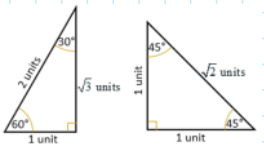
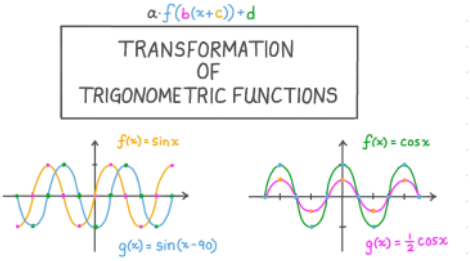
Thursday, Apr. 4th



	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$



Plan For Today:

1. Any questions about anything?
   - *6.2-6.3 Check-in quiz*
2. Continue Chapter 6: The Unit Circe & Trigonometry
   - 6.1: Trigonometric Functions
   - 6.2: Trig Functions of Acute Angles
   - 6.3: Trig Functions of General & Special Angles
   - **6.4: Graphing Basic Trig Functions**
   - **6.5: Applications of Periodic Functions**
3. Do Practice Questions from Workbook

Plan Going Forward:

1. Finish going through Ch6 practice questions in textbook.

   - CHAPTER 6 PROJECT (PART A HANDOUT & PART B IN DESMOS) DUE TUESDAY, APR. 9TH
     - https://student.desmos.com/activitybuilder/student-greeting/65f089483694a5f29f2b2f77
   - CHAPTER 6 QUIZ ON TUESDAY, APR. 9TH

2. Tuesday after the test, we will start Ch7 Trig Identities (7.1-7.2)

   - CHAPTER 7 PROJECT DUE THURSDAY, APR. 18TH
   - CHAPTER 7 QUIZ ON THURSDAY, APR. 18TH

- UNIT 4 EXAM ON TUESDAY, APRIL 23RD - LAST CLASS...
  - Start 12:30pm
  - 12 Multiple Choice & 20 marks on the Written
  - ~1 hour
  - Closed-book - no notes
  - Rewrite on Thursday, Apr. 25th

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.
Anurita Dhiman = adhiman@sd35.bc.ca

Apr. 4, 2024 **Name:** KEY **TOTAL = ____ / 7 marks**

Check-in Quiz Section 6.2-6.3: Trig Ratios and Special Angles

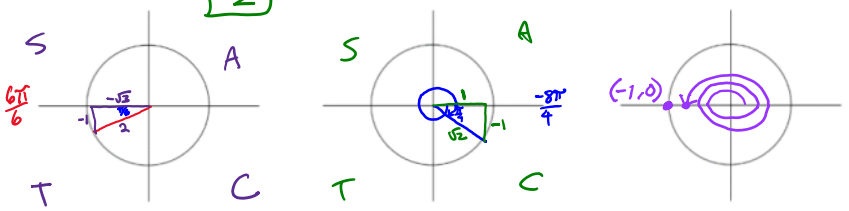
Complete the following questions SHOWING ALL WORK and steps where applicable.

1. Determine the exact value of each of the following (no calculator) and show the location on the unit circle provided: (1 mark)

a) $\cos\left(\frac{7\pi}{6}\right)$ $= -\frac{\sqrt{3}}{2}$

b) $\sin\left(-\frac{9\pi}{4}\right)$ $= \frac{-1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$

c) $\tan(5\pi)$ $= \frac{y}{x} = \frac{0}{-1} = 0$

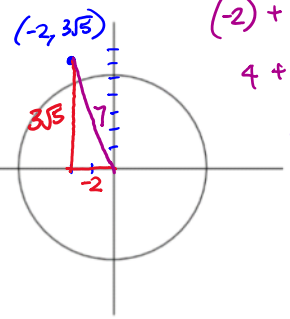


2. Given the point, $P(-2, 3\sqrt{5})$, is on the unit circle terminal arm for angle θ, determine the value of all six trigonometric ratios. (3 marks)

$\cos\theta = -\frac{2}{7}$ $\quad \sec\theta = -\frac{7}{2}$

$\sin\theta = \frac{3\sqrt{5}}{7}$ $\quad \csc\theta = \frac{7}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{7\sqrt{5}}{15}$ or

$\tan\theta = -\frac{3\sqrt{5}}{2}$ $\quad \cot\theta = -\frac{2}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{15}$ or



$(-2)^2 + (3\sqrt{5})^2 = r^2$

$4 + 45 = r^2$

$\sqrt{49} = \sqrt{r^2}$

$r = \pm 7$

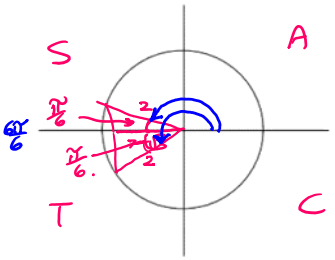
$r = 7$

3. Determine the measure of all angles that satisfy the following conditions: (3 marks)

a) $\cos\theta = -\frac{\sqrt{3}}{2}$, $0 \le \theta < 2\pi$ cosine is negative in quadrants: QII + QIII

adj / hyp

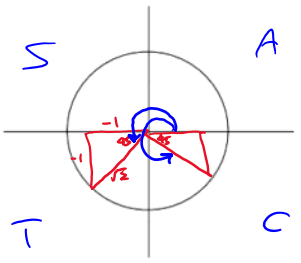
$\theta_R = \frac{\pi}{6}$



if you answer in degrees here - zero marks -

$\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

b) $\sin\theta = -\frac{1}{\sqrt{2}}$, $0^\circ \le \theta < 360^\circ$ sine is negative in quadrants: QIII + QIV

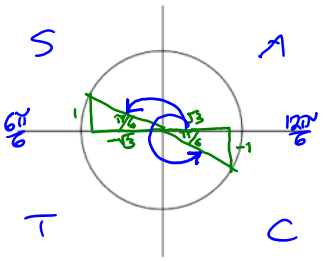


$\theta = 225^\circ, 315^\circ$

c) $\cot\theta = \frac{-\sqrt{3}}{1}$, $0 \le \theta < 2\pi$ tangent is negative in quadrants: QII, QIV

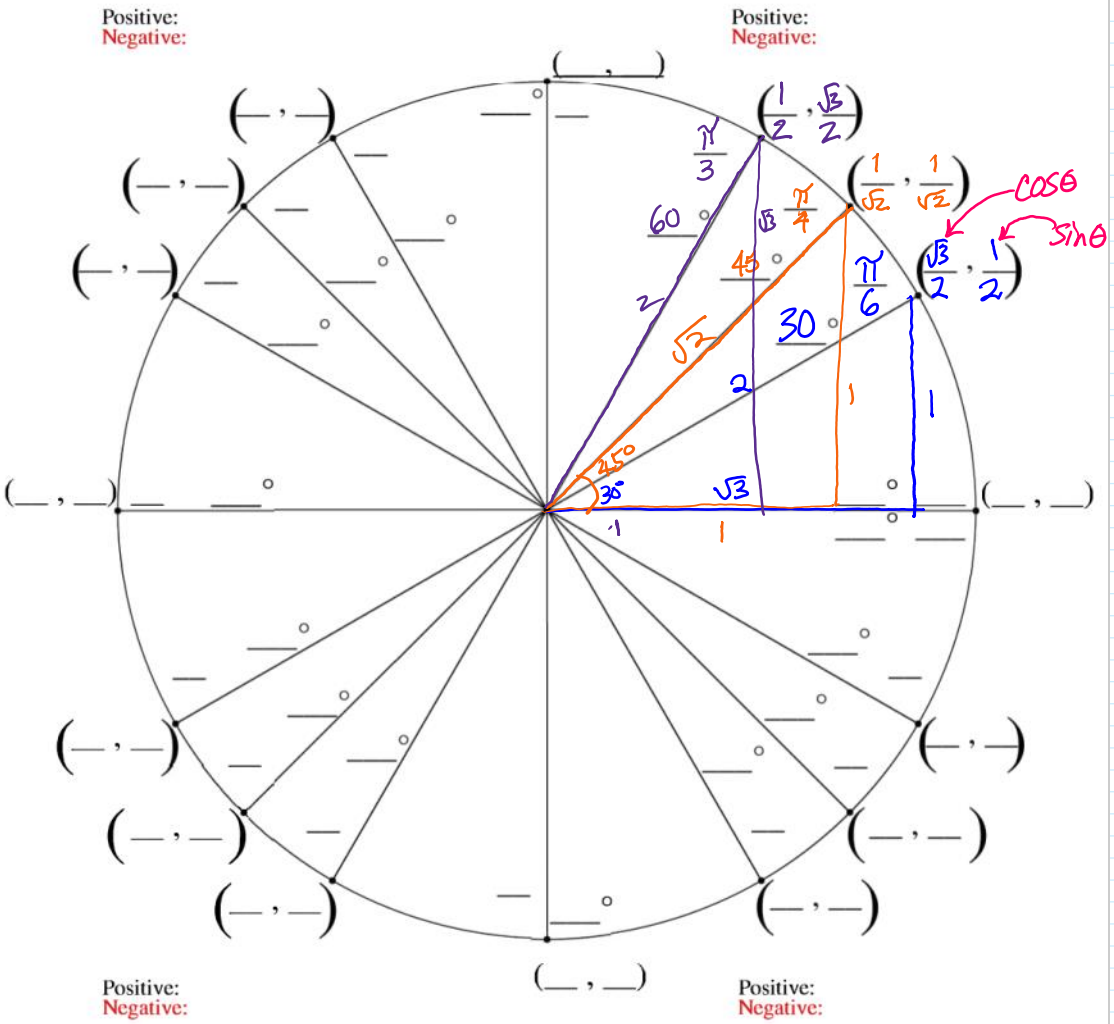
A

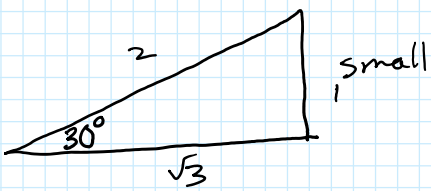
10

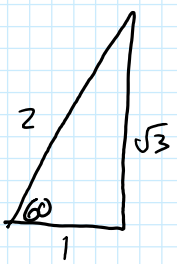


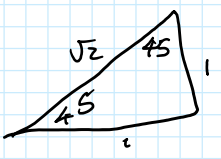
$\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

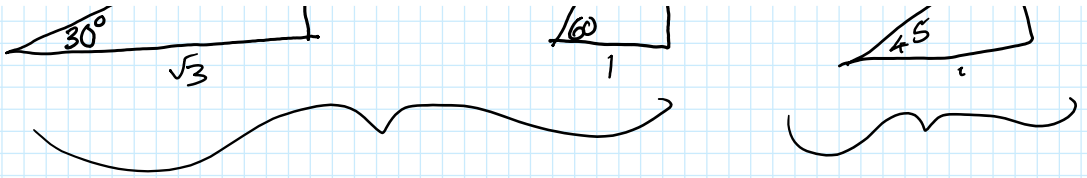
The Unit Circle

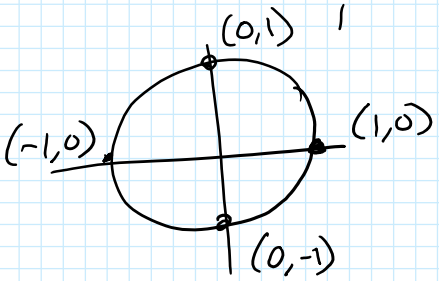






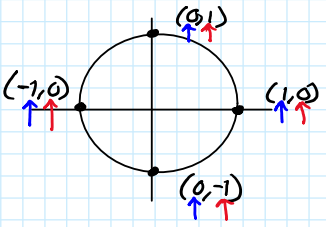


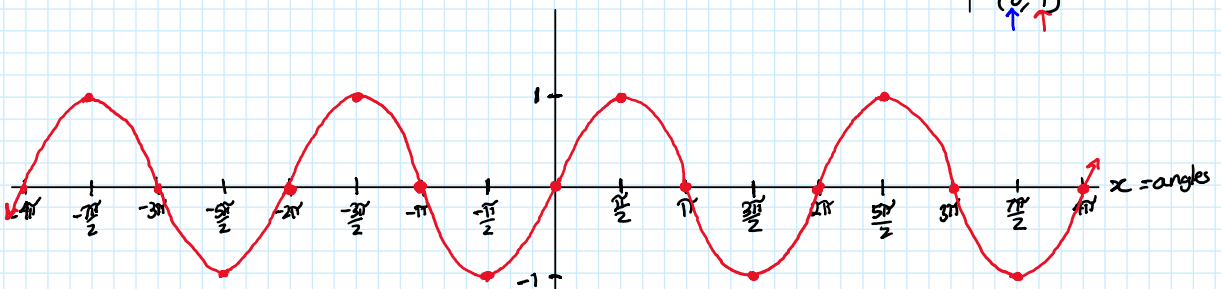




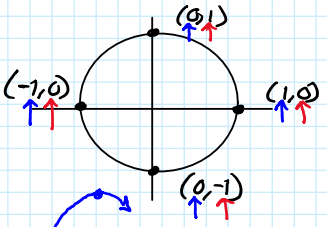
6.4 Graph Trig Functions.

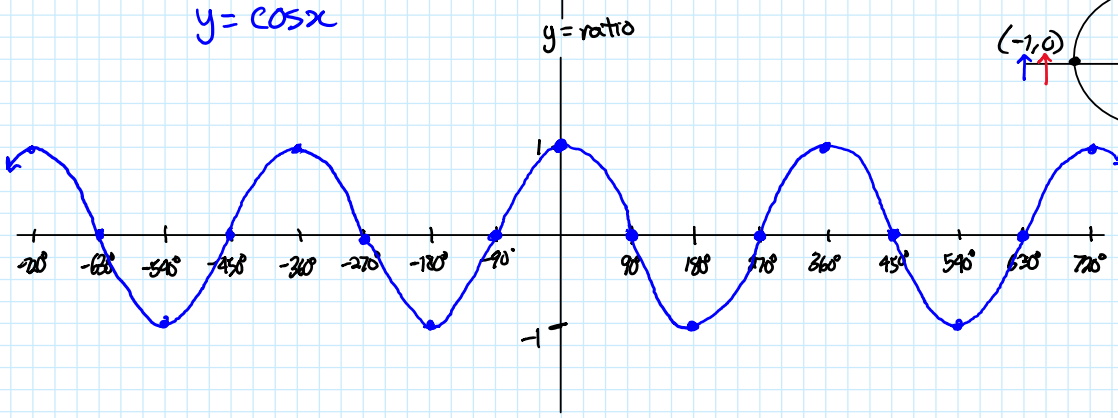
$y = \sin x$ $y = \cos x$





$y = \cos x$





$$y = a \overset{\text{(cos)}}{\sin} b(x - c) + d$$

- **a** → vertical stretch = amplitude ↓ distance above & below midline
- **b** → horizontal stretch by $\frac{1}{b}$ = period $\times \frac{1}{b}$ gives new period. $p = 2\pi \times \frac{1}{b} \rightarrow \frac{2\pi}{b}$
- **c** → right/left = phase shift. $x - \frac{\pi}{4} = \frac{\pi}{4}$ right; $x + 30° = 30°$ left.
- **d** → up/down: = adjust midline up + down. ↳ midline is = d

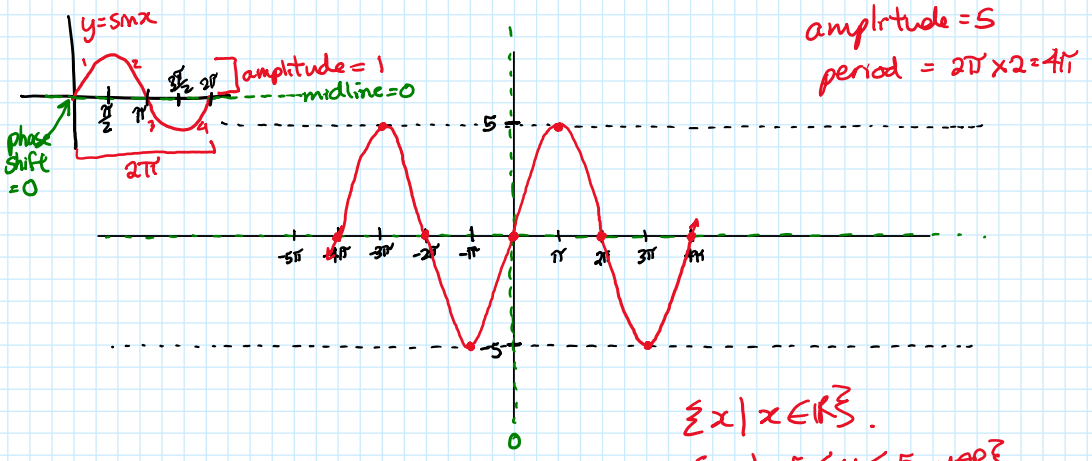
distance above & below midline

gives new period.

$p = 2\pi \times \frac{1}{b} \rightarrow \frac{2\pi}{b}$

$p = 360^\circ \times \frac{1}{b} \rightarrow \frac{360^\circ}{b}$

Ex: 1 $y = 5 \sin \frac{1}{2}x$ graph $-4\pi \le x \le 4\pi$.



$\{x \mid x \in \mathbb{R}\}$.

$\{y \mid -5 \le y \le 5, y \in \mathbb{R}\}$.

Ex2. $y = -6 \cos 2\left(x - \frac{\pi}{6}\right) - 3$ 2 cycles.

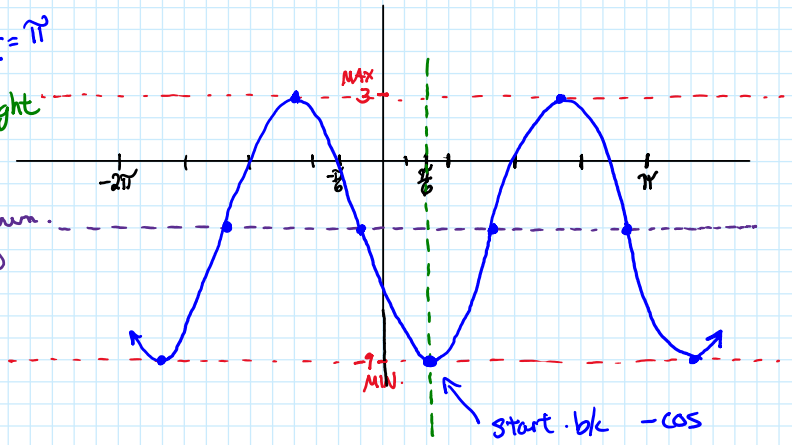
refl. over x-axis.

amp = 6

period = $2\pi \times \frac{1}{2} = \pi$

phase shift = $\frac{\pi}{6}$ right

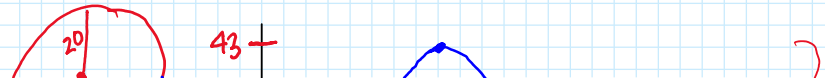
midline = -3 (vertical displacement) 3 down.



$\{y \mid -9 \le y \le 3, y \in \mathbb{R}\}$.

6.5 Applications.

p.288 Ex 3 Ferris Wheel.



p.288 Ex 3 [illegible]

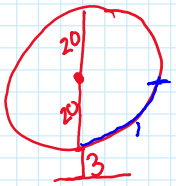
a) radius = 20 m

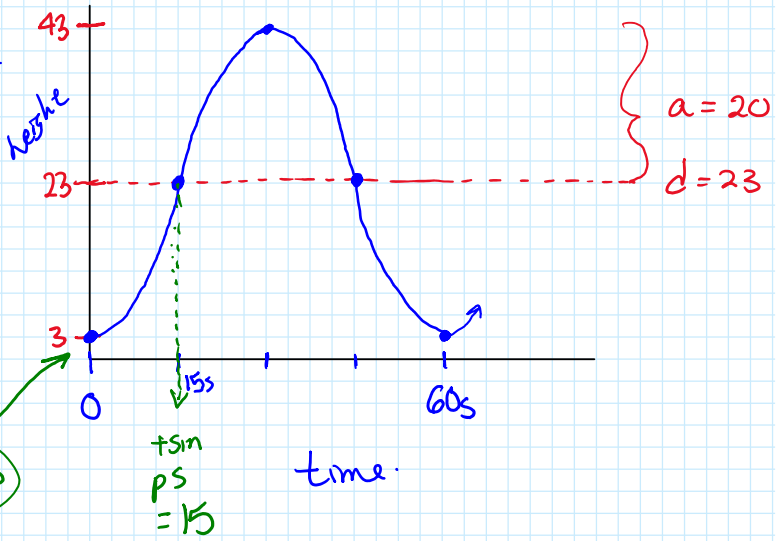
rotation = 60 s

enter at lowest point = 3 m

cos h = height t = time

b) h = 30 m → find t





amp. 20

d = 23

$p = 60 \longrightarrow b = \frac{2\pi}{p}$

$b = \frac{2\pi}{60}$ or $\frac{\pi}{30}$

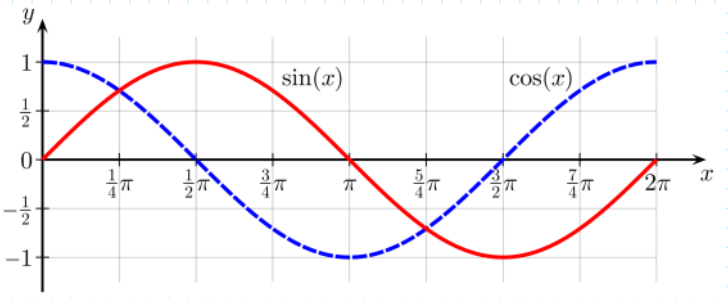
p.s. - cos.

$h(t) =$

$$h = -20\cos\frac{\pi}{30}t + 23$$

Explore Learning Gizmo: (5 min per day)

https://tinyurl.com/yckn3e3p



Sine Function

Domain: $(-\infty, \infty)$ **Range:** $[-1, 1]$

y-intercept: 0

x-intercepts: $n\pi, n \in \mathbb{Z}$

Continuity: continuous on $(-\infty, \infty)$

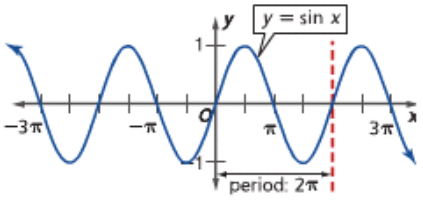
Symmetry: origin (odd function)

Extrema: maximum of 1 at $x = \frac{\pi}{2} + 2n\pi, n \in \mathbb{Z}$

minimum of -1 at $x = \frac{3\pi}{2} + 2n\pi, n \in \mathbb{Z}$

End Behavior: $\lim_{x \to -\infty} \sin x$ and $\lim_{x \to \infty} \sin x$ do not exist.

Oscillation: between -1 and 1



Cosine Function

Domain: $(-\infty, \infty)$ **Range:** $[-1, 1]$

y-intercept: 1

x-intercepts: $\frac{\pi}{2}n, n \in \mathbb{Z}$

Continuity: continuous on $(-\infty, \infty)$

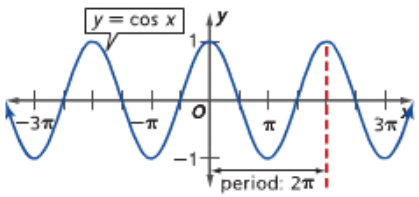
Symmetry: y-axis (even function)

Extrema: maximum of 1 at $x = 2n\pi$, $n \in \mathbb{Z}$

minimum of -1 at $x = \pi + 2n\pi$, $n \in \mathbb{Z}$

End Behavior: $\lim_{x \to -\infty} \cos x$ and $\lim_{x \to \infty} \cos x$ do not exist.

Oscillation: between -1 and 1



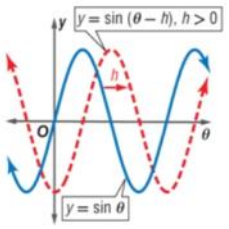
Term	Definition/Explanation
Amplitude	Half the vertical distance from the maximum height to the minimum height of the function.
Interval	The domain of one cycle; written as $[x_b, x_e]$, where x_b is the beginning and x_e is the end.
Period	The horizontal length of one repeating pattern of the function.
Phase Shift or Horizontal Shift	The horizontal distance a function is moved.
Vertical Shift	The vertical distance a function is moved.
Interval	The horizontal starting point and ending point of one complete period of a cyclical trigonometric function.

MATHguide.com

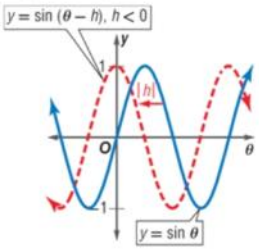
KeyConcept Phase Shift

Words The phase shift of the functions $y = a \sin b(\theta - h)$, $y = a \cos b(\theta - h)$, and $y = a \tan b(\theta - h)$ is h, where $b > 0$.

Models



If $h > 0$, the shift is h units to the right.



If $h < 0$, the shift is $|h|$ units to the left.

Examples

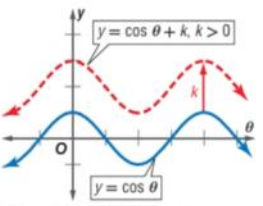
$y = \cos(\theta - 90°)$ The phase shift is 90° to the right.

$y = \tan(\theta + 30°)$ The phase shift is 30° to the left.

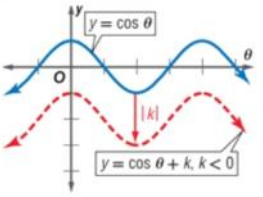
KeyConcept Vertical Shift

Words The vertical shift of the functions $y = a \sin b\theta + k$, $y = a \cos b\theta + k$, and $y = a \tan b\theta + k$ is k.

Models



If $k > 0$, the shift is k units up.



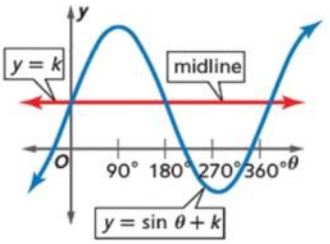
If $k < 0$, the shift is $|k|$ units down.

Examples

$y = \sin\theta + 4$ The vertical shift is 4 units up.

$y = \tan\theta - 3$ The vertical shift is 3 units down.

https://slideplayer.com/slide/10269911/



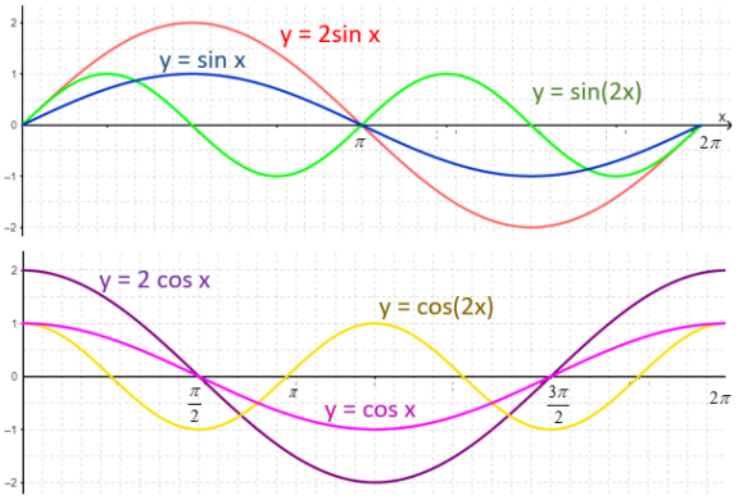
Transform Sine and Cosine Graphs

y = A sin(B(x-k)) + c
y = A cos(B(x-k)) + c

The amplitude is |A|

The period is $\frac{2\pi}{|B|}$

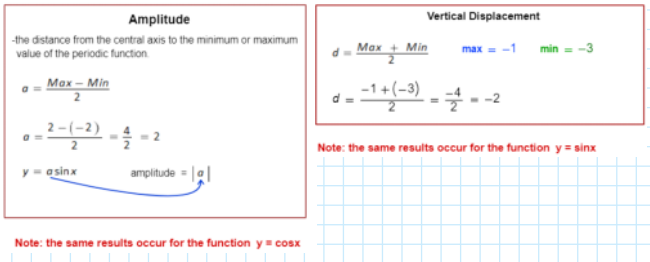
k is horizontal shift
c is vertical shift



Graphing Transformations on the Sine and Cosine Function

Steps:

1. Make sure the equation is written in standard transformation form:
   - $y = a\sin b(x-c)+d$ **and** $y = a\cos b(x-c)+d$
2. List all characteristics (in radians or degrees depending on the question):
   - Amplitude = a
   - Vertical Displacement = d
   - Period = 2pi/b
   - Phase Shift = c
3. Determine the Midline, Maximum and Minimum for the y-axis scale based on the amplitude and vertical displacement.



Note: the same results occur for the function y = cosx

Note: the same results occur for the function y = sinx

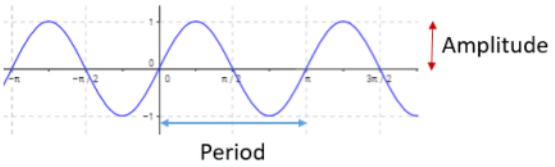
4. Determine the period and phase shift for the x-axis scale.
   - Use 4,8,12,16 squares on the grid to equal the length of the period
   - Divide the period by the number of squares to determine the length of one square then label the x-axis to easily find the phase shift
5. Place the first point at the beginning of the cycle at the phase shift.
   - On the midline for sine
   - On the max point for +cosine
   - On the min point for -cosine
6. Divide the number of square you gave for the length of the period by 4, then count that many squares for each max, midline and min point for one complete cycle.
7. Continue the pattern to graph at least 2 cycles.

Transformation of Trigonometric Graphs

$$y = A\sin\left[B\left(x - C\right)\right] + D$$

$|A|$ is the amplitude

The period is $\frac{2\pi}{B}$



Phase (horizontal) shift is C

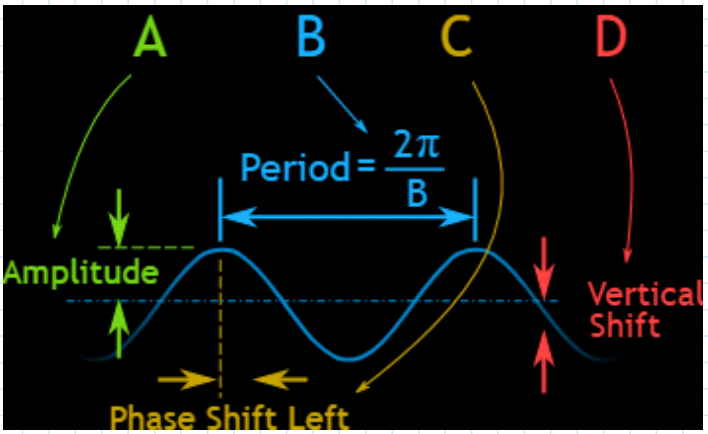
Vertical shift is D

The same applies for the Cosine Function.

For the Tangent Function, the period is $\frac{\pi}{B}$

Rational Periods

- ✧ x-axis scale is divided into integers, not into radians or degree units.
- ✧ RECALL:

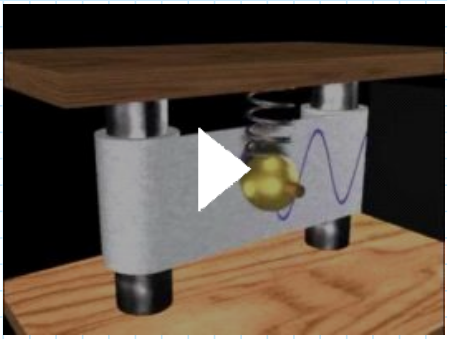


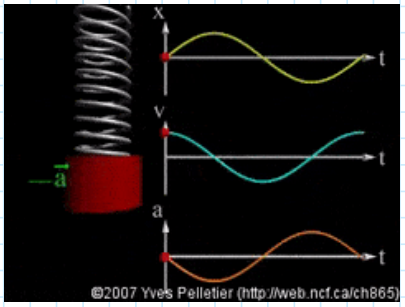
Modelling Real-life Period Functions

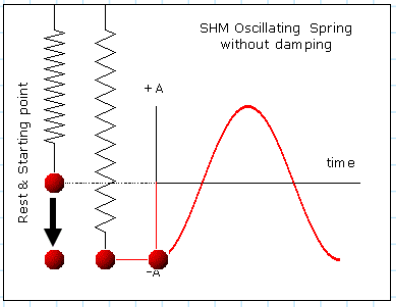
Bouncing Mass on a Spring:
https://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=421

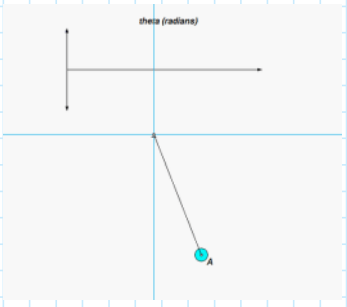
Tides:
https://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=368

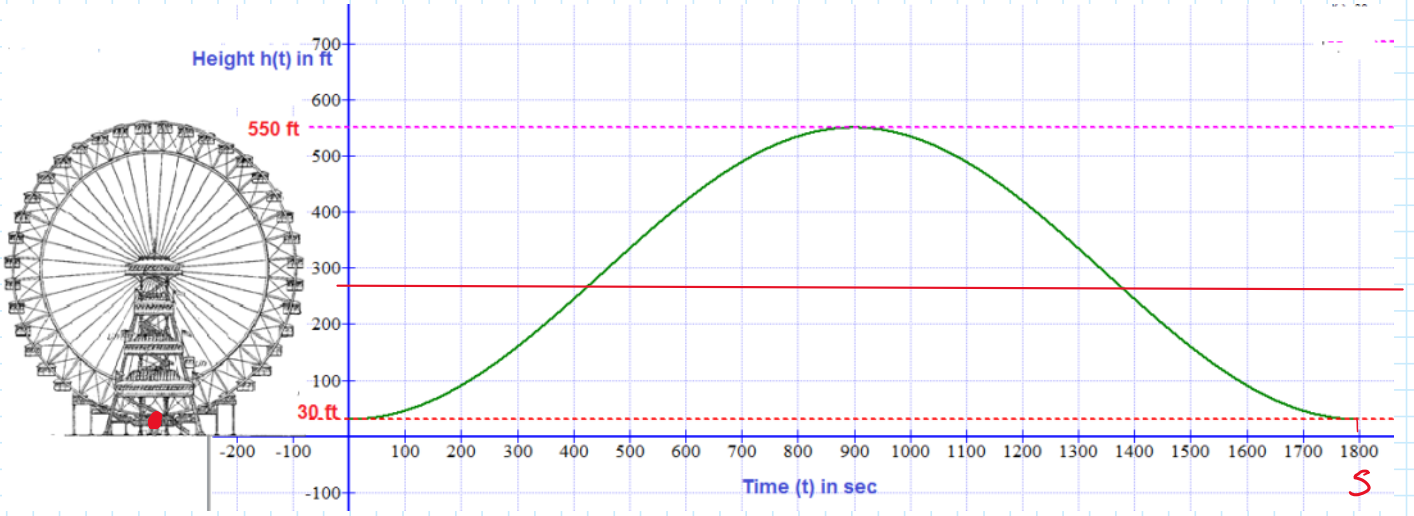
pendulum sinusoidal curve animation










Height h(t) in ft
700
600
550 ft
500
400
300
200
100
30 ft
-100
-200
-100
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500
1600
1700
1800
Time (t) in sec
S