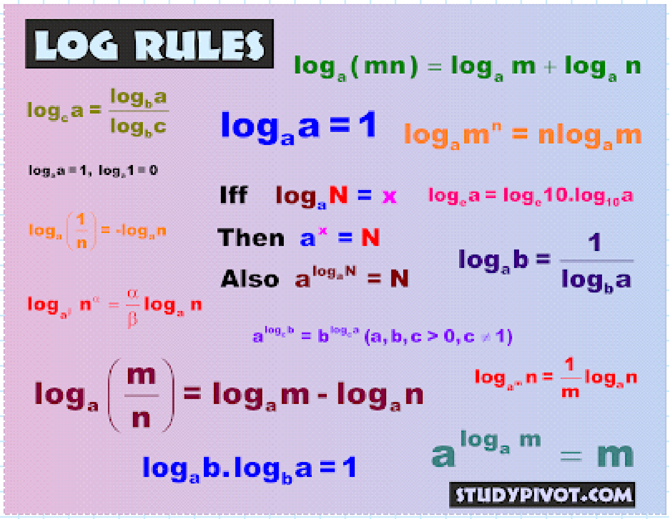
Tuesday, Nov. 7th

Plan For Today:

1. Questions from 8.1 & basic Log graph?
   - Do 8.1 Check-in Quiz
2. Continue Chapter 8: Logarithmic Functions
   - ✓ 8.1: Understanding Logarithms
   - **8.2: Transformations of Logarithmic Functions**
   - **8.3: Laws of Logarithms**
   - 8.4: Logarithmic & Exponential Functions
3. Work on Practice Questions from Workbook



Plan Going Forward:

1. Finish going through 8.2-8.3 and chapter practice questions in workbook and start working on review handout.

- **8.2-8.3 CHECK-IN QUIZ ON THURSDAY, NOV. 9TH**

2. We will finish Ch8 Logarithms and Review Ch7 & 8 on Thursday after long weekend for the Unit 3 Exam.

- CHECK-IN QUIZ ON THURSDAY, NOV. 16TH
- CHAPTER 8 PROJECT DUE TUESDAY, NOV. 21ST (TRY FOR THURSDAY, NOV. 16TH)
- **UNIT 3 EXAM ON TUESDAY, NOV. 21ST**

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class.
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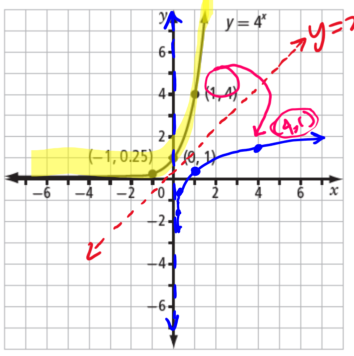
Tuesday, Nov. 7th In-Class Notes

Review

1. For each exponential graph below, sketch the inverse function and state the domain, range, x-intercept, and equation of the vertical asymptote. Then, write the equation of the inverse function.

a)



$y = \log_4 x$

① $y = 4^x$ → inverse → ② $y = \log_4 x$

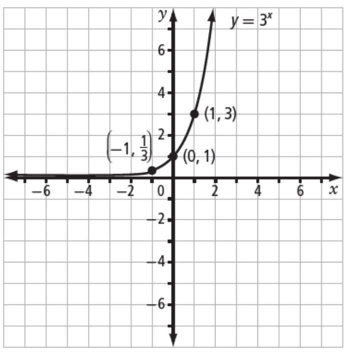
x	y
−2	1/16
−1	1/4
0	1
1	4
2	16

$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

$4^{-1} = \frac{1}{4}$

x	y
1/16	−2
1/4	−1
1	0
4	1
16	2

b)



$y = \log_3 x$

x	y
1/9	−2
1/3	−1
1	0
3	1
9	2

$3^{-2} = \frac{1}{9}$

$3^{-1} = \frac{1}{3}$

c) $16^{\frac{1}{2}} = 4$

d) $8^{-2} = \frac{1}{64}$

e) $10^{-2} = 0.01$

f) $27^{\frac{2}{3}} = 9$

g) $12^x = 2y$

h) $2^{2x-5} = y - 1$

$\log_2 (y-1) = 2x-5$

3. Express in exponential form.

a) $\log_2 32 = 5$

b) $\log_8 512 = 3$

$8^3 = 512$

c) $\log_5 625 = 4$

d) $\log 1000 = 3$

a) $4^x = 64$

b) $10^{2x} = 1\,000\,000$

c) $\log_2 x = 4$ — $x > 0$

d) $\log_5 x = -2$ — $x = 5^{-2}$ → $x = \frac{1}{5^2}$ → $x = \frac{1}{25}$ ✓

Restriction $x > 0$

e) $\log_4 256 = x$ — common base — $\log_4 4^4 = x$ — $x = 4$

f) $\log_{16} 4 = x$ — No

g) $\log_x 81 = 4$ — $x > 0, x \neq 1$

h) $\log_x 6 = \frac{1}{2}$ — $x > 0, x \neq 1$ — $x^{\frac{1}{2}} = 6$ (index radical) — $(\sqrt{x})^2 = (6)^2$ — $x = 36$

i) $\log_x \frac{1}{25} = -2$

j) $\log_x \frac{1}{64} = -3$

8.2 Transformations

$$y = \log_c x \longrightarrow y = a\log_c(b(x-h)) + k$$

- a: vertical stretch by a; $-a$ = reflection in x-axis
- b: horizontal stretch by $\frac{1}{b}$; $-b$ = refl. in y-axis
- h: right/left + $x = h$ asymptote (vertical asymptote) VA
- k: up/down

Ex: $y = -2\log_3(3(x+4)) + 3$

① Base Function $y = \log_3 x$

② stretchs + reflections: HC $\frac{1}{3}$, VE 2, refl. in x-axis

③ 4 left, 3 up

Start $y = 3^x$ → inverse $y = \log_3 x$

x	y
-2	$\frac{1}{9}$ ← $3^{-2} = \frac{1}{3^2}$
-1	$\frac{1}{3}$ ← $3^{-1} = \frac{1}{3}$
0	1 ← $3^0 = 1$
1	3 ← $3^1 = 3$
2	9 ← $3^2 = 9$

x	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2

$\frac{1}{3}x$	$-2y$
$\frac{1}{27}$ ($\frac{1}{9} \times \frac{1}{3}$)	4
$\frac{1}{9}$	2
$\frac{1}{3}$	0
1 ($3 \times \frac{1}{3}$)	-2
3 ($9 \times \frac{1}{3}$)	-4

$\frac{1}{3}x - 4$	$-2y + 3$
$\frac{1}{27} - \frac{108}{27}$ → $-\frac{107}{27}$	7
$\frac{1}{9} - \frac{36}{9}$ → $-\frac{35}{9}$	5
$\frac{1}{3} - \frac{12}{3}$ → $-\frac{11}{3}$	3
$1 - 4$ → -3	1
$3 - 4$ → -1	-1

plot coordinates on graph + include asymptote.

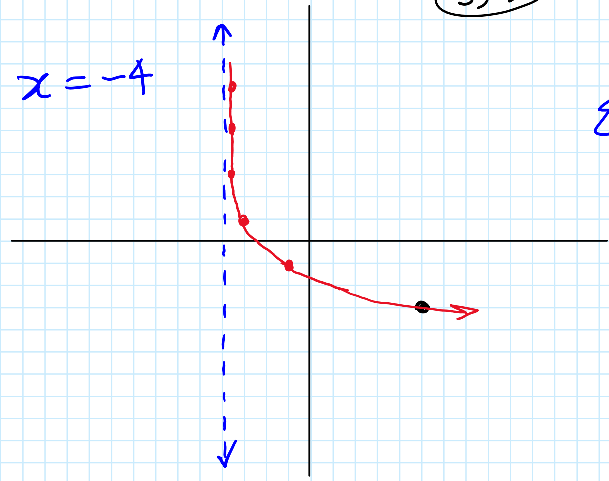
3, 27 ⟶ 27, 3 ⟶ 9, -6

3,27 → 27,3 → 9,-6 → (5,-3)

$9 \times \frac{1}{3}$ → 3 | -4

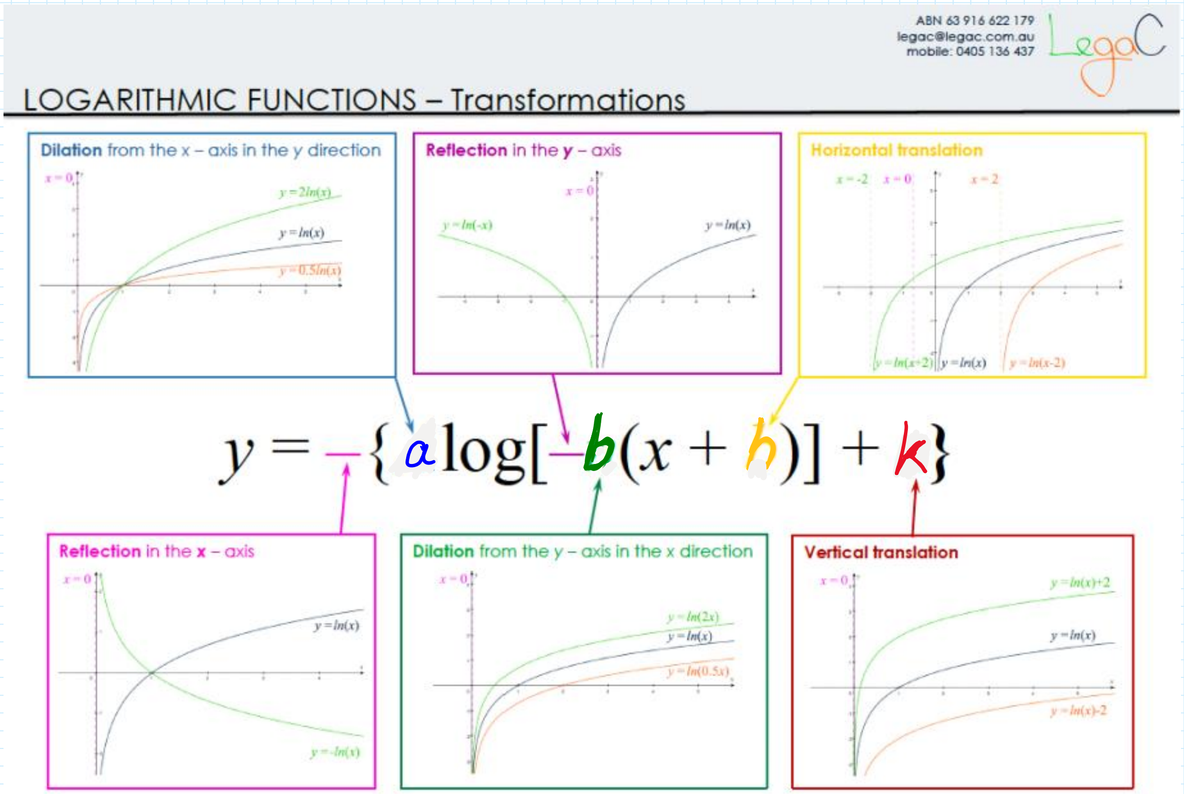
3-4 → (-1 | -1)

* asymptote : $x = -4$



$\{x \mid x > -4, x \in \mathbb{R}\}$

$\{y \mid y \in \mathbb{R}\}$

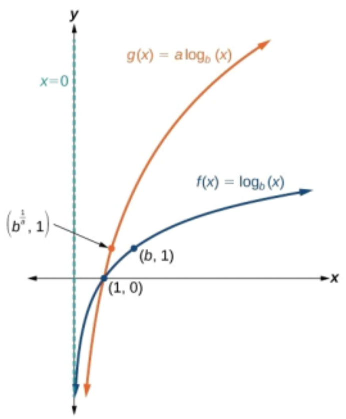


Transformations $y = \log x$

Transformation	f(x) Notation	Examples
Horizontal Translation Graph shifts left or right.	$f(x-h)$	$g(x)=\log(x-3) \to$ 3 units right $g(x)=\log(x+4) \to$ 4 units left
Vertical Translation Graph shifts up or down.	$f(x)+k$	$g(x)=\log x+4 \to$ 4 units up $g(x)=\log x-5 \to$ 5 units down
Reflection Graph flips over x-axis.	$-f(x)$	$g(x)=-\log x \to$ over x-axis
Reflection Graph flips over y-axis.	$f(-x)$	$g(x)=\log(-x) \to$ over y-axis
Horizontal Shrink Graph shrinks toward y-axis.	$f(ax), a>1$	$g(x)=\log 2x \to$ shrink by $\frac{1}{2}$
Horizontal Stretch Graph stretches away from y-axis.	$f(ax),\ 0<a<1$	$g(x)=\log\frac{x}{2} \to$ stretch by 2
Vertical Stretch [redacted] Graph stretches away from x-axis.	$a\cdot f(x),\ a>1$	$g(x)=2\cdot\log x \to$ stretch by 2
Vertical Shrink Graph shrinks toward x-axis.	$a\cdot f(x),\ 0<a<1$	$g(x)=\frac{1}{2}\log x \to$ shrink by $\frac{1}{2}$

Vertical Stretch

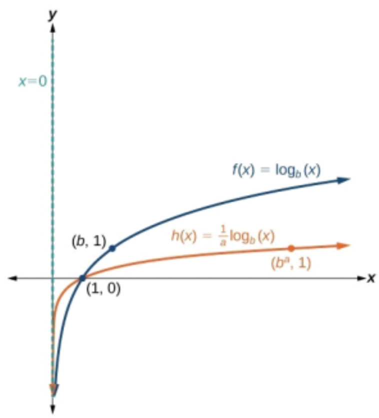
$g(x) = a\log_b(x)$, $a > 1$



- The asymptote remains $x = 0$.
- The x-intercept remains $(1, 0)$.
- The domain remains $(0, \infty)$.
- The range remains $(-\infty, \infty)$.

Vertical Compression

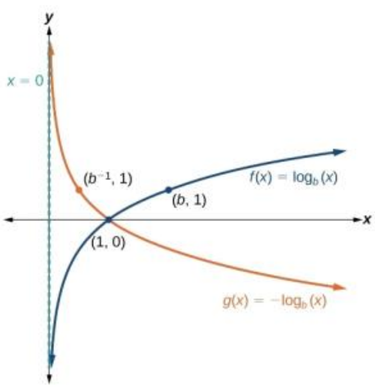
$h(x) = \frac{1}{a}\log_b(x)$, $a > 1$



- The asymptote remains $x = 0$.
- The x-intercept remains $(1, 0)$.
- The domain remains $(0, \infty)$.
- The range remains $(-\infty, \infty)$.

Reflection about the *x*-axis

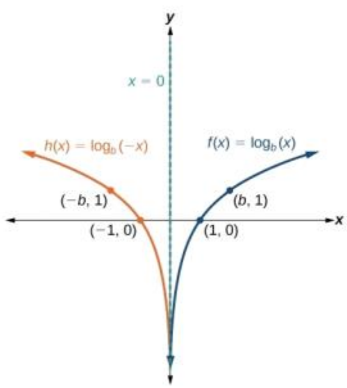
$g(x) = \log_b(x)$, $b > 1$



- The reflected function is decreasing as x moves from zero to infinity.
- The asymptote remains $x = 0$.
- The x-intercept remains $(1, 0)$.
- The key point changes to $(b^{-1}, 1)$
- The domain remains $(0, \infty)$.
- The range remains $(-\infty, \infty)$.

Reflection about the *y*-axis

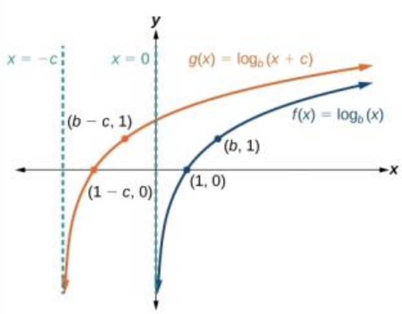
$h(x) = \log_b(-x)$, $b > 1$



- The reflected function is decreasing as x moves from negative infinity to zero.
- The asymptote remains $x = 0$.
- The x-intercept changes to $(-1, 0)$.
- The key point changes to $(-b, 1)$
- The domain changes to $(-\infty, 0)$.
- The range remains $(-\infty, \infty)$.

Shift left

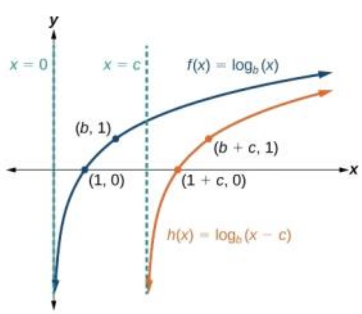
$g(x) = \log_b(x + c)$



- The asymptote changes to $x = -c$.
- The domain changes to $(-c, \infty)$.
- The range remains $(-\infty, \infty)$.

Shift right

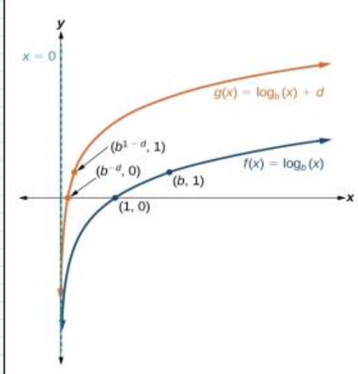
$h(x) = \log_b(x - c)$



- The asymptote changes to $x = c$.
- The domain changes to (c, ∞).
- The range remains $(-\infty, \infty)$.

Shift up

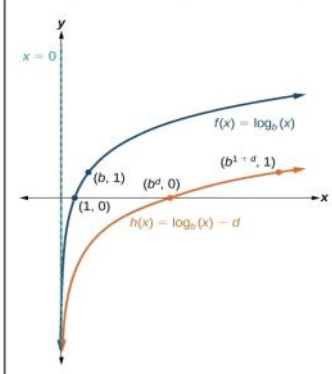
$g(x) = \log_b(x) + d$



- The asymptote remains $x = 0$.
- The domain remains to $(0, \infty)$.
- The range remains $(-\infty, \infty)$.

Shift down

$h(x) = \log_b(x) - d$



- The asymptote remains $x = 0$.
- The domain remains to $(0, \infty)$.
- The range remains $(-\infty, \infty)$.

8.2 Transformations of Logarithmic Functions

KEY IDEAS

- To represent real-life situations, you may need to transform the basic logarithmic function, $y = \log_b x$, by applying reflections, stretches, and translations. These transformations should be performed in the same manner as those applied to any other function.
- The effects of the parameters a, b, h, and k in $y = a\log_c (b(x - h)) + k$ on the graph of the logarithmic function $y = \log_c x$ are described in the table.

Parameter	Effect
a	Vertically stretch by a factor of $\lvert a \rvert$ about the x-axis. Reflect in the x-axis if $a < 0$.
b	Horizontally stretch by a factor of $\left\lvert \frac{1}{b} \right\rvert$ about the y-axis. Reflect in the y-axis if $b < 0$.
h	Horizontally translate h units. vertical asymptote $x = h$
k	Vertically translate k units.

- Only parameter h changes the vertical asymptote and the domain. None of the parameters changes the range.

Working Example 1: Translations of a Logarithmic Function

a) Sketch the graph of $y = \log_4 (x + 4) - 5$.

b) State the
- domain and range
- x-intercept
- y-intercept
- equation of the asymptote

Solution

Base Function

a) Begin with the graph of $y = \log_4 x$. Identify key points, such as (1, 0), (4, 1), and (16, 2).

inverse $y = 4^x$

x	y
$\frac{1}{16}$	−2
$\frac{1}{4}$	−1
1	0
4	1
16	2

$4^{-2} = \frac{1}{4^2}$

Identify the transformations.

The graph moves 4 units to the left and 5 units down.

$$y = \log_4 (x + 4) - 5$$

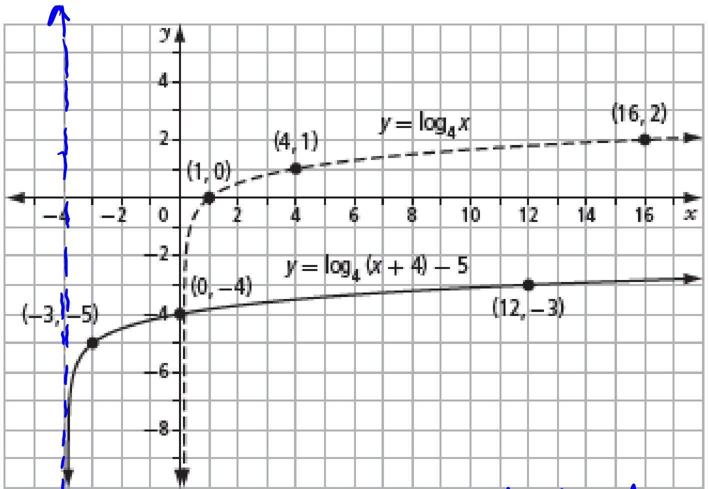
4 left 5 down.

4 left, 5 down.

In mapping notation, the key points are transformed as follows:

Key points: (x, y) maps to $(x - 4, y - 5)$

(x, y)	→	$(x - 4, y - 5)$
(1, 0)	→	(−3, −5)
(4, 1)	→	(0, −4)
(16, 2)	→	(12, −3)



required to draw asymptote ☆ always dashed line

b) The domain of $y = \log_4 x$ is $\{x \mid x > 0, x \in R\}$. Since the graph is translated 4 units left, this changes the domain to $\{x \mid x > -4, x \in R\}$. The range of the transformed function is not changed and is $\{y \mid y \in R\}$.

To determine the x-intercept, set $\log_4 (x + 4) - 5 = 0$. Then, solve for x.

$\log_4 (x + 4) = 5$

$x + 4 =$ 4^5 (write in exponential form)

$x =$ 1020

To confirm that the y-intercept is −4, substitute $x = 0$.

$y = \log_4 (0 + 4) - 5$

$y =$ $\log_4 4$ $- 5$

$y =$ −4

☆ Recall:
- to determine x-intercepts, make y = 0 + solve for x
- to determine y-int, make x = 0 + solve for y.

The graph of $y = \log_4 x$ has a vertical asymptote at $x = 0$. Since the graph is translated

$y =$ −4 ✓

The graph of $y = \log_4 x$ has a vertical asymptote at $x = 0$. Since the graph is translated 4 units left, the asymptote is translated as well.

Thus, $y = \log_4 (x + 4) - 5$ has a vertical asymptote at $x = -4$.

Working Example 2: Reflections and Stretches of Logarithmic Functions

Sketch the graph of each of the following. State any invariant points.

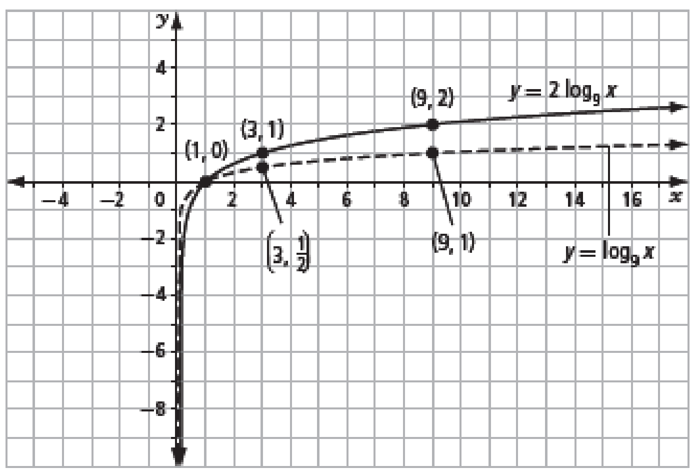
a) $y = 2 \log_9 x$ b) $y = -\log_2 4x$

Solution

a) The coefficient of 2 indicates a vertical stretch by a factor of 2. Choose key points on the graph of $y = \log_9 x$. Then, use mapping notation to show the transformation of those points.

Key points: (x, y) maps to $(x, 2y)$

(x, y)	→	$(x, 2y)$
(1, 0)	→	(1, 0)
$\left(3, \frac{1}{2}\right)$	→	(3, 1)
(9, 1)	→	(9, 2)

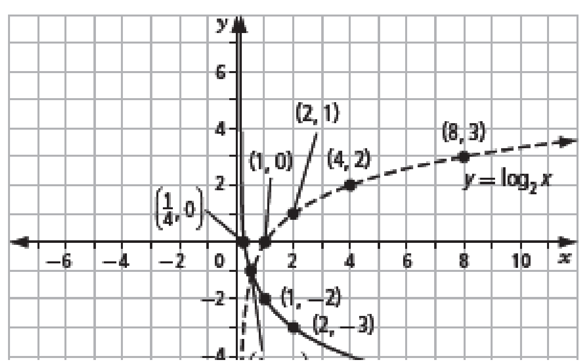


The invariant point is (1, 0).

b) The transformations are a reflection in x-axis and a horizontal stretch by a factor of $\frac{1}{4}$. Using key points on $y = \log_2 x$, use mapping notation to express the transformations.

Key points: (x, y) maps to $\left(\frac{1}{4}x, -y\right)$

(x, y)	→	$\left(\frac{1}{4}x, -y\right)$
(1, 0)	→	$\left(\frac{1}{4}, 0\right)$
(2, 1)	→	$\left(\frac{1}{2}, -1\right)$
(4, 2)	→	(1, −2)

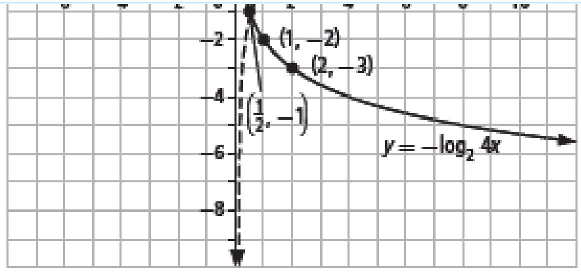


(4, 2)	→	(1, −2)
(8, 3)	→	(2, −3)

$8 \times \frac{1}{4}$

There are no invariant points on the graph.

no invariant for horizontal transformations



Why does this set of transformations have no invariant points?

Working Example 3: Combine Transformations

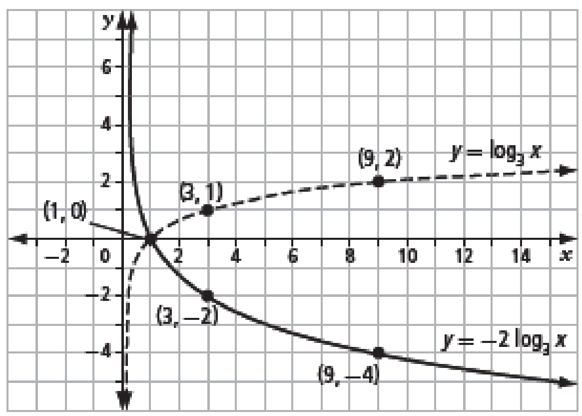
Sketch the graph of $y = -2\log_3(x - 3) + 5$.

Solution

First, apply all stretches and reflections to $y = \log_3 x$, in any order. Then, apply translations. Consider the effect of a reflection in the x-axis and a vertical stretch by a factor of 2 on the key points (1, 0), (3, 1), and (9, 2).

Key points: (x, y) maps to $(x, -2y)$

$y = \log_3 x$	$y = -2\log_3 x$
(1, 0)	(1, 0)
(3, 1)	(3, −2)
(9, 2)	



Since the graph is translated 3 units right and 5 units up, apply these translations to the points.

Key points: (x, y) maps to $(x + 3, y + 5)$

$y = -2\log_3 x$	$y = -2\log_3(x - 3) + 5$
(1, 0)	(4, 5)
(3, −2)	
(9, −4)	

base function

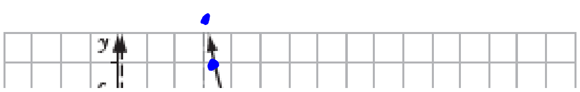
$y = \log_3 x$ ($y = 3^x$ inversed)

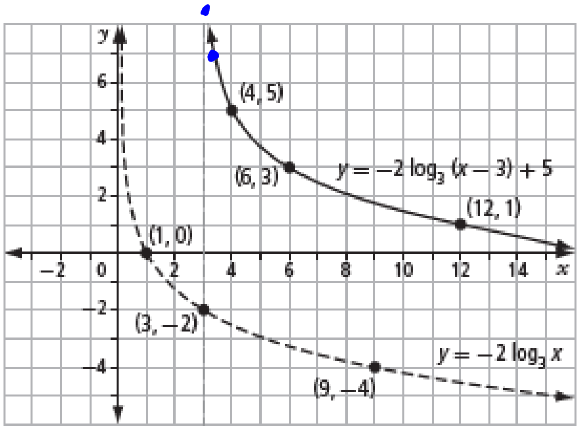
VE by 2, ref. in x-axis; 3 right, 5 up

x	y
1/9	−2
1/3	−1
1	0
3	1
9	2

x	$-2y$
1/9	4
1/3	2
1	0
3	−2
9	−4

$x+3$	$-2y+5$
$+\frac{27}{9}$ → $\frac{28}{9}$	9
$+\frac{9}{3}$ → $\frac{10}{3}$	7
4	5
6	3
12	1





3	1
9	2

3	-2
9	-4

4	5
6	3
12	1

See pages 384–389 of *Pre-Calculus 12* for more examples.

TRY #3 p271 + #7,8

Log Graphs Practice

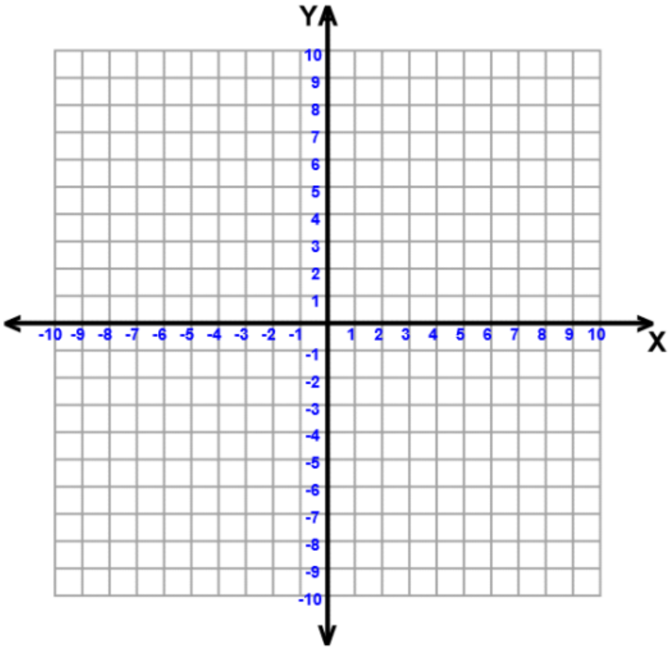
Name : ______________________________ Score : ____________

Teacher : ____________________________ Date : ____________

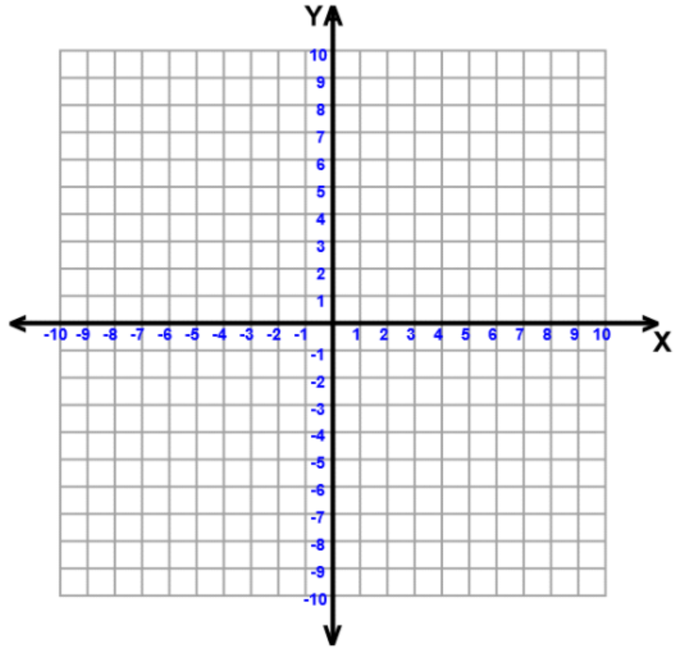
Graphing Logarithms

Give the domain and range of each function, then graph.

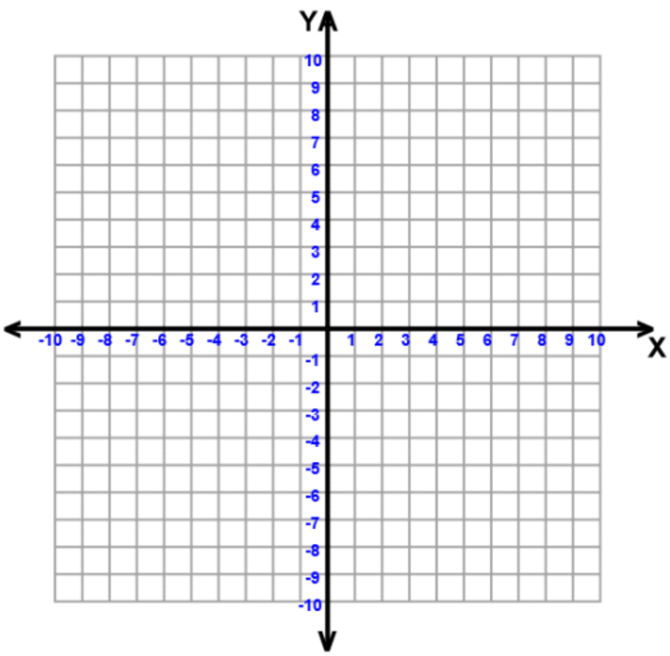
1) $y = \log_9 (x + 4) + 3$



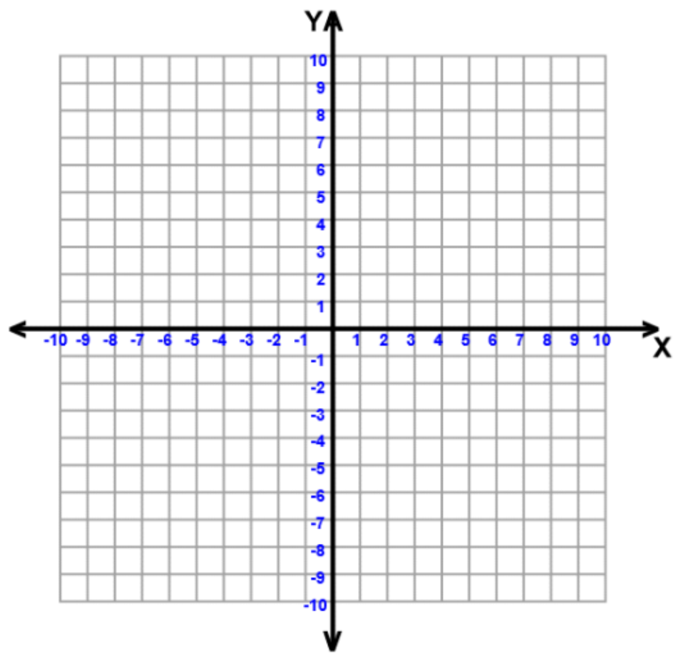
2) $y = \log_5 (x - 2) + 5$



3) $y = \log_7 (x - 4) - 2$



4) $y = \log_4 (x - 2) - 5$



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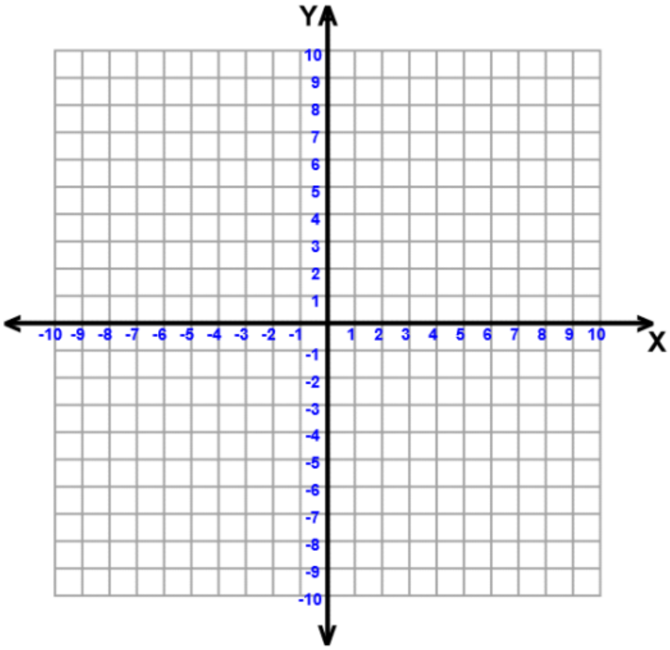
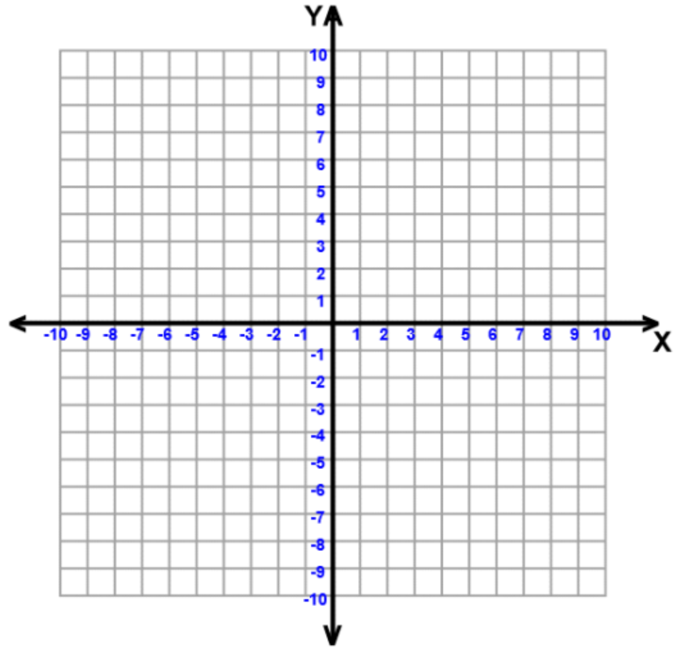
Graphing Logarithms

Give the domain and range of each function, then graph.

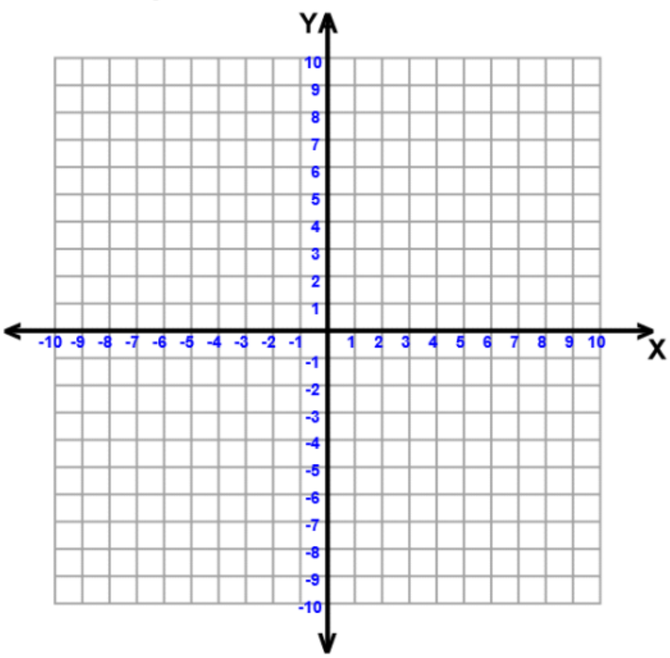
5) $y = \log_5 (4x - 5) - 2$



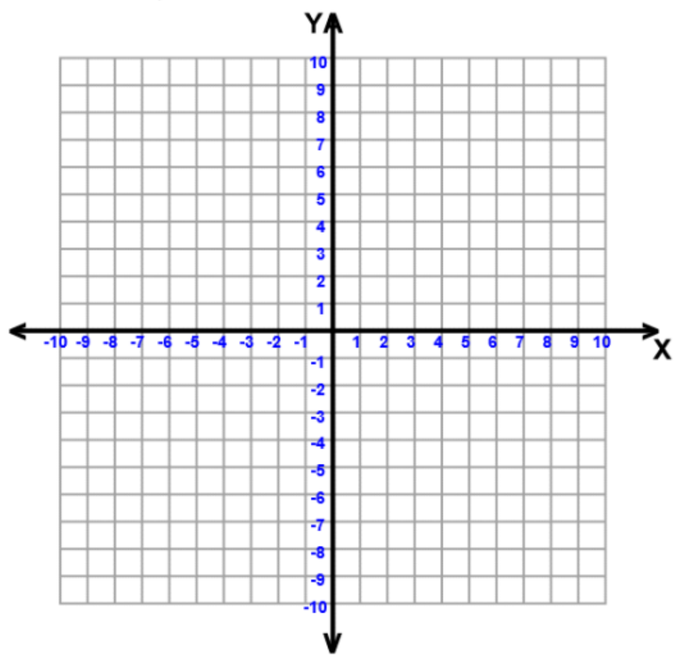
6) $y = \log (4x - 3) - 4$



7) $y = \log_8 (2x + 5) + 3$



8) $y = \log_7 (4x + 4) - 3$



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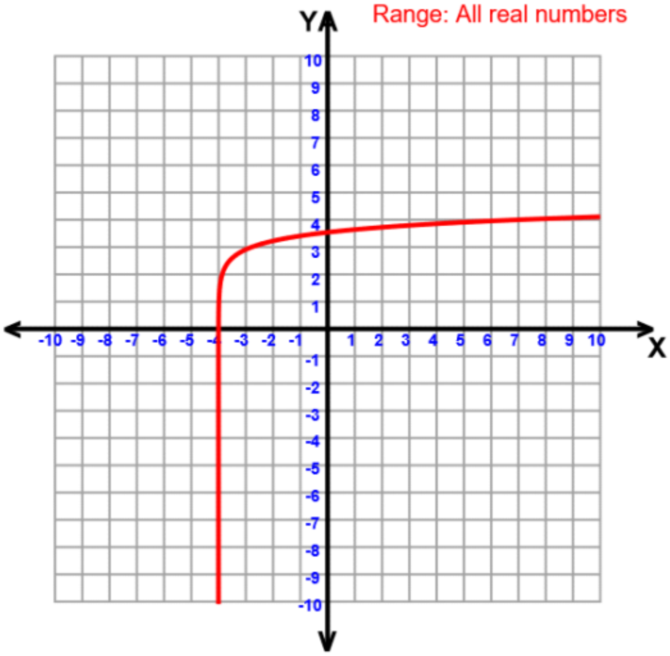
Graphing Logarithms

Give the domain and range of each function, then graph.

1) $y = \log_9 (x + 4) + 3$

Domain: x > -4

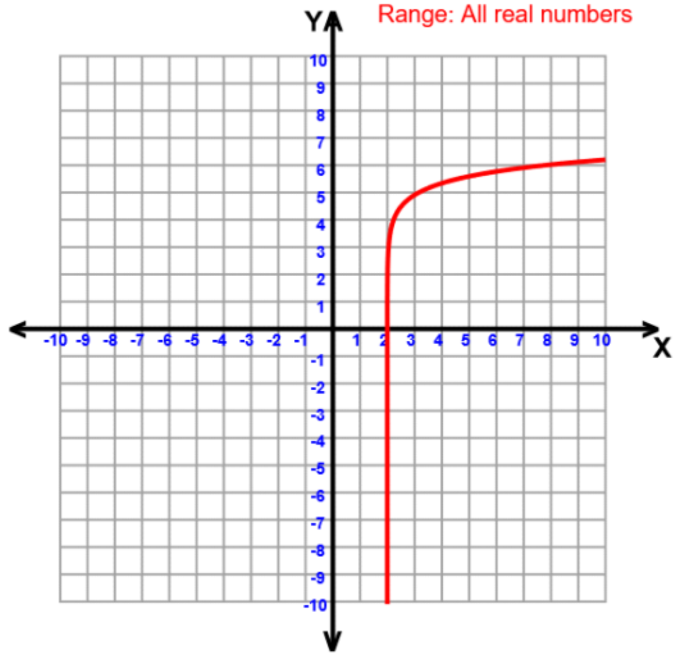
Range: All real numbers



2) $y = \log_5 (x - 2) + 5$

Domain: x > 2

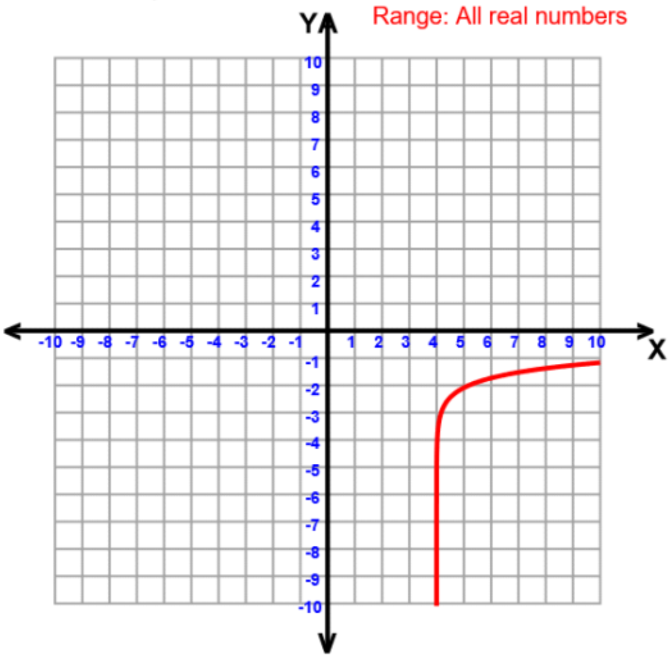
Range: All real numbers



3) $y = \log_7 (x - 4) - 2$

Domain: x > 4

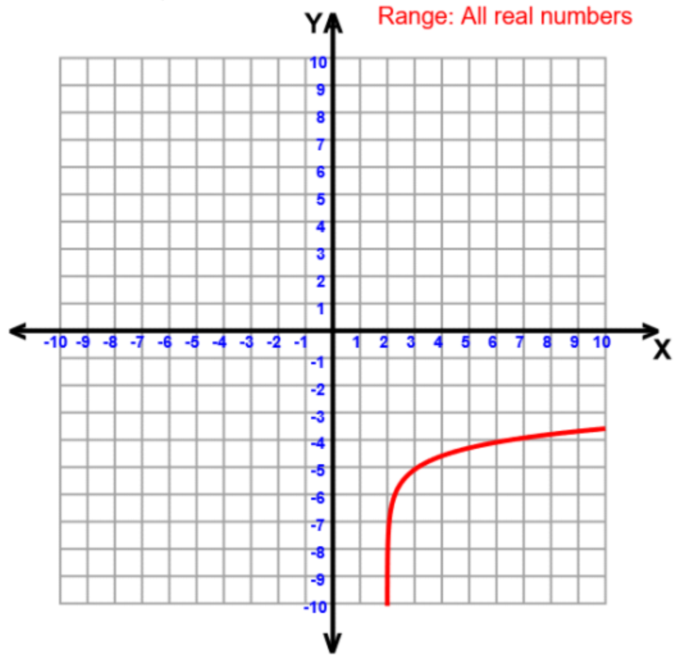
Range: All real numbers



4) $y = \log_4 (x - 2) - 5$

Domain: x > 2

Range: All real numbers



Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

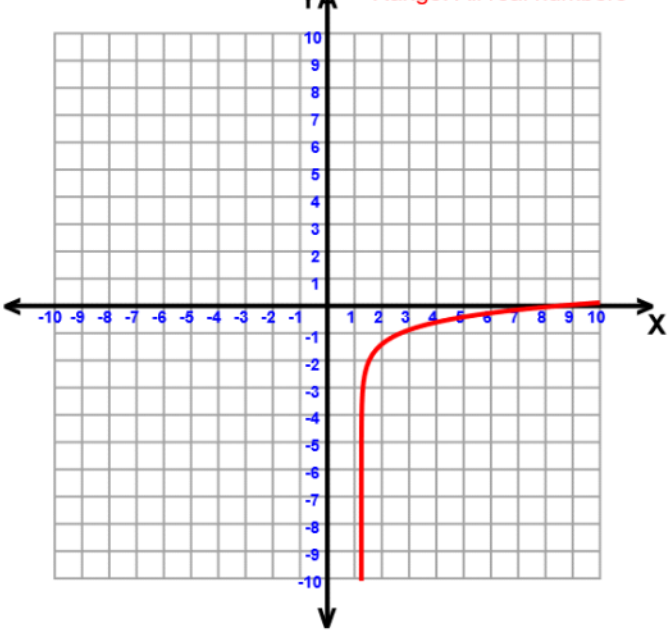
Graphing Logarithms

Give the domain and range of each function, then graph.

5) $y = \log_5 (4x - 5) - 2$

Domain: $x > \frac{5}{4}$

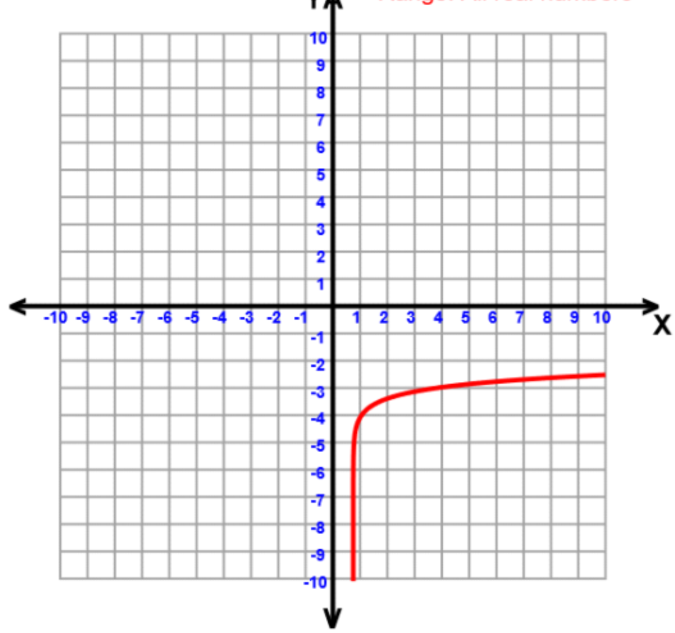
Range: All real numbers



6) $y = \log (4x - 3) - 4$

Domain: $x > \frac{3}{4}$

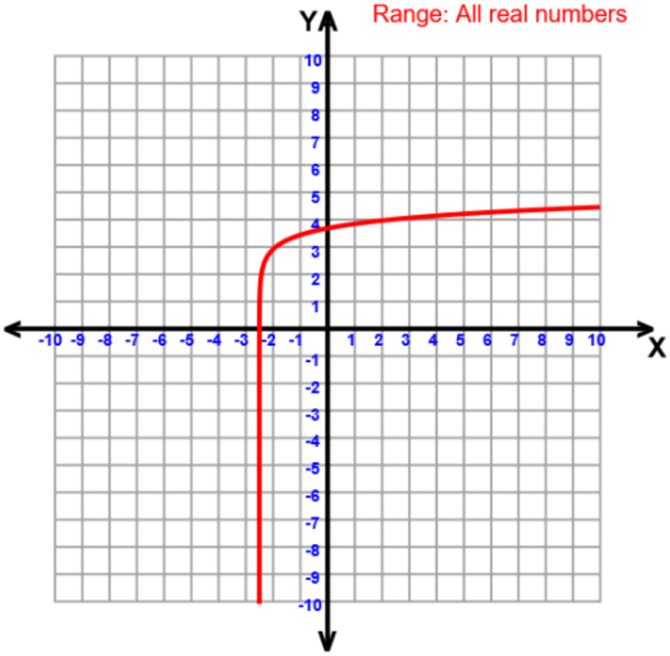
Range: All real numbers



7) $y = \log_8 (2x + 5) + 3$

Domain: $x > \frac{-5}{2}$

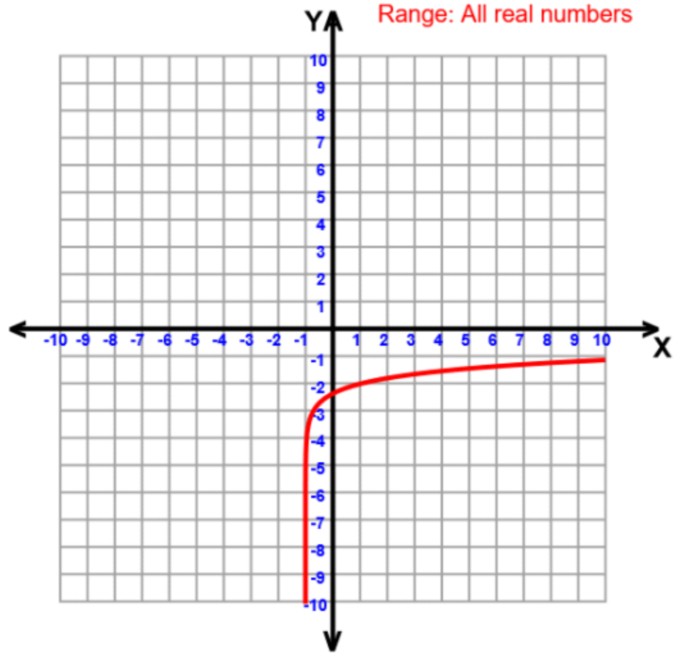
Range: All real numbers



8) $y = \log_7 (4x + 4) - 3$

Domain: $x > -1$

Range: All real numbers



8.3: Laws of Logarithms

Rule of Logarithms

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Rule Name	Property
Log of 1	$\log_b 1 = 0$
Log of the same number as base	$\log_b b = 1$
Product Rule	$\log_b(mn) = \log_b m + \log_b n$
Quotient Rule	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
Power Rule	$\log_b m^n = n \log_b m$
Change of Base Rule	$\log_a b = \frac{\log_c b}{\log_c a}$ (OR) $\log_a b \,.\, \log_c a = \log_c b$
Equality Rule	$\log_b a = \log_b c \Rightarrow a = c$
Number Raised to Log	$b^{\log_b x} = x$
Other Rules	$\log_{b^n} a^m = \frac{m}{n} \log_b a$ $-\log_b a = \log_b \frac{1}{a}$ (OR) $= \log_{\frac{1}{b}} a$

Rule 1: $\log_b(M \cdot N) = \log_b M + \log_b N$

Rule 2: $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

Rule 3: $\log_b\left(M^k\right) = k \cdot \log_b M$

Rule 4: $\log_b(1) = 0$

Rule 5: $\log_b(b) = 1$

Rule 6: $\log_b\left(b^k\right) = k$

Rule 7: $b^{\log_b(k)} = k$

Where:

$b > 0$ but $b \neq 1$, and M, N, and k are real numbers but M and N must be positive!

- **The Law of Logarithms for Powers (Power Law)** = $\log_a x^n = n \log_a x$

- **The Law of Logarithms for Roots** = $\log_x \sqrt[n]{x^m} = \log_a x^{\frac{m}{n}} = \frac{m}{n} \log_a x$

- **The Multiplication Law of Logs (Product Law)**= $\log_a xy = \log_a x + \log_a y$

- **The Division Law of Logs (Quotient Law)**= $\log_a \frac{x}{y} = \log_a x - \log_a y$

$$5\log_3(x)+2\log_3(4x)-\log_3(8x^5)=\log_3(x^5)+\log_3\left((4x)^2\right)-\log_3(8x^5)$$
$$=\log_3(x^5)+\log_3(16x^2)-\log_3(8x^5)$$
$$=\log_3(x^5\cdot 16x^2)-\log_3(8x^5)$$
$$=\log_3(16x^7)-\log_3(8x^5)$$
$$=\log_3\left(\frac{16x^7}{8x^5}\right)$$
$$5\log_3(x)+2\log_3(4x)-\log_3(8x^5)=\log_3(2x^2)$$

$$\log_6\left(\frac{36m^3}{\sqrt{n}}\right)=\log_6(36m^3)-\log_6(\sqrt{n})$$
$$=\log_6(36)+\log_6(m^3)-\log_6\left(n^{\frac{1}{2}}\right)$$
$$=\log_6(6^2)+3\log_6(m)-\frac{1}{2}\log_6(n)$$
$$=2\log_6(6)+3\log_6(m)-\frac{1}{2}\log_6(n)\quad \text{Rule 5}\Rightarrow\log_6(6)=1$$
$$=2(1)+3\log_6(m)-\frac{1}{2}\log_6(n)$$
$$\log_6\left(\frac{36m^3}{\sqrt{n}}\right)=2+3\log_6(m)-\frac{1}{2}\log_6(n)$$

$$2\log_5(m)+3\log_5(k)-8\log_5(y)=\log_5(m^2)+\log_5(k^3)-\log_5(y^8)$$
$$=\log_5(m^2\cdot k^3)-\log_5(y^8)$$
$$=\log_5(m^2k^3)-\log_5(y^8)$$
$$2\log_5(m)+3\log_5(k)-8\log_5(y)=\log_5\left(\frac{m^2k^3}{y^8}\right)$$

$$3+\frac{1}{2}\log_4(x)+\frac{1}{2}\log_4(y)=3+\log_4\left(x^{\frac{1}{2}}\right)+\log_4\left(y^{\frac{1}{2}}\right)$$

$$=3+\log_4\left(x^{\frac{1}{2}}\cdot y^{\frac{1}{2}}\right)$$

$$=3+\log_4\left(\sqrt{x}\cdot\sqrt{y}\right)$$

$$=3+\log_4\left(\sqrt{xy}\right)$$

$$=3\cdot\log_4(4)+\log_4\left(\sqrt{xy}\right) \quad \text{Since } \log_4(4)=1$$

$$=\log_4\left(4^3\right)+\log_4\left(\sqrt{xy}\right)$$

$$=\log_4\left(4^3\cdot\sqrt{xy}\right)$$

$$3+\frac{1}{2}\log_4(x)+\frac{1}{2}\log_4(y)=\log_4\left(64\sqrt{xy}\right)$$

- **Common Base Law =** $\log_a a^x = x$ **OR** $a^{\log_a x} = x$

$$\log_a N = x$$

$$a^x = N$$

$$a^{\log_a N} = N$$

$a^x = N$	$\log_a N = x$
$2^3 = 8$	$\log_2 8 = 3$
$3^4 = 81$	$\log_3 81 = 4$
$5^3 = 125$	$\log_5 125 = 3$
$10^4 = 10000$	$\log_{10} 10000 = 4$
$7^1 = 7$	$\log_7 7 = 1$
$5^0 = 1$	$\log_5 1 = 0$

Log Laws

- **The Law of Logarithms for Powers (Power Law) =** $\log_a x^n = n\log_a x$

- **The Law of Logarithms for Roots =** $\log_x \sqrt[n]{x^m} = \log_a x^{\frac{m}{n}} = \frac{m}{n}\log_a x$

- **The Multiplication Law of Logs (Product Law)=** $\log_a xy = \log_a x + \log_a y$

- **The Division Law of Logs (Quotient Law)=** $\log_a \frac{x}{y} = \log_a x - \log_a y$

- **Change of Base Law=** $\log_a x = \frac{\log x}{\log a}$

- **Common Base Law =** $\log_a a^x = x$ **OR** $a^{\log_a x} = x$

8.3 Laws of Logarithms

KEY IDEAS

- Let P be any real number, and M, N, and c be positive real numbers with $c \neq 1$. Then, the following laws of logarithms are valid.

Name	Law	Description
Product	$\log_c MN = \log_c M + \log_c N$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
Quotient	$\log_c \frac{M}{N} = \log_c M - \log_c N$	The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor.
Power	$\log_c M^P = P \log_c M$	The logarithm of a power of a number is the exponent times the logarithm of the number.

- Many quantities in science are measured using a logarithmic scale. Two commonly used logarithmic scales are the decibel scale and the pH scale.

Working Example 1: Use the Laws of Logarithms to Expand Expressions

Expand each expression using the laws of logarithms.

a) $\log_4 \frac{x^3y}{4z}$ **b)** $\log_5 \sqrt{xy^3}$ **c)** $\log \frac{100\sqrt[3]{x^4}}{y^2}$

Solution

a) $\log_4 \frac{x^3y}{4z} = \log_4 x^3y - \log_4 4z$ quotient law ÷ → −

$= \log_4 x^3 + \log_4 y - (\log_4 4 + \log_4 z)$ product law × → +

power law $= 3\log_4 x + \log_4 y - 1 - \log_4 z$

Why does $\log_4 4 = 1$?

$\log_4 4^1 = 1$

b) $\log_5 \sqrt{xy^3} = \log_5 (xy^3)^{\frac{1}{2}}$ $\sqrt{x} = x^{\frac{1}{2}}$

$= \frac{1}{2}\log_5 (xy^3)$ power law

$= \frac{1}{2}(\log_5 x + \log_5 y^3)$ product law

$= \frac{1}{2}(\log_5 x + 3\log_5 y)$ powerlaw

$= \frac{1}{2}\log_5 x + \frac{3}{2}\log_5 y$ simplify.

$= \frac{1}{2}(\log_5 \underline{\quad} + \underline{\quad} \log_5 \underline{\;y\;})$ power law

$= \frac{1}{2}\log_5 \underline{\;x\;} + \underline{\;\frac{3}{2}\;}\log_5 \underline{\;y\;}$ simplify.

$= \boxed{\frac{1}{2}\log_5 x + \frac{3}{2}\log_5 y}$

c) $\log \frac{100\sqrt[3]{x^4}}{y^2} = \log 100\sqrt[3]{x^4} - \log y^2$ quotient law

$= \log 100 + \log x^{\boxed{\frac{4}{3}}} - \underline{\log y^2}$ product law

$\log_{10} 10^2 = 2$ $\quad = \boxed{2 + \underline{\frac{4}{3}\log x} - \underline{2\log y}}$ power law

Working Example 2: Write Expressions With a Single Logarithm

Rewrite each expression using a single logarithm. State the restrictions on the variable.

a) $\log_2 x^3 - 4\log_2 x - \log_2 \sqrt{x}$

b) $4\log_6 y^2 + \log_6 y - \frac{2}{3}\log_6 y$

c) $\log(x - 3) + \log(x + 4)$

Solution

a) $\log_2 x^3 - 4\log_2 x - \log_2 \sqrt{x}$ radical to fraction exponent

$= \log_2 x^3 - \log_2 \underline{\;x^4\;} - \log_2 x^{\frac{1}{2}}$ power law

$= \log_2 \frac{x^3}{\boxed{x^4 \cdot x^{\frac{1}{2}}}}$ (+ in numerator, − in denominator) quotient law

$\log_2\left(\frac{\frac{x^3}{x^4}}{x^{\frac{1}{2}}}\right)$ $\quad \log_2\left(\frac{x^3}{x^4} \times \frac{1}{x^{\frac{1}{2}}}\right)$

$= \log_2 \frac{1}{x^{\frac{3}{2}}}$, $x > 0$

exponent rules: $x^{3-(4+\frac{1}{2})}$, $x^{3-\frac{9}{2}}$, $x^{-\frac{3}{2}} \rightarrow \frac{1}{x^{\frac{3}{2}}}$

b) $4\log_6 (y^2)^4 + \log_6 y - \frac{2}{3}\log_6 y^{2/3}$ power law

$= \log_6 \underline{\;y^8\;} + \log_6 y - \log_6 \underline{\;y^{2/3}\;}$

$= \log_6 \frac{y^8 y}{y^{\frac{2}{3}}}$ (multiply, divide) product + quotient law

$= \underline{\log_6 y^{\frac{25}{3}}}$, $y > 0$

or $\frac{25}{3}\log_6 y$ ✓

$y^{8+1-\frac{2}{3}} = y^{9-\frac{2}{3}} = y^{\frac{27}{3}-\frac{2}{3}} = y^{\frac{25}{3}}$

c) $\log(x - 3) + \log(x + 4)$

product law $= \log[(x - 3)(x + 4)]$ FOIL

$= \log\underline{(x^2 + x - 12)}$, $x > 3$ ✓

Restrictions: $x - 3 > 0$, $x + 4 > 0$; $x > 3$, $x > -4$

Working Example 3: Evaluate Expressions With the Laws of Logarithms

Evaluate each expression.

Evaluate each expression.

a) $\log_4 8 + \log_4 32$

b) $\log_6 216\sqrt[4]{36}$

Solution

a) $\log_4 8 + \log_4 32$

$= \log_4 (8 \times 32)$

$= \log_4$ ______

$=$ ______

b) $216 = 6^{\square}$ and $\sqrt[4]{36} = 6^{\square}$

Rewrite $\log_6 216\sqrt[4]{36}$.

$\log_6 216\sqrt[4]{36} = \log_6 6^3 6^{\frac{1}{2}}$

$=$ ______

$=$ ______

See pages 395–399 of *Pre-Calculus 12* for more examples.