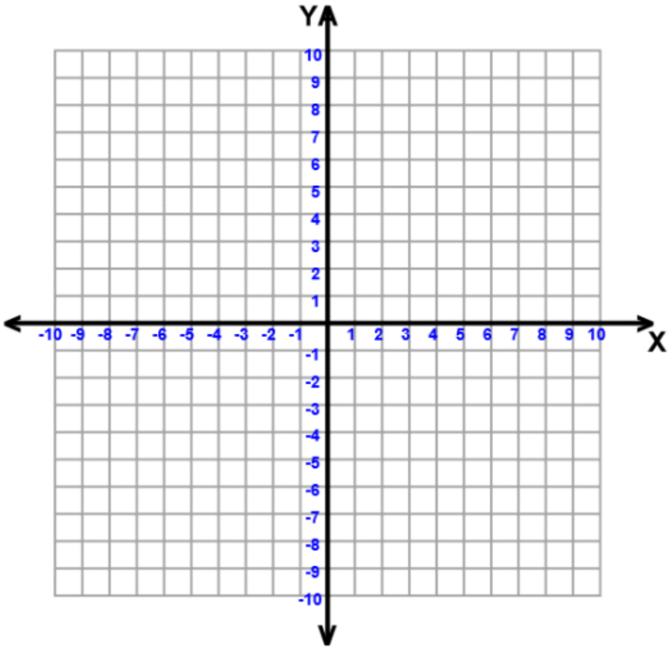
Name : ______________________________ Score : ____________

Teacher : ____________________________ Date : ____________

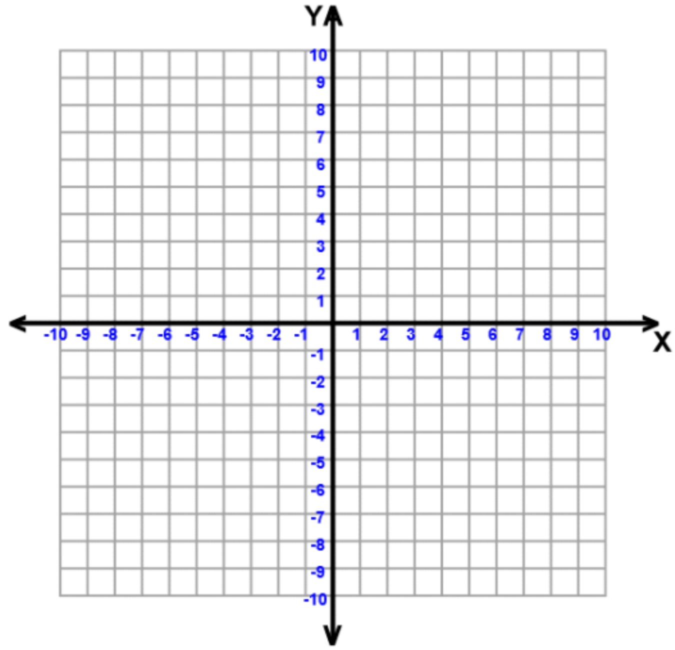
Graphing Logarithms

Give the domain and range of each function, then graph.

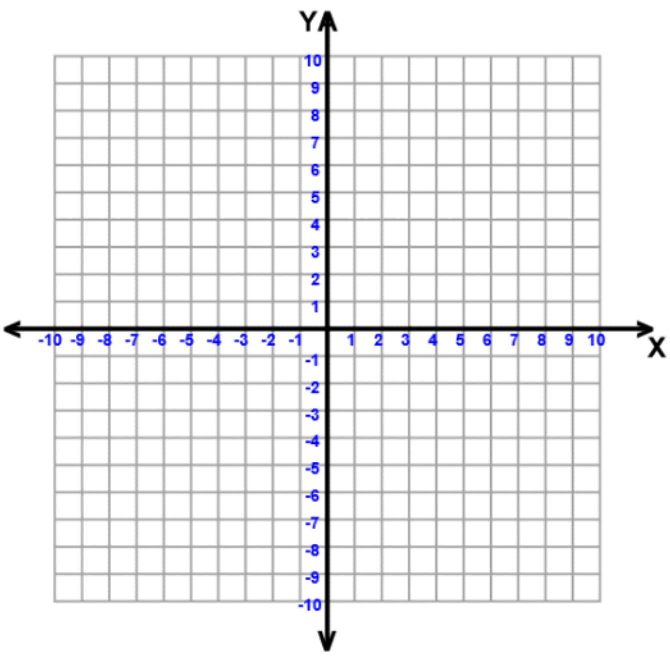
5) $y = \log_5 (4x - 5) - 2$



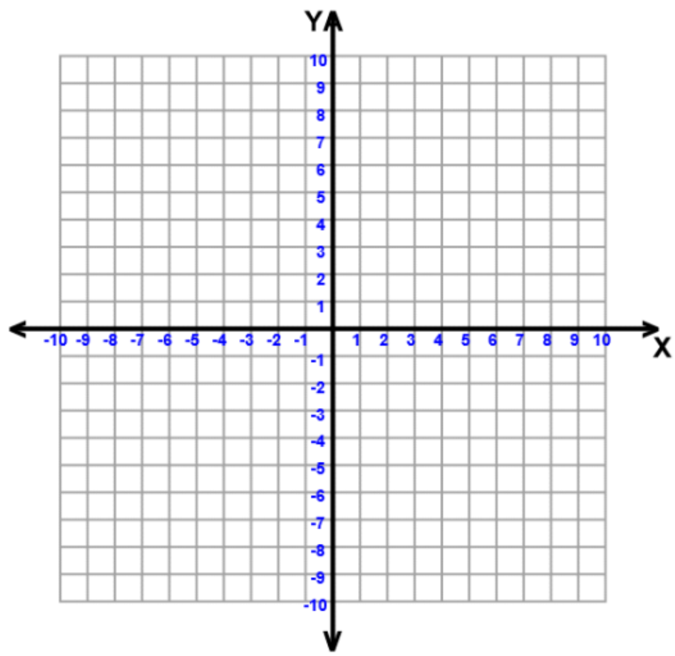
6) $y = \log (4x - 3) - 4$



7) $y = \log_8 (2x + 5) + 3$



8) $y = \log_7 (4x + 4) - 3$



Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

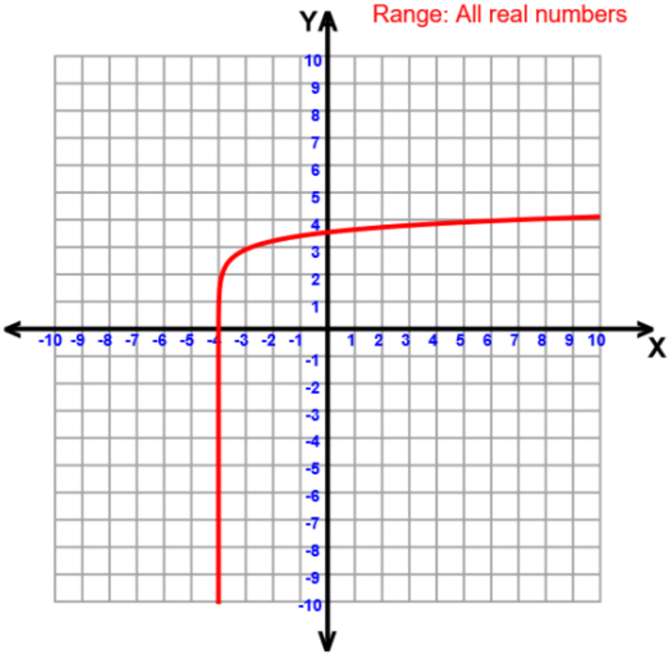
Graphing Logarithms

Give the domain and range of each function, then graph.

1) $y = \log_9 (x + 4) + 3$

Domain: x > -4

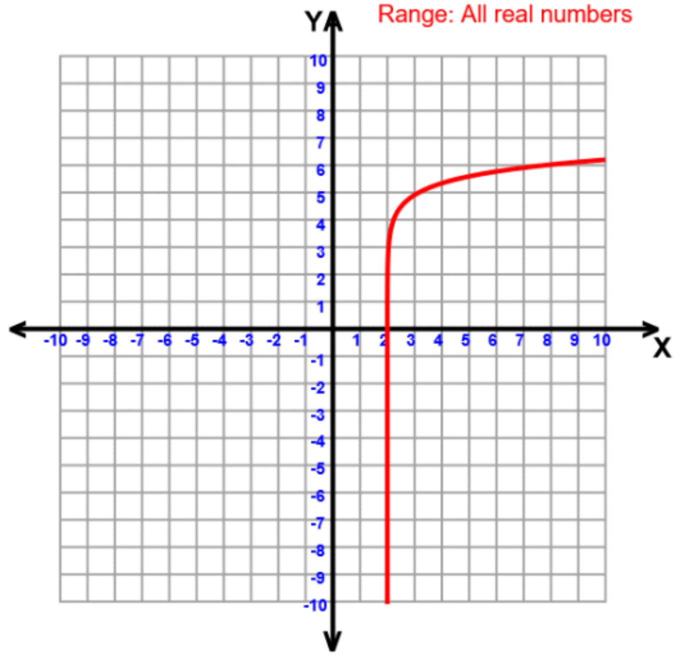
Range: All real numbers



2) $y = \log_5 (x - 2) + 5$

Domain: x > 2

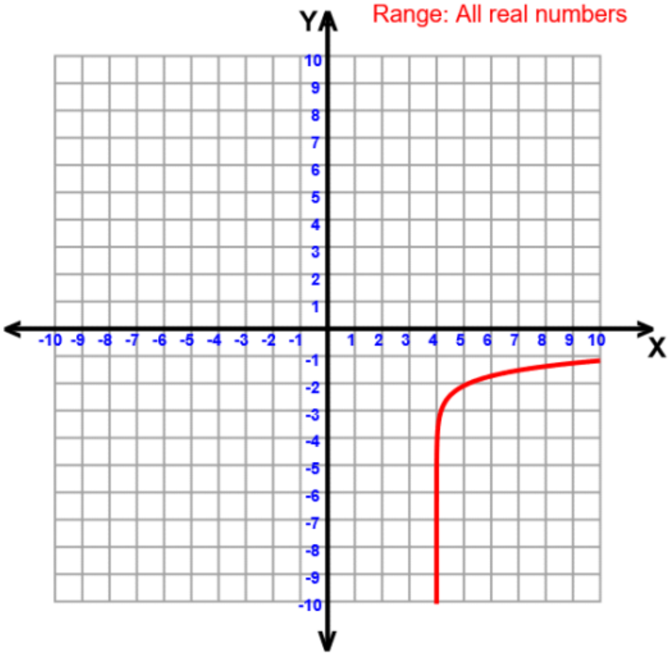
Range: All real numbers



3) $y = \log_7 (x - 4) - 2$

Domain: x > 4

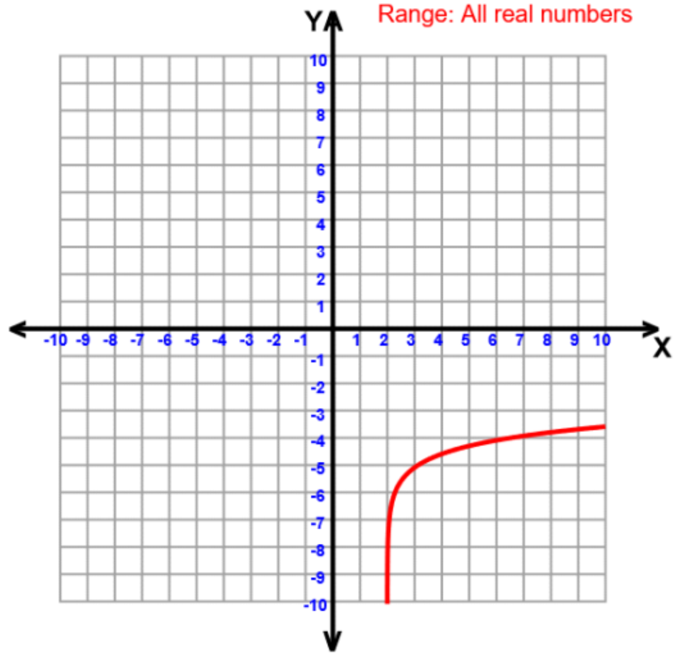
Range: All real numbers



4) $y = \log_4 (x - 2) - 5$

Domain: x > 2

Range: All real numbers



Name : ______________________________ Score : ____________

Teacher : ____________________________ Date : ____________

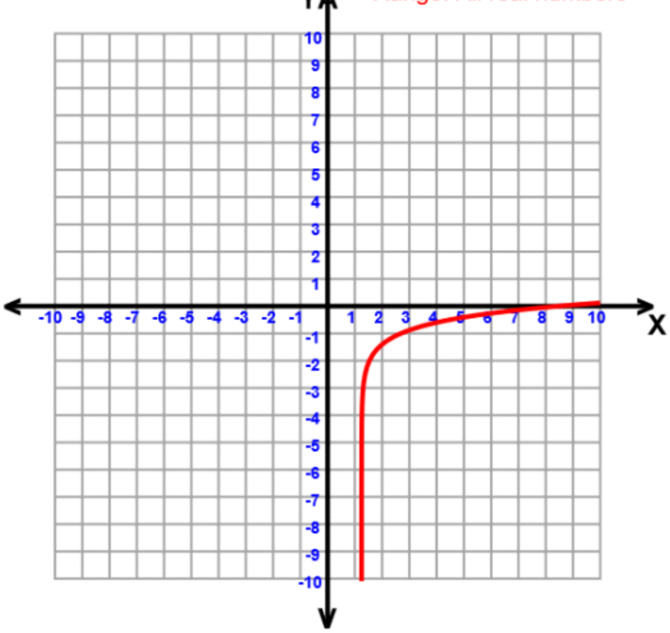
Graphing Logarithms

Give the domain and range of each function, then graph.

5) $y = \log_5 (4x - 5) - 2$

Domain: $x > \frac{5}{4}$

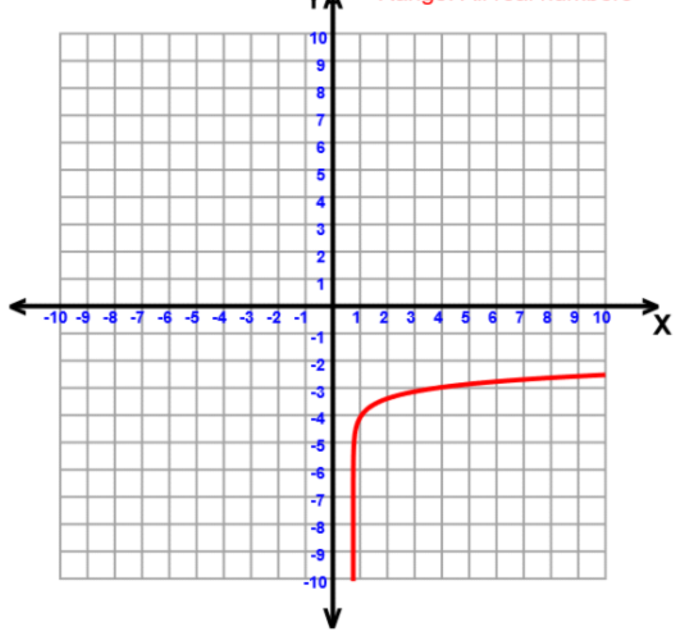
Range: All real numbers



6) $y = \log (4x - 3) - 4$

Domain: $x > \frac{3}{4}$

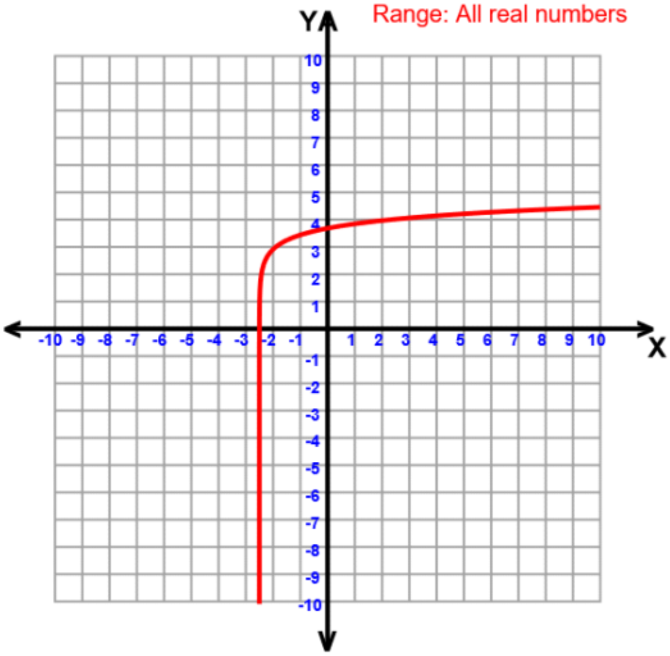
Range: All real numbers



7) $y = \log_8 (2x + 5) + 3$

Domain: $x > \frac{-5}{2}$

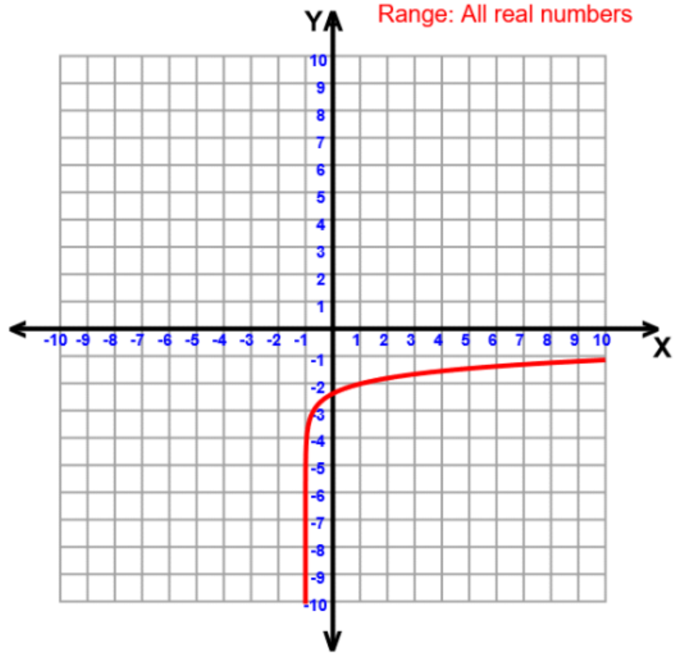
Range: All real numbers



8) $y = \log_7 (4x + 4) - 3$

Domain: $x > -1$

Range: All real numbers



Understanding Logarithms

What is a log?
A logarithm is an exponent.

$$\log_b(a) = c \iff b^c = a$$

Logarithmic Form		Exponential Form
$\log_3 x = 5$	➡	$3^5 = x$

Both forms use the same base.

The logarithm is equal to the exponent.

Changing between log and exponent form:

Convert each to logarithmic form.

a) $2^m = n$ (base) $\log_2 n = m$

b) $10^{x-1} = 1000$ → $\log 1000 = x - 1$

c) $(x+1) = 3^{z+1}$ → $\log_3(x+1) = z + 1$

More:
p.263 # 2 + #3

Convert each to exponential form.

a) $\log_2 x = 3$ → $x = 2^3$

b) $\log_{x-1} 3 = 4$ → $(x-1)^4 = 3$

c) $\log_x (x+2) = 2$ → $x^2 = x+2$

Evaluating and solving logarithms by changing to exponential form.
But there is a short-cut in some cases!

Try solving these:

BOOT the BASE

a) $\log_x 27 = 3$ → $\sqrt[3]{x^3} = \sqrt[3]{27}$

$\boxed{x = 3}$

★ Note:
log have restrictions
$y = \log_c x$
$x > 0$
base is also restricted
$c > 0,\ c \neq 1$

Extra #4-7
p. 264

b) $\log_2 (2x-5) = 4$ → $2x-5 = 2^4$

$2x-5 = 16$ (+5)

$2x = 21$ (÷2)

$\boxed{x = \frac{21}{2}}$

★ Restrictions
$2x-5 > 0$
$x > \frac{5}{2}$

c) $\log_{(x+1)} (2x-1) = 2$

$(x+1)^2 = 2x-1$

$(x+1)(x+1)$

$x^2 + 2x + 1 = 2x - 1$
$-2x \quad +1$

$x^2 + 2 = 0$
-2

$\sqrt{x^2} = \sqrt{-2}$

$x = \varnothing$ undefined

$\boxed{x = \text{no solution}}$ (no real roots)

- **Change of Base Law=** $\log_a x = \frac{\log x}{\log a}$

$$\log_7(10) = \frac{\log(10)}{\log(7)}$$
$$\approx \frac{1}{0.8451}$$
$$\approx 1.183$$

Try to evaluate these logs:

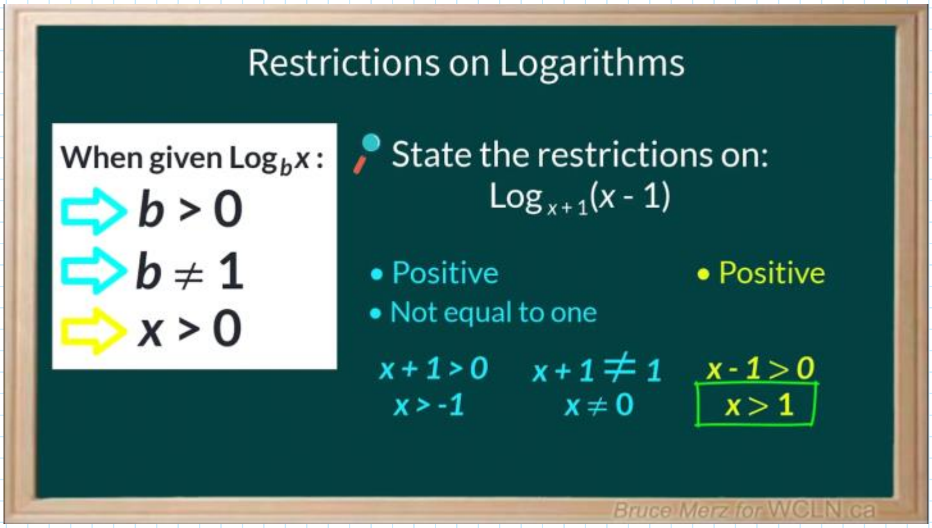
a) $\log_2 64$

b) $\log_4 0.0625$

c) $\log_{\frac{1}{2}} 32$

d) $\log_{1.2} 223.87$

Logarithms have restrictions (non-permissible values = NPVs)



Graphing a Logarithmic Function

Graphing a logarithmic function is the same as graphing the inverse of the exponential function with the same base.

Recall: to graph the inverse, switch the x and y for each coordinate and plot (the graph reflects over the line y=x and the domain and range also inverse)

Ex. Graph the function $y = \log_2 x$

1. First determine the points on the function $y = 2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

Step① = graph exponential function with the same base as log function

2. Inverse each coordinate and the asymptote to become $x = 0$

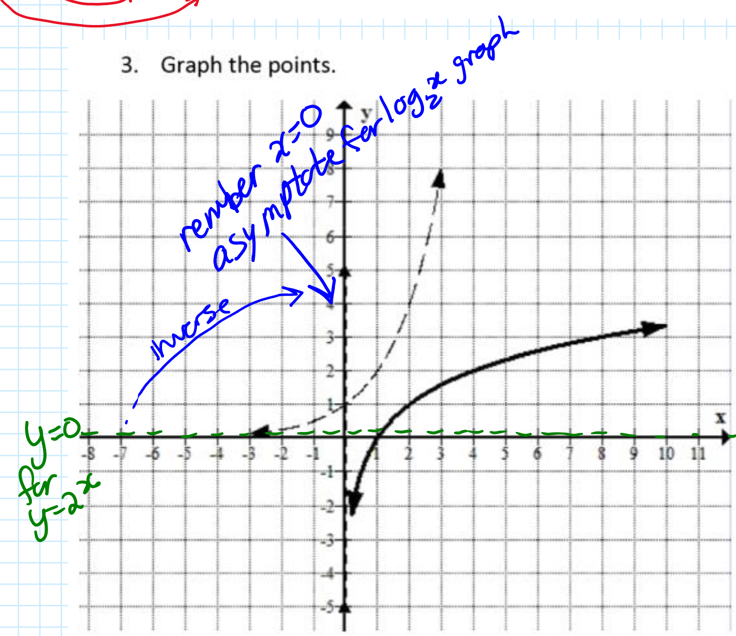
$y=2^x$

x	y
-2	¼
-1	½
0	1
1	2
2	4

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

Step② = inverse the coordinates from $y=2^x$ to get $y=\log_2 x$

3. Graph the points.



Step③ Graph the inverse to toget $y=\log_2 x$ graph.

Try graphing the following in the same steps as above:

a) $y = \log_3 x$ (base 3)

b) $y = \log_{\left(\frac{1}{2}\right)} x$ (base $\frac{1}{2}$)

step 1 step 2 ① ② inverse

step 1

$y = 3^x$

x	y
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

$3^{-2} \to \frac{1}{3^2}$

$3^0 \to 1$

asymptote $y=0$

step 2

inverse $\Rightarrow y = \log_3 x$

x	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2

asymptote $x=0$

① $y = \frac{1}{2}^x$

x	y
-3	8
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

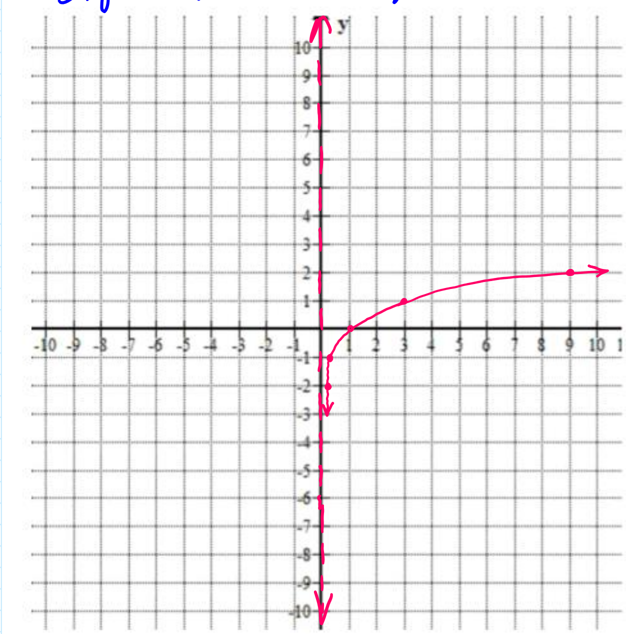
$\left(\frac{1}{2}\right)^{-3} = 2^3$

② inverse $y = \log_{\frac{1}{2}} x$

x	y
8	-3
4	-2
2	-1
1	0
$\frac{1}{2}$	1
$\frac{1}{4}$	2

Step 3 ☆ inverse asymptote

③ Graph.

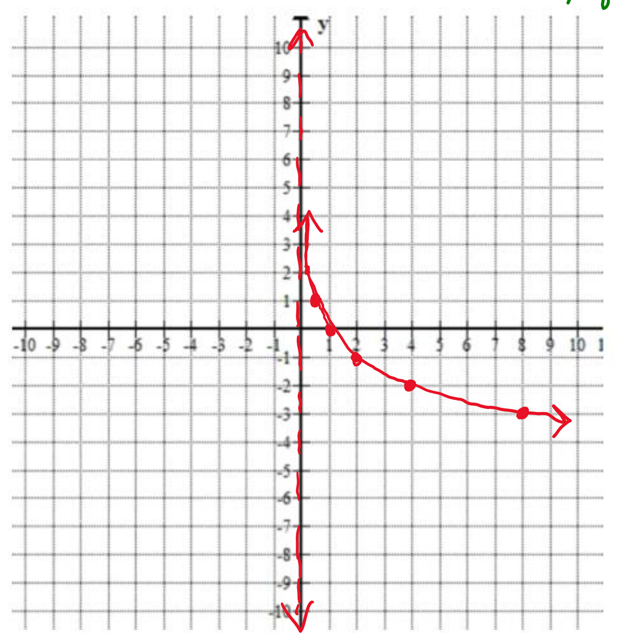


$x=0$

$\{x \mid x>0, x \in \mathbb{R}\}$

$\{y \mid y \in \mathbb{R}\}$

asymptote: $x=0$ (VA)



VA: $x=0$

$\{x \mid x>0, x \in \mathbb{R}\}$

$\{y \mid y \in \mathbb{R}\}$

asymptote VA $x=0$

Laws of Logarithms

Rule of Logarithms

cuemath
THE MATH EXPERT

Rule Name	Property
Log of 1	$\log_b 1 = 0$
Log of the same number as base	$\log_b b = 1$
Product Rule	$\log_b(mn) = \log_b m + \log_b n$
Quotient Rule	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
Power Rule	$\log_b m^n = n \log_b m$
Change of Base Rule	$\log_a b = \frac{\log_c b}{\log_c a}$ (OR) $\log_a b \cdot \log_c a = \log_c b$
Equality Rule	$\log_b a = \log_b c \Rightarrow a = c$
Number Raised to Log	$b^{\log_b x} = x$
Other Rules	$\log_{b^n} a^m = \frac{m}{n} \log_b a$ $-\log_b a = \log_b \frac{1}{a}$ (OR) $= \log_{\frac{1}{b}} a$

Rule 1: $\log_b(M \cdot N) = \log_b M + \log_b N$

Rule 2: $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

Rule 3: $\log_b\left(M^k\right) = k \cdot \log_b M$

Rule 4: $\log_b(1) = 0$

Rule 5: $\log_b(b) = 1$

Rule 6: $\log_b\left(b^k\right) = k$

Rule 7: $b^{\log_b(k)} = k$

Where:

$b > 0$ but $b \neq 1$, and M, N, and k are real numbers but M and N must be positive!

- **The Law of Logarithms for Powers (Power Law)** = $\log_a x^n = n \log_a x$
- **The Law of Logarithms for Roots** = $\log_x \sqrt[n]{x^m} = \log_a x^{\frac{m}{n}} = \frac{m}{n} \log_a x$
- **The Multiplication Law of Logs (Product Law)**= $\log_a xy = \log_a x + \log_a y$
- **The Division Law of Logs (Quotient Law)**= $\log_a \frac{x}{y} = \log_a x - \log_a y$

$$5\log_3(x)+2\log_3(4x)-\log_3(8x^5)=\log_3(x^5)+\log_3((4x)^2)-\log_3(8x^5)$$
$$=\log_3(x^5)+\log_3(16x^2)-\log_3(8x^5)$$
$$=\log_3(x^5\cdot 16x^2)-\log_3(8x^5)$$
$$=\log_3(16x^7)-\log_3(8x^5)$$
$$=\log_3\left(\frac{16x^7}{8x^5}\right)$$
$$5\log_3(x)+2\log_3(4x)-\log_3(8x^5)=\log_3(2x^2)$$

$$\log_6\left(\frac{36m^3}{\sqrt{n}}\right)=\log_6(36m^3)-\log_6(\sqrt{n})$$
$$=\log_6(36)+\log_6(m^3)-\log_6\left(n^{\frac{1}{2}}\right)$$
$$=\log_6(6^2)+3\log_6(m)-\frac{1}{2}\log_6(n)$$
$$=2\log_6(6)+3\log_6(m)-\frac{1}{2}\log_6(n) \quad \text{Rule 5} \Rightarrow \log_6(6)=1$$
$$=2(1)+3\log_6(m)-\frac{1}{2}\log_6(n)$$
$$\log_6\left(\frac{36m^3}{\sqrt{n}}\right)=2+3\log_6(m)-\frac{1}{2}\log_6(n)$$

$$2\log_5(m)+3\log_5(k)-8\log_5(y)=\log_5(m^2)+\log_5(k^3)-\log_5(y^8)$$
$$=\log_5(m^2\cdot k^3)-\log_5(y^8)$$
$$=\log_5(m^2k^3)-\log_5(y^8)$$
$$2\log_5(m)+3\log_5(k)-8\log_5(y)=\log_5\left(\frac{m^2k^3}{y^8}\right)$$

$$3+\frac{1}{2}\log_4(x)+\frac{1}{2}\log_4(y)=3+\log_4\left(x^{\frac{1}{2}}\right)+\log_4\left(y^{\frac{1}{2}}\right)$$

$$=3+\log_4\left(x^{\frac{1}{2}}\cdot y^{\frac{1}{2}}\right)$$

$$=3+\log_4\left(\sqrt{x}\cdot\sqrt{y}\right)$$

$$=3+\log_4\left(\sqrt{xy}\right)$$

$$=3\cdot\log_4(4)+\log_4\left(\sqrt{xy}\right) \quad \text{Since } \log_4(4)=1$$

$$=\log_4\left(4^3\right)+\log_4\left(\sqrt{xy}\right)$$

$$=\log_4\left(4^3\cdot\sqrt{xy}\right)$$

$$3+\frac{1}{2}\log_4(x)+\frac{1}{2}\log_4(y)=\log_4\left(64\sqrt{xy}\right)$$

- **Common Base Law =** $\log_a a^x = x$ **OR** $a^{\log_a x} = x$

$\log_a N = x$	$a^x = N$	$\log_a N = x$
$a^x = N$	$2^3 = 8$	$\log_2 8 = 3$
$a^{\log_a N} = N$	$3^4 = 81$	$\log_3 81 = 4$
	$5^3 = 125$	$\log_5 125 = 3$
	$10^4 = 10000$	$\log_{10} 10000 = 4$
	$7^1 = 7$	$\log_7 7 = 1$
	$5^0 = 1$	$\log_5 1 = 0$

Log Laws

- **The Law of Logarithms for Powers (Power Law) =** $\log_a x^n = n\log_a x$

- **The Law of Logarithms for Roots =** $\log_x \sqrt[n]{x^m} = \log_a x^{\frac{m}{n}} = \frac{m}{n}\log_a x$

- **The Multiplication Law of Logs (Product Law)=** $\log_a xy = \log_a x + \log_a y$

- **The Division Law of Logs (Quotient Law)=** $\log_a \frac{x}{y} = \log_a x - \log_a y$

- **Change of Base Law=** $\log_a x = \frac{\log x}{\log a}$

- **Common Base Law =** $\log_a a^x = x$ **OR** $a^{\log_a x} = x$