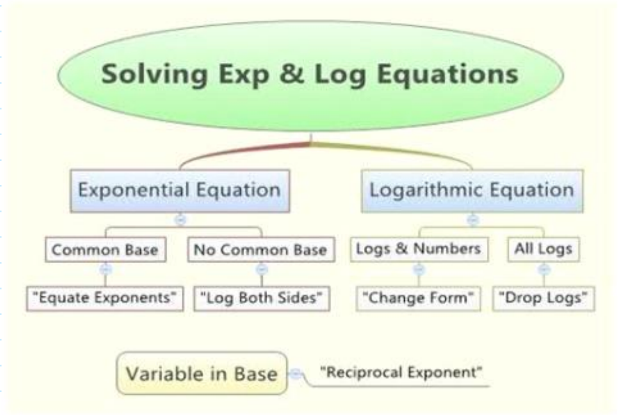
Thursday, Nov. 16th

Plan For Today:

1. Questions & Review from 8.4?
   - ✵ Do 8.4 Check-in Quiz Solving Log Equations
2. Finish Chapter 8: Logarithmic Functions
   - ✓ 8.1: Understanding Logarithms
   - ✓ 8.2: Transformations of Logarithmic Functions
   - ✓ 8.3: Laws of Logarithms
   - ✷ **8.4: Logarithmic & Exponential Functions (finish)**
3. Work on Practice Questions from Workbook



Plan Going Forward:

1. Finish going through all of the Ch8 questions in workbook and finish working on review handout. Complete Ch7 & 8 review and practice.

   - **CHAPTER 8 PROJECT DUE TUESDAY, NOV. 21ST (TRY FOR TODAY)**
   - **UNIT 3 EXAM ON TUESDAY, NOV. 21ST**

2. Start Ch9 after the exam on Tuesday.

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at anurita.weebly.com after class. Anurita Dhiman = adhiman@sd35.bc.ca

Thursday, Nov. 16th In-Class Notes

Review

4. Solve.

a) $3\log_6 x = \log_6 9 + \log_6 24$

① single logs

$\log_6 x^3 = \log_6(9\times24)$

$\cancel{\log_6} x^3 = \cancel{\log_6} 216$

$\sqrt[3]{x^3} = \sqrt[3]{216}$

$\boxed{x = 6}$ ✓

Restrictions.

$x > 0$

d) $5\log_3 x - \log_3 x = 8$

$\log_3 x^5 - \log_3 x = 8$

$\div$

$\log_3\left(\frac{x^5}{x}\right) = 8$

$\log_3 x^4 = 8$

$x^4 = 3^8$ not 8^3

$\sqrt[4]{x^4} = \sqrt[4]{6561}$

$\boxed{x = 9}$ ✓

$x > 0$

$\log 6^x$ $\log 6x$

$\log_6 x$

8.4 Logarithmic and Exponential Equations

KEY IDEAS

- When solving a logarithmic equation algebraically, start by applying the laws of logarithms to express one side or both sides of the equation as a single logarithm.
- Some useful properties are listed below, where c, L, $R > 0$ and $c \neq 1$.
  - If $\log_c L = \log_c R$, then $L = R$.
  - The equation $\log_c L = R$ can be written with logarithms on both sides of the equation as $\log_c L = \log_c c^R$.
  - The equation $\log_c L = R$ can be written in exponential form as $L = c^R$.
  - The logarithm of zero or a negative number is undefined. To identify whether a root is extraneous, substitute the root into the original equation and check whether all of the logarithms are defined.
- You can solve an exponential equation algebraically by taking logarithms of both sides of the equation. If $L = R$, then $\log_c L = \log_c R$, where c, L, $R > 0$ and $c \neq 1$. Then, apply the power law for logarithms to solve for an unknown.
- You can solve an exponential equation or a logarithmic equation using graphical methods.
- Many real-world situations can be modelled with an exponential or a logarithmic equation. A general model for many problems involving exponential growth or decay is

$$\text{Final quantity} = \text{initial quantity} \times (\text{change factor})^{\text{number of changes}}$$

Working Example 1: Solve Logarithmic Equations

Solve.

a) $\log_4 (5x + 1) = \log_4 (x + 17)$

b) $\log (5x) - \log (x - 1) = 1$

c) $\log_6 (x - 3) + \log_6 (x + 6) = 2$

Solution

a) Since $\log_4 (5x + 1) = \log_4 (x + 17)$, $5x + 1 = x + 17$.
So, $4x = 16$ and $x = 4$.
Check $x = 4$ in the original equation.

Left Side	Right Side
$\log_4 (5(4) + 1)$ $= \log_4 21$	$\log_4 (4 + 17)$ $= \log_4 21$

Left Side = Right Side

log₄ on both sides of = sign are common ∴ cancel

$\cancel{\log_4}(5x+1) = \cancel{\log_4}(x+17)$

$5x + 1 = x + 17$
$-x \quad\quad -1$

$4x = 16$
$/4 \quad /4$

$\boxed{x = 4}$ ✓

Restrictions.
$5x + 1 > 0$ $\quad x + 17 > 0$
$x > -\frac{1}{5}$ $\quad x > -17$

$\boxed{x = 4}$ ✓

Restrictions: $5x+1>0$, $x+17>0$; $x>-\frac{1}{5}$, $x>-17$

b) Method 1: Solve Algebraically by Rewriting in Exponential Form

Using the laws of logarithms, rewrite $\log(5x) - \log(x-1)$ as $\log$ $\left(\frac{5x}{x-1}\right)$.

quotient law

Then, rewrite $\log \frac{5x}{x-1} = 1$ in exponential form.

$$\frac{5x}{x-1} = 10^1$$

Multiply both sides by $(x - 1)$.

$$\frac{5x}{x-1}(x-1) = 10(x-1)$$
$$5x = 10x - 10$$
$$-5x = -10$$
$$x = 2$$

Restrictions

$5x>0$ $x-1>0$

$x>0$ $x>1$

$$\log(5x) - \log(x-1) = 1$$
$$\log_{10}\left(\frac{5x}{x-1}\right) = 1$$

log on one side ∴ boot base

$$(x-1)\frac{5x}{x-1} = 10^1(x-1)$$
$$5x = 10x - 10$$
$$-5x = -10$$
$$\boxed{x = 2}$$ ✓

Check:

Left Side	Right Side
$\log(5(\mathbf{2})) - \log(\mathbf{2} - 1)$ $= \log 10 - \log 1$ $= 1 - 0$ $= 1$	1

Left Side = Right Side

~~**Method 2: Solve Algebraically by Writing Each Side as a Logarithm**~~

Begin by rewriting 1 as $\log_{10}$ ______.

So, the equation is $\log(5x) - \log(x-1) = \log 10$.

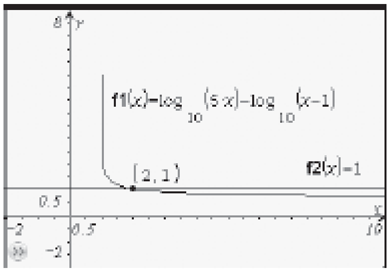
Solve for x.

$$\log(5x) - \log(x-1) = 10$$
$$\log \frac{5x}{x-1} = \log 10$$
$$\frac{5x}{x-1} = 10$$
$$5x = 10(x-1)$$
$$5x = 10x - 10$$
$$-5x = -10$$
$$x = 2$$

Compare the algebra in Method 1 to the algebra in Method 2. How are the methods similar? How are they different?

~~**Method 3: Solve Graphically**~~

Use technology to graph the equations $y = \log(5x) - \log(x-1)$ and $y = 1$ in the same window. Then, find the point of intersection.



The solution is $x = 2$.

c) Using the laws of logarithms, rewrite the left side of the equation.

$\log_6 (x - 3) + \log_6 (x + 6) = 2$

Product law $\log_6 (x - 3)(x + 6) =$ <u>2</u>

Method 1: Rewrite in Exponential Form (Root Base)

In exponential form, the equation is equivalent to $(x - 3)(x + 6) = 6^2$.

Expand and simplify the left side of the equation.

<u>$x^2 + 3x - 18$</u> $= 36$

Subtract 36 from each side.

<u>$x^2 + 3x - 54$</u> $= 0$

Factor the left side.

$(x +$ <u>9</u> $)(x -$ <u>6</u> $) = 0$

$x =$ <u>-9</u> or $x =$ <u>6</u>

R

The original equation is defined for $x > 3$ and $x > -6$. In other words, both conditions are met when $\boxed{x > 3}$. Thus, $x = -9$ is an extraneous root. Check $x = 6$.

Left Side	Right Side
$\log_6 (6 - 3) + \log_6 (6 + 6)$ $= \log_6 3 + \log_6 12$ $= \log_6 36$ $= 2$	2

Left Side = Right Side

$$\log_6(x-3) + \log_6(x+6) = 2$$

product law

$$\log_6(x-3)(x+6) = 2$$

Root Base

$$(x-3)(x+6) = 6^2$$

FOIL

$$x^2 + 3x - 18 = 36$$

-36

Solve: $x^2 + 3x - 54 = 0$

AC

-54

$+9, -6$ $= 3$

$(x+9)(x-6) = 0$ factor

$x + 9 = 0$ -9

$x \neq -9$ extraneous

$\boxed{x = 6}$

Restrictions: $x - 3 > 0$ $x + 6 > 0$

$x > 3$ $x > -6$

~~**Method 2: Rewrite as a Logarithm**~~

Written as a logarithm with base 6, 2 = ________.

$$\log_6 (x - 3) + \log_6 (x + 6) = \log_6 36$$
$$\log_6 (x - 3)(x + 6) = \log_6 36$$
$$(x - 3)(x + 6) = 36$$

________________ $= 36$

________________ $= 0$

Factoring,

________________ $= 0$

So, $x = -9$ or $x = 6$.
Since $x > 3$, the solution is $x = 6$.

Since $x > 3$, the solution is $x = 6$.

Working Example 2: Solve Exponential Equations

Solve. Express your answer as an exact value and as a decimal correct to two decimal places.

a) $5^x = 200$ **b)** $8^{2x-3} = 15\,109$ **c)** $3^{2x} = 7^{x+1}$

★ since no common base, use logs to solve

→ IF common base possible, ch7 method

ex: $2^{x+1} = 4^x$

$2^{x+1} = (2^2)^x$

$\cancel{2}^{x+1} = \cancel{2}^{4x}$

$x+1 = 4x$ $-x$

$\frac{3x}{3} = \frac{1}{3}$

$\boxed{x = \frac{1}{3}}$

Solution

a) Method 1: Use the Power Law of Logarithms

Take the logarithms of both sides of the equation:

power law

$\log 5^x = \log 200$

$x \cancel{\log 5} = \log 200$
$\frac{x \log 5}{\log 5} = \frac{\log 200}{\log 5}$

$\boxed{x = \dfrac{\log 200}{\log 5}}$ Exact ★ Required

As a decimal, $x \approx 3.29$. Approximate ★ if question asks.

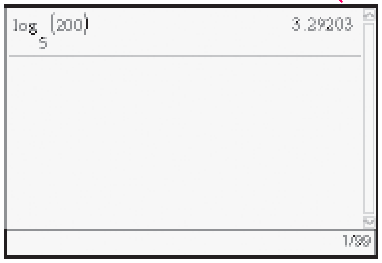
Method 2: Write in Logarithmic Form

$5^x = 200$ is the same as $\boxed{\log_5 200 = x}$. An exact value for x is $\log_5 200$.
Use technology to calculate the value to two decimal places: $\log_5 200 = 3.29$.

Exact

$x = \log_5 200$ ✓

approx.



b) Take the logarithm of each side: $\log 8^{2x-3} = \log 15\,109$.
Then, use the power law of logarithms.

power law

$(2x-3)\log 8 = \log 15\,109$

only x

$2x - 3 = \dfrac{\log 15\,109}{\log 8}$ → $x = \dfrac{\frac{\log 15109}{\log 8} + 3}{2}$

$x = \dfrac{1}{2}\left(\dfrac{\log 15\,109}{\log 8} + 3\right)$

or $x = \dfrac{\log 15109}{2\log 8} + \dfrac{3}{2}$

$x \approx 3.81$

c) Since $3^{2x} = 7^{x+1}$, $\log 3^{2x} = \log 7^{x+1}$. ← take log of both sides

Use the power law of logarithms.

$2x \log 3 = (x+1)\log 7$ ← power law

$2x \log 3 = x \log 7 + \log 7$ ← expand brackets

$2x \log 3 - x \log 7 = \log 7$ ← move x's to one side

Factor. - common factor x

$2x \log 3 - x \log 7 = \log 7$ ← move x's to one side

Factor. - common factor x

$x(2 \log 3 - \log 7) = \log 7$ ← factor x

$$\boxed{x = \frac{\log 7}{2 \log 3 - \log 7}}$$ ← divide to solve for x

exact

Written as a decimal, $x \approx 7.74$. approx.

p.286

Working Example 3: Model Exponential Growth

$A = A_0 (b)^{\frac{t}{n}}$

A town has a current population of 12 468. The population is growing by 2% per year.

A_0

a) Write an exponential equation to model the population growth. $b = 1 + 0.02$ $n = 1$

b) What will be the town's population in eight years?

c) When will the population first reach 20 000?

$A = 12468(1.02)^t$

Solution

a) If P is the population of the town and t is time, in years, then $P = 12\,468(1.02)^t$.

b) In eight years, the population will be $P =$ $12468(1.02)^8$, or 14 608. $t = 8$

c) When the population reaches 20 000, $12\,468(1.02)^t = 20\,000$.

$\frac{20000}{12468} = \frac{\cancel{12468}(1.02)^t}{12468}$

Divide each side by 12 468: $1.02^t =$ $\frac{20000}{12468}$ not 1.60

Take the logarithm of each side: $\log 1.02^t =$ $\log\left(\frac{20000}{12468}\right)$

Apply the power law of logarithms: $\frac{t \log 1.02}{\log 1.02}$ $=$ $\frac{\log\left(\frac{20000}{12468}\right)}{\log 1.02}$

~~IF $t = \frac{\log(1.6)}{\log(1.02)} = 23.73\text{yr}$~~

$t =$ $\frac{\log\left(\frac{20000}{12468}\right)}{\log 1.02}$ → $t = 23.86\text{yr}$.

Written as a decimal, t is approximately 23.86. It will take about 23.9 years until the town reaches a population of 20 000.

Working Example 4: Model Exponential Decay

A business invests $450 000 in new equipment. For tax purposes, the equipment is considered to depreciate in value by 20% each year. ← $n = 1$

A_0

$b = 1 - 0.2 \rightarrow b = 0.8$

a) Write an exponential equation to model the value of the equipment.

b) What will be the value of the equipment in three years?

c) When will the value first drop to $100 000?

$A = 450{,}000(0.8)^t$

Solution

a) If V denotes the value of the equipment, and t is time, in years, then $V = 450\,000(0.8)^t$.

b) $V = 450\,000(0.8)^3$ ← $t = 3$ (in 3yr)

$\boxed{= 230\,400}$

The equipment will be worth $230 400 in three years.

$\frac{100000}{450000} = \frac{450000(0.8)^t}{450000}$

c) $450\,000(0.8)^t = 100\,000$

The equipment will be worth \$230 400 in three years.

$$\frac{100000}{450000} = \frac{450000(0.8)^t}{450000}$$

c) $450\,000(0.8)^t = 100\,000$

Divide each side by 450 000: $\frac{10}{45}$ = $(0.8)^t$

Take the logarithm of each side: $\log\left(\frac{10}{45}\right)$ = $\log 0.8^t$

Use the power law of logarithms to rewrite the equation: $\frac{\log\left(\frac{10}{45}\right)}{\log 0.8}$ = $\frac{t \log \cancel{0.8}}{\log 0.8}$

Divide each side by log 0.8: t = $\frac{\log\left(\frac{10}{45}\right)}{\log 0.8}$

After 6.74 years, the equipment will have a value of \$100 000 for tax purposes. $\boxed{t = 6.74 \text{ yr}}$

See pages 406–411 of *Pre-Calculus 12* for more examples.

Using log laws to solve the log equation:

SOLVING LOG EQUATIONS:

1. Use the log laws to condense each side of the = sign to a single log or number.

$$log_a b = log_a c \quad OR \quad log_a b = C$$

2. A) If one log on each side, cancel the logs.

$$log_a b = log_a c$$
$$\cancel{log_a} b = \cancel{log_a} c$$
$$b = c$$

B) If log on one side and a number on the other side, BOOT the log to change to exponential form.

$$log_a b = C$$

$$a^C = b$$

3. Solve the equation.
4. Write restrictions or do a check to determine if there are any extraneous roots.

Solving a logarithmic equation by changing to exponential form = BOOT THE LOG

Solve the log equation by combining to a common of base:

Recall solving an exponential by changing to a common base. You can then make the exponents equal and solve.

What if you can't get a common base? Log both side and use the power law to solve.

SOLVING EXPONENTIAL EQUATIONS WITH LOGS:

1. Simplify equation by trying to get a single base on both sides of the equal sign.

$$M^{a+b} = N^{c+d}$$

Recall: if there is a single common base on each side of the = sign, cancel the bases and make the exponents equal to solve.

$$M^{a+b} = M^{c+d}$$

$$\cancel{M}^{a+b} = \cancel{M}^{c+d}$$

$$a + b = c + d$$

2. If you cannot get a common base, take the log of both sides.

$$\log M^{a+b} = \log N^{c+d}$$

3. Use the power law to bring the exponent to the front of the log.

$$\log M^{a+b} = \log N^{c+d}$$

$$(a+b)\log M = (c+d)\log N$$

4. Expand the brackets by distribution, collect the common variables to one side, factor and solve for x.

$$(a+b)\log M = (c+d)\log N$$

$$a\log M + b\log M = c\log N + d\log N$$

Exponential Equation with Different Bases

1. Isolate the exponential part of the equation. If there are two exponential parts put one on each side of the equation.
2. Take the logarithm of each side of the equation.
3. Apply power property to rewrite the exponent.
4. Solve for the variable.

Example:

$$3^x - 1 = 4$$
$$3^x = 5$$
$$\log 3^x = \log 5$$
$$x \log 3 = \log 5$$
$$x = \frac{\log 5}{\log 3}$$

Example:

$$5^{x-1} - 2^x = 0$$
$$5^{x-1} = 2^x$$
$$\log 5^{x-1} = \log 2^x$$
$$(x-1)\log 5 = x \log 2$$
$$x \log 5 - \log 5 = x \log 2$$
$$x \log 5 - x \log 2 = \log 5$$
$$x(\log 5 - \log 2) = \log 5$$
$$x = \frac{\log 5}{\log 5 - \log 2}$$

Solve $6^x - 15 = 0$

1. Isolate the exponential expression of the equation.

$$6^x = 15$$

2. Take the common logarithm of each side.

$$\log 6^x = \log 15$$

3. Use the power property

$$x \log 6 = \log 15$$

4. Solve for the variable

$$x = \frac{\log 15}{\log 6}$$

Exponential Equations with Different Bases

Did you say different bases? Now that could be a little more difficult.

Let's open the books and give it a try.

	$2^{x+1} = 8^2$	$9^{x+1} = 27^x$
Write each base as a power of the same base	$2^{x+1} = (2^3)^2$	$(3^2)^{x+1} = (3^3)^x$
Simplify the exponents	$2^{x+1} = 2^6$	$3^{2x+2} = 3^{3x}$
Drop the bases and set the exponents equal	$x + 1 = 6$	$2x + 2 = 3x$
Solve the resulting equation	$\boxed{x = 5}$	$\boxed{x = 2}$

More Exponential Equations with Different Bases

Solve for x to the nearest hundredth: $3^x = 21$

Just for fun, let's look at that in logarithmic form. $x = \log_3 21$

This looks like that change of base stuff.

	$3^x = 21$	$2^{3x} = 7^2$
Turn it into a log equation	$\log 3^x = \log 21$	$\log 2^{3x} = \log 7^2$
Apply the logarithm laws	$x \log 3 = \log 21$	$3x \log 2 = 2 \log 7$
Isolate the variable	$x = \dfrac{\log 21}{\log 3}$	$x = \dfrac{2 \log 7}{3 \log 2}$
Use your calculator to solve	$\boxed{x = 2.77}$	$\boxed{x = 1.87}$

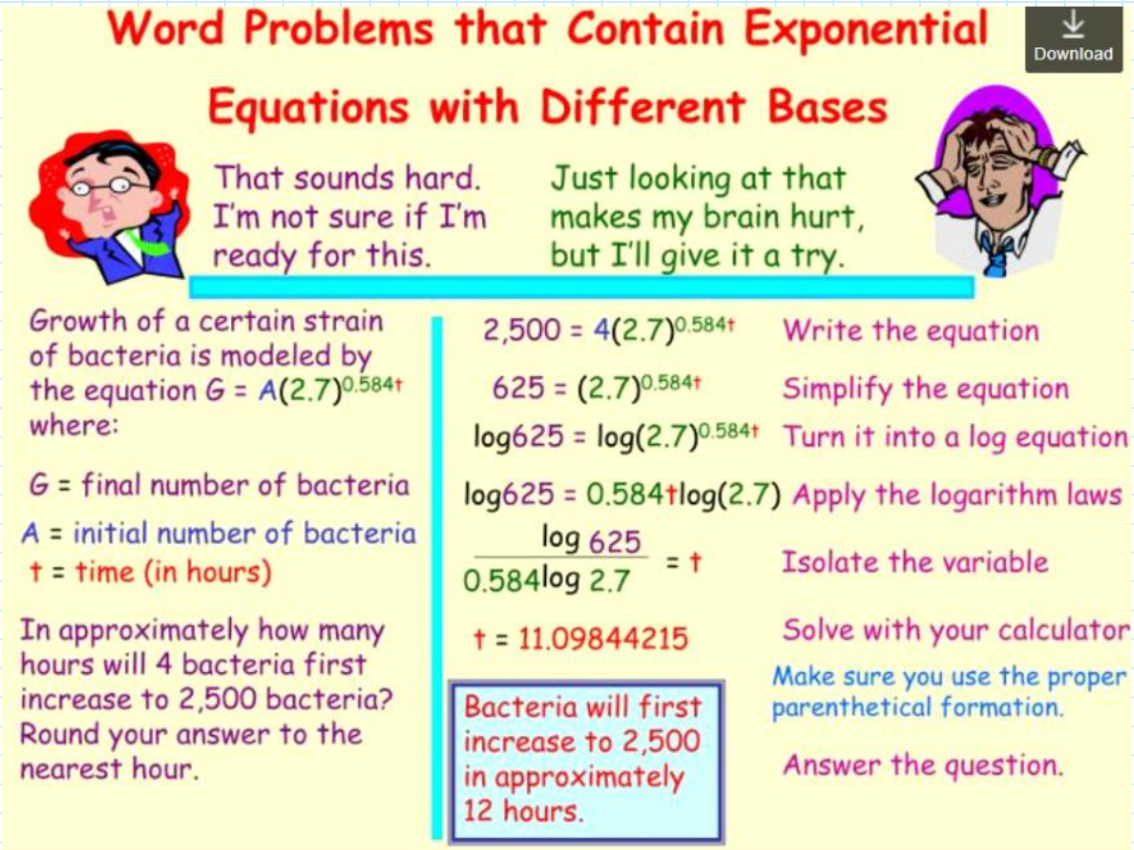
Make sure you use the proper parenthetical formation.

Revisiting Exponential Application Questions & Formulae

Compound Interest: $A = A_{\circ}\left(1+\frac{r}{n}\right)^{nt}$

General Growth/Decay: $A = A_{\circ}\left(b\right)^{\frac{t}{n}}$

General Earthquake/pH: $I = \left(10\right)^{high-low}$



JAN02 30

That's a Def-Con 3 problem. It's worth 4 points on the regents exam.

Remember: Don't worry about the words, just look for numbers, formulas, and equations.

Depreciation (the decline in cash value) on a car can be determined by the formula $V = C(1 - r)^t$, where V is the value of the car after t years, C is the original cost of the car, and r is the rate of depreciation. If a car's cost, when new, is $15,000, the rate of depreciation is 30%, and the value of the car now is $3,000, how old is the car to the *nearest tenth of a year*?

$3{,}000 = 15{,}000(1 - .30)^t$ Write the equation

$.2 = (1 - .30)^t$ Simplify the equation

$.2 = (.7)^t$

$\log(.2) = \log(.7)^t$ Turn it into a log equation

$\log(.2) = t\log(.7)$ Apply the logarithm laws

$\frac{\log(.2)}{\log(.7)} = t$ Isolate the variable

$t = 4.512338026$ Solve with your calculator

Make sure you use the proper parenthetical formation.

The car is approximately 4.5 years old — Answer the question.

JAN06 32

Difficulty level
DefCon 3
4 points

The current population of Little Pond, New York is 20,000. The population is *decreasing*, as represented by the formula $P = A(1.3)^{-0.234t}$, where P = final population, t = time, in years, and A = initial population.

What will the population be 3 years from now? Round your answer to the *nearest hundred people*.

To the *nearest tenth of a year*, how many years will it take for the population to reach half the present population?

Part 1

$P = 20{,}000(1.3)^{-0.234(3)}$ Plug in the given v

$P = 16{,}635.72614$ Solve with your calculator

The population will be approximately 16,600 — Answer the question.

Part b

$10{,}000 = 20{,}000(1.3)^{-0.234t}$ Write the equation

$1 = 2(1.3)^{-0.234t}$ Simplify the equation

$\log 1 = \log 2(1.3)^{-0.234t}$ Turn it into a log equation

$\log 1 = \log 2 + \log(1.3)^{-0.234t}$ Apply the logarithm laws

$\log 1 = \log 2 - 0.234t\log(1.3)$

$\log 1 - \log 2 = -0.234t\log(1.3)$ Isolate the variable

$\frac{\log 1 - \log 2}{-0.234\log 1.3} = t$

$t = 11.2903$ Solve with your calculator

Make sure you use the proper parenthetical formation.

It will take approximately 11.3 years — Answer the question.

Ch8 Extra Practice

Extra Practice for Chapter 8

1. Solving the following equations. Show restrictions and final answer in a box.

a) $log_2(x-2) + log_2(x-1) = 2$

$\times$

$\log_2 (x-2)(x-1) = 2$

$(x-2)(x-1) = 2^2$

$x^2 - 3x + 2 - 4 = 0$

$x^2 - 3x - 2 = 0$

① Factor ② Quadratic Formula

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2(1)}$

$x = \frac{3 \pm \sqrt{17}}{2}$

$\boxed{x = \frac{3+\sqrt{17}}{2}}$ ✓

$x \neq \frac{3-\sqrt{17}}{2}$ extraneous.

R: $x-2>0$ $\quad x-1>0$

$x>2$ $\quad x>1$

b) $log_3(2x+5) - log_3(x+2) = log_3 4$

$\div$

$\cancel{\log_3}\left(\frac{2x+5}{x+2}\right) = \cancel{\log_3} 4$

$\frac{2x+5}{x+2} = 4(x+2)$

$2x+5 = 4x+8$

$-4x \qquad -5$

$-2x = 3$

$\boxed{x = -\frac{3}{2}}$ ✓

R

$2x+5>0 \quad x+2>0$

$x > -\frac{5}{2} \quad x>-2$

2. Write the following in expanded form:

a) $log_5\left(\frac{25A^3}{\sqrt[4]{BC^2}}\right)$ $\div$

① $\log_5 25A^3 - \log_5 \sqrt[4]{BC^2}$

② $\log_5 25 + \log_5 A^3 - (\log_5 \sqrt[4]{B} + \log_5 C^2)$

③ $\log_5 5^2 + 3\log_5 A - \log_5 B^{\frac{1}{4}} - 2\log_5 C$

④ $\boxed{2 + 3\log_5 A - \frac{1}{4}\log_5 B - 2\log_5 C}$

2. Write the following in condensed form as a single logarithm:

a) $2log_5(x) - \frac{1}{2}log_5 y + 3\,log_5 z$

① $\log_5 x^2 - \log_5 y^{\frac{1}{2}} + \log_5 z^3$

$\div \qquad \times$

② $\log_5\left(\frac{x^2}{\sqrt{y}} \cdot \frac{z^3}{1}\right)$ → ③ $\boxed{\log_5\left(\frac{x^2 z^3}{\sqrt{y}}\right)}$

Extra Practice for Chapter 8

$\log_2$

3. Write a log base two equation in standard form after the following transformations:

Vertical stretch by a factor of 3 and a reflection over the *x*-axis, a Horizontal stretch of ½, 3 units right and 5 down.

$a=3$ $\quad -a \quad$ $b=2$ $\quad h=3$

$k=-5$

$$\boxed{y=-3\log_2(2(x-3))-5}$$

4. Solving the following equations:

a) $3^{(x-1)} = 9(27)^{(2-x)}$

common base

$$3^{x-1} = 3^2\cdot(3^3)^{2-x}$$

ch7 method

$$3^{x-1} = 3^2\cdot 3^{6-3x}$$

no Restrictions.

$$\cancel{3}^{x-1} = \cancel{3}^{8-3x}$$

$$x-1 = 8-3x$$

$+3x \quad +1$

$$4x = 9$$

$$\boxed{x=\frac{9}{4}}\ \checkmark$$

b) $2^{(x-3)} = 3(5)^{(2-x)}$

no common base : log both sides

ch8 method.

$$(\ )\log[2^{x-3}] = \log[3(5)^{2-x}]$$

$+$

$$(x-3)\log 2 = \log 3 + (2-x)\log 5$$

$$x\log 2 - 3\log 2 = \log 3 + 2\log 5 - x\log 5$$

$$x\log 2 + x\log 5 = \log 3 + 2\log 5 + 3\log 2$$

$$x\,\underbrace{(\log 2+\log 5)}_{(\ )} = \underbrace{\log 3 + 2\log 5 + 3\log 2.}_{(\ )}$$

Exact

$$\boxed{x = \frac{\log 3 + 2\log 5 + 3\log 2}{\log 2 + \log 5.}}$$

$x \approx 2.78$

IF want to condense, use log laws

$$x = \frac{\log(3\cdot 5^2\cdot 2^3)}{\log(2\cdot 5)}$$

$$x = \frac{\log 600}{\cancel{\log 10}_{=1}} \rightarrow \boxed{x = \log 600}$$

$x \approx 2.78$

$A = A_0(b)^{t/n}$

Extra Practice for Chapter 8

5. A radioactive substance decays with a half-life of 4 days. How long would it take for the sample to decay to 24% of the original amount?

$b = 0.5$ or $\frac{1}{2}$ $n = 4$ $t = ?$

$A = 0.24$ (24) $A_0 = 1.00$ (100)

$$\log 0.24 = \log 1(0.5)^{\frac{t}{4}}$$

$$(4)\log 0.24 = \frac{t}{4}\log 0.5 \ (4)$$

$$\frac{4\log 0.24}{\log 0.5} = \frac{t\log 0.5}{\log 0.5}$$

$$t = \frac{4\log 0.24}{\log 0.5}$$

$$\boxed{t = 8.24 \text{ days.}}$$

6. If an earthquake in Town A is 22567 time more intense than an earthquake in Town B which had a magnitude of 2.3 on the Richter scale, what is the magnitude of the earthquake in Town A?

$$I = 10^{R_H - R_L} \longrightarrow \log 22567 = \log 10^{R_A - 2.3} \quad \cancel{2.3 - R}$$

$$\log 22567 = (R_A - 2.3)\cancel{\log 10}$$

$$\log 22567 = R_A - 2.3$$

$+2.3$

$$R_A = \log 22567 + 2.3$$

$$\boxed{R_A = 6.7 \text{ Magnitude for Town A}}$$

7. If a population of 4 rats grew to a population of 10,000 rats in 12 months, what is the doubling time of the rats?

$t = 12$ $b = 2$ $n = ?$

$$A = A_0(b)^{t/n}$$

$$\frac{10000}{4} = \frac{4(2)^{\frac{12}{n}}}{4}$$

$$\log 2500 = \log 2^{\frac{12}{n}}$$

$$(n)\log 2500 = \frac{12}{n}\log 2 \cdot (n)$$

$$\frac{n\log 2500}{\log 2500} = \frac{12\log 2}{\log 2500}$$

$$n = \frac{12\log 2}{\log 2500}$$

$$\boxed{n = 1.06 \text{ months}}$$