KEY

Extra Practice for Chapter 8.4

1. Solving the following equations. Show restrictions and final answer in a box.

a) $log_2(x-2) + log_2(x-1) = 2$

① Product Law
② change to exponential form
③ solve
④ check

$\log_2 (x-2)(x-1) = 2$

$(x-2)(x-1) = 2^2$

FOIL

$x^2 - 3x + 2 = 4$

$-4 \quad -4$

$x^2 - 3x - 2 = 0$

quadratic formula.

$\Delta = -3$

$x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2(1)}$

$x = \dfrac{3 \pm \sqrt{17}}{2}$ $\quad x \approx 3.56$ $\quad x \approx -0.56$

$\boxed{x = \dfrac{3+\sqrt{17}}{2}}$

$\log_c A = Z$

$A = C^Z$

Check

$\log_2\left(\frac{3\pm\sqrt{17}}{2} - 2\right)$

$\log_2\left(\frac{3\pm\sqrt{17}}{2} - 1\right)$

Restriction

$x - 2 > 0 \quad x - 1 > 0$

$\underline{x > 2} \quad \underline{x > 1}$

b) $log_3(2x+5) - log_3(x+2) = log_3 4$

① Quotient Law
② Cancel Logs
③ solve

$\cancel{\log_3}\left(\frac{2x+5}{x+2}\right) = \cancel{\log_3} 4$

$\frac{2x+5}{x+2} = 4$

$2x + 5 = 4(x+2)$

$2x + 5 = 4x + 8$

$-4x \quad -5 \quad -4x \quad -5$

$-2x = 3$ $\rightarrow$ $\boxed{x = -\frac{3}{2}}$

$\log_c A = \log_c B$

$A = B$

Restrictions

$2x + 5 > 0 \quad x + 2 > 0$

$x > -\frac{5}{2} \quad x > -2$

(-2.5)

2. Solving the following equations:

a) $3^{(x-1)} = 9(27)^{(2-x)}$

$3^{(x-1)} = 3^2 \cdot (3^3)^{2-x}$

$3^{x-1} = 3^2 \cdot 3^{6-3x}$ ADD

$\cancel{3}^{x-1} = \cancel{3}^{8-3x}$

$x - 1 = 8 - 3x$

$+3x \quad +1$

$\rightarrow 4x = 9$

$\frac{4x}{4} = \frac{9}{4}$

$\boxed{x = \frac{9}{4}}$

b) $2^{(x-3)} = 3(5)^{(2-x)}$

$\log 2^{x-3} = \log 3(5)^{2-x}$

Product Law

$(x-3)\log 2 = \log 3 + \log 5^{2-x}$

$(x-3)\log 2 = \log 3 + (2-x)\log 5$ Expand

$x\log 2 - 3\log 2 = \log 3 + 2\log 5 - x\log 5$ collect like terms

$x\log 2 + x\log 5 = \log 3 + 2\log 5 + 3\log 2$ x on left

$x(\log 2 + \log 5) =$ "

$x = \dfrac{(\log(3) + 2\log(5) + 3\log(2))}{(\log(2) + \log(5))} \approx 2.78$ ✓

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Applications

3. A scientist started with a culture of 20 bacteria in a dish. He noticed that after 80 hours, there were 1800 bacteria. What is the doubling time of this bacteria?

$$\frac{1800}{20} = \frac{20(2)^{80/n}}{20}$$

$$\log 90 = \log 2^{80/n}$$

$$n \log 90 = \frac{80}{n} \log 2$$

$$\frac{n \log 90}{\log 90} = \frac{80 \log 2}{\log 90}$$

$$n = \frac{80 \log 2}{\log 90} = \boxed{12.32 \text{ hr.}}$$

4. At the beginning of the year, you deposit $1000 into a bank account, with an annual interest rate of 5%. Assume no other deposits or withdrawals are made and the interest rate stays constant.

a) what will be the value of the account after 5 years if interest is compounded annually?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$n=1$

$$A = 1000(1 + 0.05)^5$$

$$= 1000(1.05)^5$$

$$\boxed{A = \$1276.28}$$

b) how long will it be when his money doubles in value?

$$\frac{2000}{1000} = \frac{1000(1.05)^t}{1000}$$

$$\log 2 = \log 1.05^t$$

$$\frac{\log 2}{\log 1.05} = \frac{t \log 1.05}{\log 1.05}$$

$$t = \frac{\log 2}{\log 1.05} = \boxed{14.21 \text{ yr}}$$

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5. When people take a particular medicine, the drug is metabolised and eliminated at a certain rate. Suppose the initial amount of a drug in the body is 200 mg and is eliminated at a rate of 30% $\underbrace{\text{per hour}}_{n=1}$.
How long will it take to reach 10 mg?

$$\frac{10}{200} = \frac{200(1-0.3)^{t/1}}{200}$$

$$\frac{1}{20} = (0.7)^t$$

$$\frac{\log\left(\frac{1}{20}\right)}{\log 0.7} = \frac{t \log 0.7}{\log 0.7}$$

$$t = \frac{\log\left(\frac{1}{20}\right)}{\log 0.7}$$

$$\boxed{t = 8.40 \text{ hr.}}$$

6. Certain bacteria, given favourable growth conditions, grow continuously at a rate of 4.6% a day. —24hrs
Find the bacterial population after thirty-six hours, if the initial population was 250 bacteria.

$$A = 250(1+0.046)^{\frac{36}{24}}$$

$$A = 250(1.046)^{3/2}$$

$$\boxed{A = 267 \text{ bacteria}}$$

(not 267.45)

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7. A penicillin solution has a half-life of 6 days. How long will it take for the concentration to drop to 70% of the initial concentration?

$$0.7 = 1\left(\tfrac{1}{2}\right)^{t/6}$$

$$\frac{6 \times \log 0.7}{\log \frac{1}{2}} = \frac{\frac{t}{6} \log \frac{1}{2}}{\log \frac{1}{2}}$$

$$t = \frac{6 \log 0.7}{\log 0.5}$$

$$\boxed{t = 3.09 \text{ days}}$$

8. What is the magnitude of the earthquake in City A if the earthquake in City B has a magnitude of 5.7 on the Richter scale and is 4500 times as intense?

$$4500 = 10^{5.7 - R_A}$$

$$\log 4500 = (5.7 - R_A) \cancel{\log 10}$$

$$\log 4500 = 5.7 - R_A$$

$$R_A = 5.7 - \log 4500$$

$$\boxed{R_A = 2.0 \text{ Magnitude in City A}}$$

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9. What is the pH of a tomato if it is 15000 times more acidic than hand soap with a pH of 9.5?

$$I = I_0 (10)^{pH_H - pH_L}$$

$$15000 = (10)^{9.5 - pH_T}$$

$$\log 15000 = (9.5 - pH_T)\cancel{\log 10}$$

1

$$\log 15000 = 9.5 - pH_T$$

$$pH_T = 9.5 - \log 15000$$

$$\boxed{pH_T = 5.3}$$

10. It is said that the eardrum can rupture at a decibel level that is 100,000,000 times as intense as the normal sound level of a vacuum at 70Db on the Decibel scale (that would be like listening to a jet at take-off). At what Db value on the scale can the eardrum rupture?

$$I = I_0 (10)^{\frac{DB_H - DB_L}{10}}$$

$$100{,}000{,}000 = 10^{\frac{DB_H - 70}{10}}$$

$$\cancel{\log 100000000} = \frac{DB - 70}{10} \cancel{\log 10}$$

7

$$10\times \quad 7 = \frac{DB - 70}{10}$$

$$70 = DB - 70$$

$$+70 \qquad +70$$

$$\boxed{DB = 140}$$