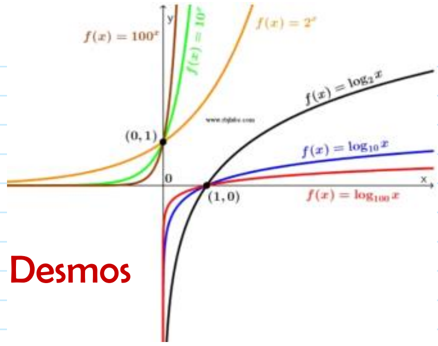
Thursday, Nov. 2nd



Plan For Today:

1. Questions from Chapter Ch7?
   - Hand-in 7.1-7.2 Check-in Quiz
   - Hand-in Ch7 Project (Part B) - Part A Desmos
   - Do Ch7 Test
2. Start Chapter 8: Logarithmic Functions
   - ✓ **8.1: Understanding Logarithms**
   - ✓ **8.2: Transformations of Logarithmic Functions**
   - 8.3: Laws of Logarithms
   - 8.4: Logarithmic & Exponential Functions
3. Work on Practice Questions from Workbook

Argument

$$b^x = a \Leftrightarrow \log_b a = x$$

base

Plan Going Forward:

1. Finish going through 8.1-8.2 and chapter practice questions in workbook and start working on review handout.

   - **8.1-8.2 CHECK-IN QUIZ ON TUESDAY, NOV. 7TH**

2. We will continue Ch8 Logarithms on Tuesday.

   - CHAPTER 8 PROJECT DUE TUESDAY, NOV. 21ST
   - UNIT 3 EXAM ON TUESDAY, NOV. 21ST

based on today (8.1) you can do #1,3,7,6 12,14,17,18

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.

Anurita Dhiman = adhiman@sd35.bc.ca

Thursday, Nov. 2nd In-Class Notes

Chapter 8 Logarithmic Functions

8.1 Understanding Logarithms

KEY IDEAS

- A logarithm is the exponent to which a fixed base must be raised to obtain a specific value.

  Example: $5^3 = 125$. The logarithm of 125 is the exponent that must be applied to base 5 to obtain 125. In this example, the logarithm is 3: $\log_5 125 = 3$.

- Equations in exponential form can be written in logarithmic form and vice versa.

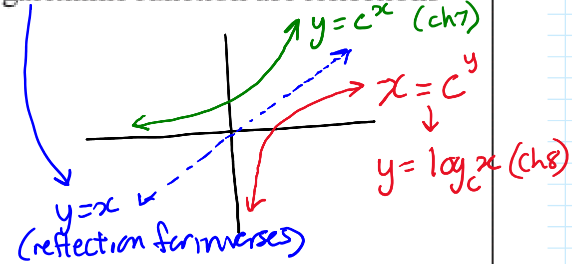
  b/c log equals exponent

Exponential Form	**Logarithmic Form**
$x = c^y$	$y = \log_c x$ → $y = \log_c x$

  exponent base argument

- The inverse of the exponential function $y = c^x$, $c > 0$, $c \neq 1$, is $x = c^y$ or, in logarithmic form, $y = \log_c x$. Conversely, the inverse of the logarithmic function $y = \log_c x$, $c > 0$, $c \neq 1$, is $x = \log_c y$ or, in exponential form, $y = c^x$.

- The graphs of an exponential function and its inverse logarithmic function are reflections of each other in the line $y = x$.



- For the logarithmic function $y = \log_c x$, $c > 0$, $c \neq 1$,
  - the domain is $\{x \mid x > 0, x \in \mathrm{R}\}$
  - the range is $\{y \mid y \in \mathrm{R}\}$
  - the x-intercept is 1
  - the vertical asymptote is $x = 0$, or the y-axis

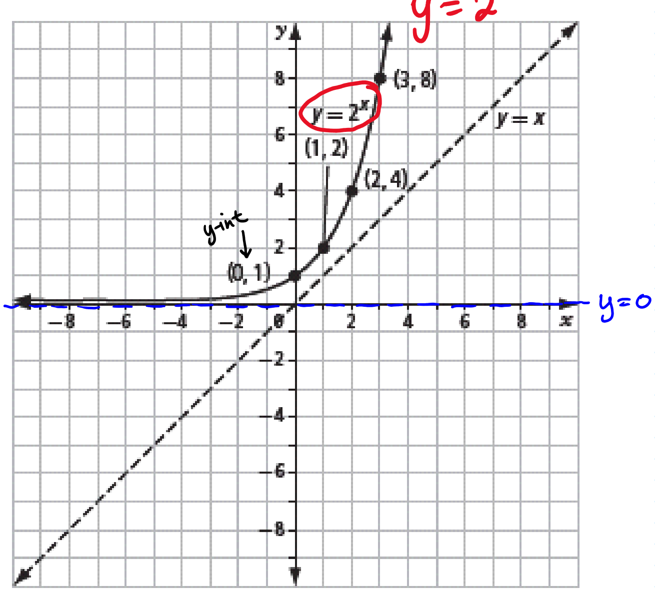
- A common logarithm has base 10. It is not necessary to write the base for common logarithms: $\log_{10} x = \log x$

Working Example 1: Graph the Inverse of an Exponential Function

The graph of $y = 2^x$ is shown at right. State the inverse of the function. Then, sketch the graph of the inverse function and identify the following characteristics of the graph:

- domain and range
- x-intercept, if it exists
- y-intercept, if it exists
- the equation of any asymptotes

$y = 2^x$



D: $\{x \mid x \in \mathrm{R}\}$.

R: $\{y \mid y > 0, y \in \mathrm{R}\}$

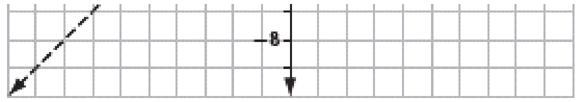
no x-int

asymptote:

no x-int

y-int: (0,1)

asymptote: y=0



Solution

Recall: $x = c^y \rightarrow y = \log_c x$

The inverse of $y = 2^x$ is $x = 2^y$.

In logarithmic form, the inverse function is $y = \log_{\boxed{2}} x$. → $\boxed{y = \log_2 x}$ is inverse of $y = 2^x$

The inverse of a function is its reflection in the line $y = x$. To graph the inverse, sketch this reflection. Alternatively, interchange each pair of coordinates on the graph of the function to obtain coordinates on the inverse graph.

For example, the point [redacted] (0,1) is on the graph of $y = 2^x$. Therefore, the point (1,0) is on the graph of the inverse.

Add the graph of the inverse of the function to the grid on the previous page.

- The domain is $\{x \mid x > 0, x \in R\}$.
- The range is $\{y \mid y \in R\}$.
- The graph has no y-intercept.
- The x-intercept is (1,0).
- The vertical asymptote is $x = 0$.

inverses of $y = 2^x$

Working Example 2: Change the Form of an Expression

For each expression in exponential form, rewrite it in logarithmic form. For each expression in logarithmic form, rewrite it in exponential form.

$x = c^y \leftrightarrow y = \log_c x$

exponential form

logarithmic form

a) $3^4 = 81$ base 3 $\quad 4 = \log_3 81$

b) $36^{\frac{1}{2}} = 6$

c) $\log_5 125 = 3$ ← exponent, base 5 $\quad 125 = 5^3$

d) $\log_{10} 10\,000 = 4$

Solution

a) The base is 3 and the exponent is 4. The logarithmic form is $\log_{\boxed{3}} 81 = 4$.

b) 36 is the base, so the logarithmic form is $\log_{36} 6 = \frac{1}{2}$.

Recall: $36^{\frac{1}{2}} = \sqrt{36} = 6$ ✓

c) The base is 5 and the exponent is 3. The exponential form is $5^3 = 125$.

d) When a base is not stated, the base of a logarithm is 10. The exponential form of this expression is $10^4 = 10000$.

Working Example 3: Evaluate and Determine a Value in Logarithmic Expressions

Evaluate each expression.

a) $\log_2 32$

b) $\log_{100} 10$

Evaluate each expression.

a) $\log_2 32$

b) $\log_{100} 10$

c) $\log_3 \sqrt{27}$

d) $\log_4 x = 3$

e) $\log_x 125 = 3$

f) $\log 0.01 = x$

Solution

common base method

a) This logarithmic expression asks which exponent is applied to base 2 to produce a result of 32. Since $2^5 = 32$, the value of the logarithmic expression is 5.

$\log_2 32 = \cancel{\log_2}(2^5) = 5$ common base can cancel log

b) Since $100^{\frac{1}{2}} = 10$, the value of $\log_{100} 10$ is $\frac{1}{2}$.

$\log_{100} 10 = \cancel{\log_{100}} 100^{\frac{1}{2}} \rightarrow = \frac{1}{2}$

c) Written as a power of 3, 27 is equal to ________ and $\sqrt{\ }$ is written as an exponent of ________. This means that $\log_3 \sqrt{27} = \frac{3}{2}$.

power rule

$\rightarrow \log_3 \sqrt{27} = \log_3 27^{\frac{1}{2}} \rightarrow \log_3 (3^3)^{\frac{1}{2}} \rightarrow \cancel{\log_3} 3^{\frac{3}{2}} = \boxed{\frac{3}{2}}$

d) In exponential form, $\log_4 x = 3$ is equivalent to x = 4^3. So, $\boxed{x = 64}$.

BOOT.

e) In exponential form, $\log_x 125 = 3$ is equivalent to $\sqrt[3]{x^3}$ = $\sqrt[3]{125}$. So, $\boxed{x = 5}$.

common base

f) 0.01 can be written as $\frac{1}{100}$, which is $10^{\boxed{-2}}$. So, $\log 0.01 =$ -2.

$\rightarrow 10^{\square}$

$\log 0.01 = \cancel{\log} \cancel{10}^{-2} \rightarrow = \boxed{-2}$

Working Example 4: An Application of Logarithmic Functions

The intensity of sound is measured in decibels (dB). The level of a sound, L, in decibels, is given by $L = 10\log\left(\frac{I}{I_0}\right)$, where I is the intensity of the sound and I_0 is the faintest sound detectable to humans. The sound level inside a particular car is 39 dB when it is idling, and 80 dB at full throttle. How many times more intense is the sound at full throttle?

Solution

Let I_i be the intensity of the sound at idle and I_f be the intensity at full throttle.

$39 = 10\log\left(\frac{I_i}{I_0}\right)$ or $3.9 = \log\left(\frac{I_i}{I_0}\right)$

$80 = 10\log\left(\frac{I_f}{I_0}\right)$ or $8 =$ ________________

Rewrite each expression in exponential form.

$\frac{I_i}{I_0} =$ ________

$\frac{I_f}{I_0} =$ ________

Then, multiply by I_0 to obtain $I_i = I_0\, 10^{3.9}$ and $I_f =$ ________.

To compare the intensities, divide I_f by I_i.

$$\frac{I_f}{I_i} = \frac{I_0 10^{\square}}{I_0 10^{\square}}$$

I_f

$I_i \quad I_0 10^{\square}$

$\dfrac{I_f}{I_i} = $ ________

The I_0 terms divide out, leaving $10^{\square} = $ ________.

Therefore, the sound at full throttle is about 12 589 times as intense as the sound at idle.

See pages 375–379 of *Pre-Calculus 12* for more examples.

8.1: Understanding Logarithms

What is a log?
A logarithm is an exponent.

$$\log_b(a) = c \iff b^c = a$$

<u>Logarithmic Form</u>		<u>Exponential Form</u>
$\log_3 x = 5$	➡	$3^5 = x$

Both forms use the same base.

The logarithm is equal to the exponent.

Changing between log and exponent form:

Convert each to logarithmic form.

a) $2^m = n$ base $\qquad \log_2 n = m$

b) $10^{x-1} = 1000 \rightarrow \log 1000 = x-1$

c) $(x+1) = 3^{z+1} \rightarrow \log_3(x+1) = z+1$

More:
p.263 # 2 + #3

Convert each to exponential form.

a) $\log_2 x = 3$ → $x = 2^3$

b) $\log_{x-1} 3 = 4$ → $(x-1)^4 = 3$

c) $\log_x (x+2) = 2$ → $x^2 = x+2$

Evaluating and solving logarithms by changing to exponential form.
But there is a short-cut in some cases!

Try solving these:

BOOT the BASE

a) $\log_x 27 = 3$ → $\sqrt[3]{x^3} = \sqrt[3]{27}$

$\boxed{x = 3}$

b) $\log_2 (2x-5) = 4$ → $2x-5 = 2^4$

$2x-5 = 16$

$+5$

$2x = 21$

$\boxed{x = \frac{21}{2}}$

c) $\log_{(x+1)} (2x-1) = 2$

☆ Note:
log have restrictions
$y = \log_c x$
$x > 0$
base is also restricted
$c > 0,\ c \neq 1$

Extra #4-7
p. 264

☆ Restrictions
$2x-5 > 0$
$x > \frac{5}{2}$

$(x+1)^2 = 2x-1$

$(x+1)(x+1)$

$x^2 + 2x + 1 = 2x - 1$

$-2x \quad +1$

$x^2 + 2 = 0$

-2

$\sqrt{x^2} = \sqrt{-2}$

$x = \varnothing$ undefined

$\boxed{x = \text{no solution}}$ (no real roots)

- **Change of Base Law=** $\log_a x = \dfrac{\log x}{\log a}$

$$\log_7(10) = \frac{\log(10)}{\log(7)}$$

$$\approx \frac{1}{0.8451}$$

$$\approx 1.183$$

Try to evaluate these logs:

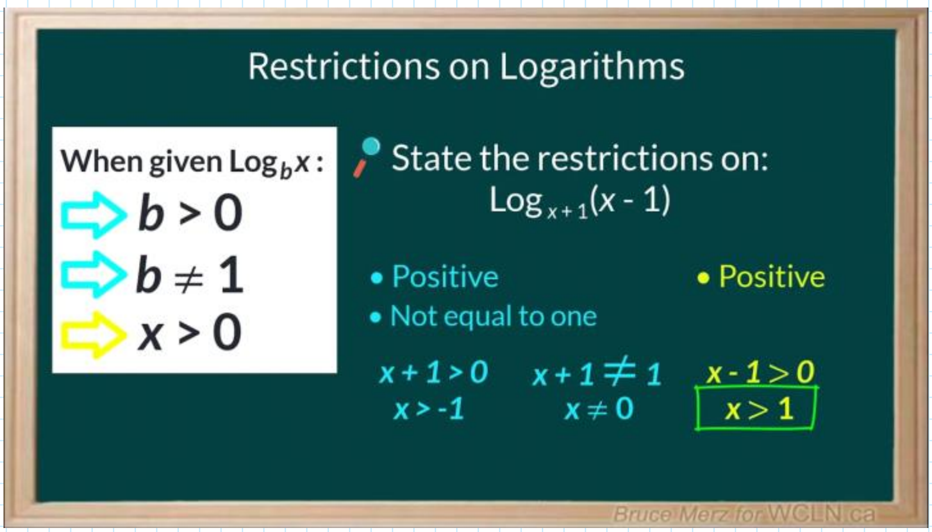
a) $\log_2 64$

b) $\log_4 0.0625$

c) $\log_{\frac{1}{2}} 32$

d) $\log_{1.2} 223.87$

Logarithms have restrictions (non-permissible values = NPVs)



Graphing a Logarithmic Function

Graphing a logarithmic function is the same as graphing the inverse of the exponential function with the same base.

Recall: to graph the inverse, switch the x and y for each coordinate and plot (the graph reflects over the line y=x and the domain and range also inverse)

Ex. Graph the function $y = \log_2 x$

1. First determine the points on the function $y = 2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

Step ① = graph exponential function with the same base as log function

2. Inverse each coordinate and the asymptote to become $x = 0$

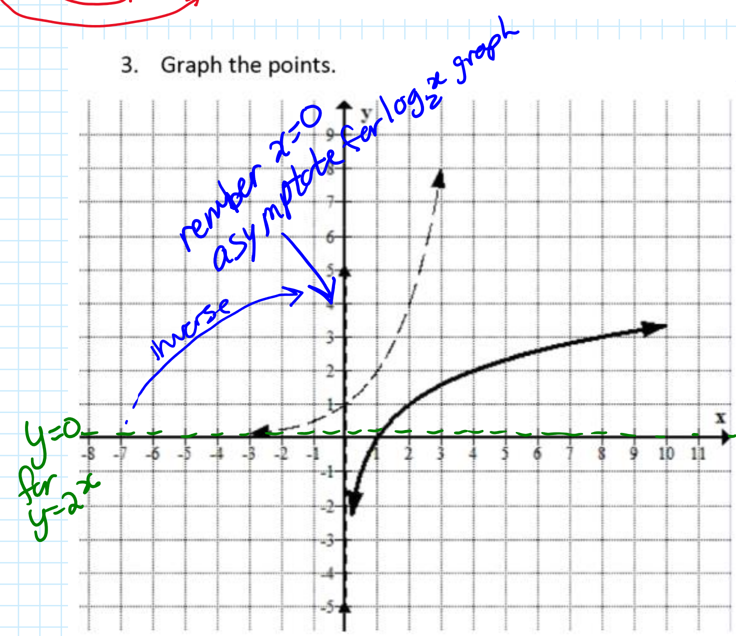
$y=2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

Step ② = inverse the coordinates from $y=2^x$ to get $y=\log_2 x$

3. Graph the points.



Step ③ Graph the inverse to toget $y=\log_2 x$ graph.

Try graphing the following in the same steps as above:

a) $y = \log_3 x$ (base 3)

b) $y = \log_{\left(\frac{1}{2}\right)} x$ (base $\frac{1}{2}$)

step 1 $y=3^x$

step 2

① x

② inverse

step 1

$y = 3^x$

x	y
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

$3^{-2} \to \frac{1}{3^2}$

$3^0 \to 1$

asymptote
$y=0$

step 2
inverse
$\Rightarrow y = \log_3 x$

x	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2

asymptote
$x=0$

① $y = \frac{1}{2}^x$

x	y
-3	8
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

$\left(\frac{1}{2}\right)^{-3} = 2^3$

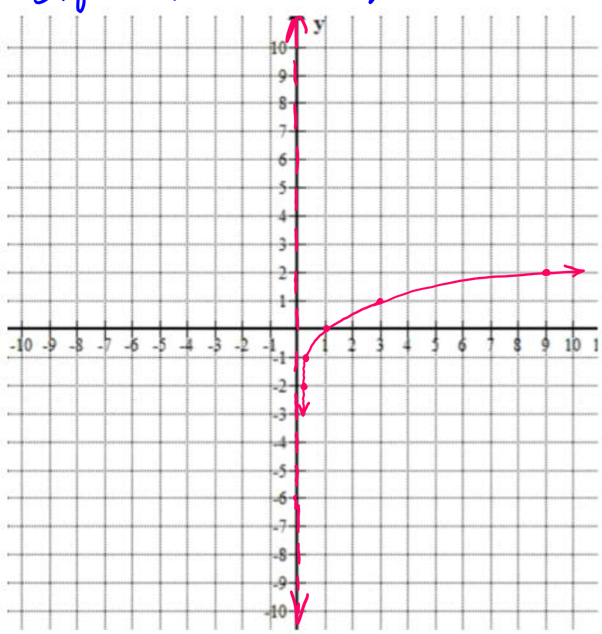
② inverse

$y = \log_{\frac{1}{2}} x$

x	y
8	-3
4	-2
2	-1
1	0
$\frac{1}{2}$	1
$\frac{1}{4}$	2

Step 3 ✩ inverse asymptote

③ Graph.

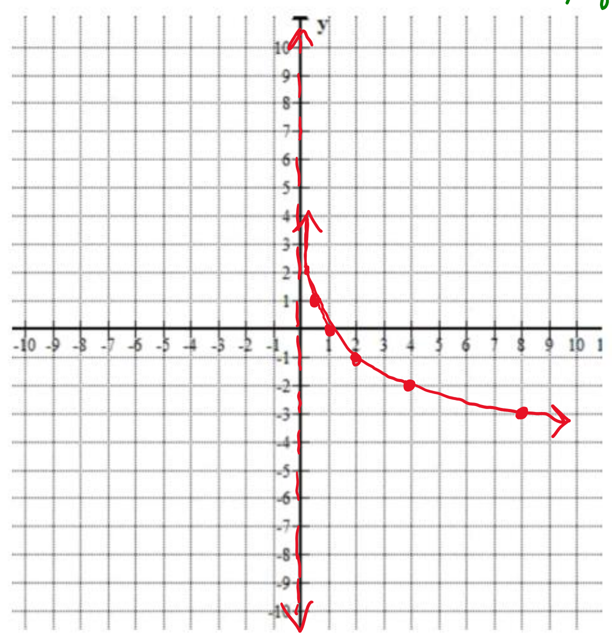


$x=0$

$\{x \mid x>0, x \in \mathbb{R}\}$

$\{y \mid y \in \mathbb{R}\}$

asymptote: $x=0$
(VA)



VA: $x=0$

$\{x \mid x>0, x \in \mathbb{R}\}$

$\{y \mid y \in \mathbb{R}\}$

asymptote $x=0$
VA