

Tuesday, Nov. 21st

## Plan For Today:

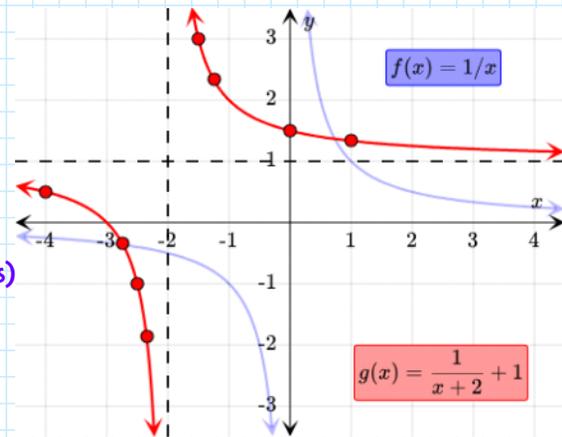
### 1. Any questions from Ch7 or Ch8?

- \* **Hand-in Ch8 Project**
- \* **Do Unit 3 Exam**
  - I will mark and go over this test on Thursday
  - Rewrite on Tuesday, Nov. 28th after class at 12:30pm

### 2. Start Chapter 9: Rational Functions

- \* **9.1: Rational Function Transformations**
- \* 9.2: Analysing Rational Functions (Characteristics of Graphs)
- \* 9.3: Graphs and Solving Rational Functions

### 3. Work on Practice Questions from Workbook



## Plan Going Forward:

1. Finish going through all of the 9.1 questions in workbook and start working on practice review handout. Skip Working Example 3. Do #1,2,4,5 in 9.1 of workbook.

● **CHECK-IN QUIZ ON 9.1 ON THURSDAY, NOV. 23RD**

- **CHAPTER 9 PROJECT DUE TUESDAY, DEC. 5TH**
  - (TRY FOR THURSDAY, NOV. 30TH)
- **CHAPTER 9 TEST ON TUESDAY, DEC. 5TH**

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at [anurita.weebly.com](http://anurita.weebly.com) after class.  
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# Tuesday, Nov. 21st In-Class Notes

# Factoring Review

## Common Factor First: GCF = Greatest Common Factor

### Factor Out the GCF

The first step to factoring is to factor out the greatest common factor (GCF) from each term.

*Example:*

$$12y^3 - 15y^2 + 6y$$

$2 \times 2 \times 3 \times y \times y \times y$        $5 \times 3 \times y \times y$        $2 \times 3 \times y$

$$3y(4y^2 - 5y + 2)$$

### Factoring Techniques

#### Factor out the GCF

$$2yx^2 - 8xy - 24y$$
$$= 2y(x^2 - 4x - 12)$$

#### Special Cases

##### Difference of Two Squares

$$x^2 - 9$$
$$= x^2 - 3^2$$
$$= (x + 3)(x - 3)$$

##### Perfect Square Trinomial

$$x^2 - 10x + 25$$
$$= (x - 5)^2$$

#### Grouping

$$4x^2 - 4x - 15$$
$$= 4x^2 - 10x + 6x - 15$$
$$= 2x(2x - 5) + 3(2x - 5)$$
$$= (2x + 3)(2x - 5)$$

## Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Examples:

$$\begin{aligned}9x^2 - 16 \\&= (3x)^2 - 4^2 \\&= (3x + 4)(3x - 4)\end{aligned}$$

$$\begin{aligned}4x^2 - 81y^2 \\&= (2x)^2 - (9y)^2 \\&= (2x + 9y)(2x - 9y)\end{aligned}$$

## Trinomial Factoring Where the Leading Coefficient 'a' = 1

$$ax^2 + bx + c$$

To factor  $x^2 + bx + c$ :

1. First arrange in descending order.
2. Use a trial-and-error procedure that looks for factors of  $c$  whose sum is  $b$ .
  - If  $c$  is positive, then the signs of the factors are the same as the sign of  $b$ .
  - If  $c$  is negative, then one factor is positive and the other is negative. (If the sum of the two factors is the opposite of  $b$ , changing the signs of each factor will give the desired factors whose sum is  $b$ .)
3. Check your result by multiplying.

Thus the factorization is

$$(x + 3)(x + 5), \text{ or } (x + 5)(x + 3)$$

by the commutative law of multiplication. In general,

$$(x + p)(x + q) = x^2 + (p + q)x + pq.$$

To factor, we can use this equation in reverse.

### Factoring Trinomials with $a = 1$

Find the two numbers that will make these equations true.

$$\square \times \square = c$$

$$\square + \square = b$$

Put the two numbers in the expression.

$$(x + \square)(x + \square)$$

$$x^2 + 2x - 8$$

$$\boxed{4} \times \boxed{-2} = -8$$

$$\boxed{4} + \boxed{-2} = 2$$

$$(x + \boxed{4})(x + \boxed{-2})$$

$$\begin{array}{l} x^2 - 5x - 24 \\ (x \quad )(x \quad ) \end{array} \begin{array}{l} -24 \cdot 1 \\ -12 \cdot 2 \\ -8 \cdot 3 \\ -6 \cdot 4 \end{array}$$

$$x^2 - 5x - 24$$

$$(x - 8)(x + 3)$$

## Factor Trinomial with Negative Leading Coefficient

When the leading coefficient of a polynomial is negative, we can factor out a common factor with a negative coefficient.

*Examples:*

$$\begin{aligned} & -5x^2 + x + 4 \\ & = -(5x^2 - x - 4) \\ & = -(5x^2 - 5x + 4x - 4) \\ & = -(5x(x-1) + 4(x-1)) \\ & = -(5x+4)(x-1) \end{aligned}$$

$$\begin{aligned} & -3x^3 + 12x^2 + 15x \\ & = -3x(x^2 - 4x - 5) \\ & = -3x(x^2 - 5x + x - 5) \\ & = -3x(x(x-5) + (x-5)) \\ & = -3x(x+1)(x-5) \end{aligned}$$

## Perfect Square Trinomials (PST)

$$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$$

*Examples:*

$$\begin{aligned} & x^2 + 12x + 36 \\ & = x^2 + (2)(6)x + 6^2 \\ & = (x + 6)^2 \end{aligned}$$

$$\begin{aligned} & 9x^2 - 12x + 4 \\ & = (3x)^2 - (3)(2)x + 2^2 \\ & = (3x - 2)^2 \end{aligned}$$

There are many methods for factoring a trinomial where  $a \neq 1$   
1. The FOIL method is the same as a 'Guess and Check' Method

## THE FOIL METHOD

To factor trinomials of the type  $ax^2 + bx + c$ ,  $a \neq 1$ , using the **FOIL method**:

1. Factor out the largest common factor.
2. Find two First terms whose product is  $ax^2$ :

$$(\square x + \quad)(\square x + \quad) = ax^2 + bx + c.$$

FOIL

3. Find two Last terms whose product is  $c$ :

$$(\quad x + \square)(\quad x + \square) = ax^2 + bx + c.$$

FOIL

4. Repeat steps (2) and (3) until a combination is found for which the sum of the Outside and Inside products is  $bx$ :

$$(\square x + \square)(\square x + \square) = ax^2 + bx + c.$$

FOIL

5. Always check by multiplying.

## TIPS FOR FACTORING $ax^2 + bx + c$ , $a \neq 1$ , USING THE FOIL METHOD

1. If the largest common factor has been factored out of the original trinomial, then no binomial factor can have a common factor (other than 1 or  $-1$ ).
2. a) If the signs of all the terms are positive, then the signs of all the terms of the binomial factors are positive.  
b) If  $a$  and  $c$  are positive and  $b$  is negative, then the signs of the factors of  $c$  are negative.  
c) If  $a$  is positive and  $c$  is negative, then the factors of  $c$  will have opposite signs.
3. Be systematic about your trials. Keep track of those you have tried and those you have not.
4. Changing the signs of the factors of  $c$  will change the sign of the middle term.

1. The AC Method here is the Decomposition method; also known as the Factor by Grouping method.

### THE *ac*-METHOD

To factor  $ax^2 + bx + c$ ,  $a \neq 1$ , using the *ac*-method:

1. Factor out the largest common factor.
2. Multiply the leading coefficient  $a$  and the constant  $c$ .
3. Try to factor the product  $ac$  so that the sum of the factors is  $b$ . That is, find integers  $p$  and  $q$  such that  $pq = ac$  and  $p + q = b$ .
4. Split the middle term. That is, write it as a sum using the factors found in step (3).
5. Factor by grouping.
6. Always check by multiplying.

## Factoring Polynomials:

### Type 2:

#### Quadratic Trinomials with a Leading coefficient $\neq 1$

##### a) Factoring by Decomposition

1. Multiply  $a$  and  $c$
2. Look for two numbers that multiply to that product and add to  $b$
3. Break down the middle term into two terms using those two numbers
4. Find the common factor for the first pair and factor it out & then find the common factor for the second pair and factor it out.
5. From the two new terms, place the common factor in one bracket and the factored out factors in the other bracket.

$$a \times c = -20$$

The 2 nos. are -20 & 1

$$\begin{aligned} & 5x^2 - 19x - 4 \\ &= 5x^2 - 20x + 1x - 4 \\ &= 5x(x - 4) + 1(x - 4) \\ &= (x - 4)(5x + 1) \end{aligned}$$

## Factor by Grouping

Example:

$$6x^2 + 15x - 21$$

$$= 3(2x^2 + 5x - 7) \quad \text{1. Factor out Greatest Common Factor}$$

$$2 \times -7 = -14 \quad +5$$

Find two numbers when multiplied get -14 and when added get +5

$$+7 \times -2 = -14$$

$$+7 + -2 = +5$$

Rewrite 5x as  $-2x + 7x$

$$= 3(2x^2 - 2x + 7x - 7)$$

2. Split the middle term into two terms.

$$= 3[2x(x-1) + 7(x-1)]$$

3. Rewrite the pairs of terms and take out the common factor.

$$= 3(x-1)(2x+7)$$

## Solving General Trinomials - the Decomposition Method

$$3x^2 - 10x + 8$$

The product is  $3 \times 8 = 24$ .

The sum is  $-10$ .

Factors of 24

1	24
2	12
3	8
4	6

$$3x^2 - 6x - 4x + 8$$

Rewrite the middle term of the polynomial using  $-6$  and  $-4$ .

( $-6x - 4x$  is just another way of expressing  $-10x$ .)

$$3x(x-2) - 4(x-2)$$

Factor by grouping.

$$(x-2)(3x-4)$$

3. This is the Short-cut Method I like to use.

### Decomposition

$$3x^2 - 19x - 14$$

$A = 3, C = -14 \rightarrow AC = -42$

Two numbers that multiply to -42 but add to -19

These numbers are: -21 and +2

Replace the middle (b) term with these two factors written with an  $x$

$$3x^2 - 21x + 2x - 14$$

Split it in half and factor each half

$$3x^2 - 21x \mid +2x - 14$$
$$3x(x - 7) + 2(x - 7)$$

Place each term in front of the brackets in its own bracket and write the other common binomial in one bracket.

$$(3x + 2)(x - 7)$$

Done!

### Short-cut Factoring

$$3x^2 - 19x - 14$$

$A = 3, C = -14 \rightarrow AC = -42$

Two numbers that multiply to -42 but add to -19

These numbers are: -21 and +2

Place the first term of the trinomial without the squared in the first spot of each factored bracket

$$(3x \quad)(3x \quad)$$

Place the two factors (-21 & +2) in the brackets to form the binomials

$$(3x - 21)(3x + 2)$$

Reduce the terms in the binomials like you would fractions

$$(\cancel{1}x - \cancel{21}_7)(3x + 2)$$
$$= (x - 7)(3x + 2)$$

Done!

## Review of Algebra and Factoring

### Common Factoring

Determine the greatest common factor by checking what the largest term divisible by all terms is (numbers and variables).

Ex.  $2x^2 - 6x \rightarrow \boxed{2x(x-3)}$

Complete the following for practice:

a)  $3x^3 - 9x^2$

b)  $-8x^3 + 2x^2 - 22x$

### Binomial Factoring with a Difference of Squares

When the 2 terms of the binomial are perfect squares and there is a subtraction between them, you can use this method for factoring. Form must be  $(a^2 - b^2)$ .

Ex.  $x^2 - 9 \rightarrow \boxed{(x+3)(x-3)}$        $4x^2 - 25y^2 \rightarrow \boxed{(2x+5y)(2x-5y)}$

Here, you put the square root of  $x$  and the square root of 9 in each bracket with different signs between them: this is the difference of squares factoring.

NOTE:  $x^2 + 9$  is a sum of squares and cannot be factored.

Complete the following:

a)  $a^2 - 16$

b)  $144 - 9y^2$

c)  $36x^2 - 49$

## Trinomial Factoring

A trinomial is in the form:  $ax^2 + bx + c$ . There are different methods for trinomial factoring; including decomposition, guess and check, short-cut factoring, box method. I will show you decomposition and short-cut factoring (I usually do short-cut factoring in class).

When a trinomial has a leading coefficient of 1, the method is simple:

$$x^2 - 4x - 5 \rightarrow C = -5.$$

Find two numbers that multiply to -5 but add to -4. Here the two numbers or factors are -5 and +1. Place these two factors in the brackets with x and you're done.

$$x^2 - 4x - 5 = \boxed{(x-5)(x+1)}$$

When the leading coefficient is not 1, use one of the following methods.

Decomposition	Short-cut Factoring	Another Short-cut Factoring	Box Method									
$3x^2 - 19x - 14$ $A = 3, C = -14 \rightarrow AC = -42$	$3x^2 - 19x - 14$ $A = 3, C = -14 \rightarrow AC = -42$	$3x^2 - 19x - 14$ $A = 3, C = -14 \rightarrow AC = -42$	$3x^2 - 19x - 14$ $A = 3, C = -14 \rightarrow AC = -42$									
Two numbers that multiply to -42 but add to -19	Two numbers that multiply to -42 but add to -19	Two numbers that multiply to -42 but add to -19	Two numbers that multiply to -42 but add to -19									
These numbers are: -21 and +2	These numbers are: -21 and +2	These numbers are: -21 and +2	These numbers are: -21 and +2									
Replace the middle (b) term with these two factors written with an x	Place the first term of the trinomial without the squared in the first spot of each factored bracket	Rewrite the trinomial with the AC as the last term	Place the first and last term in the box in the first and last spot									
$3x^2 - 21x + 2x - 14$ Split it in half and factor each half	$(3x \quad)(3x \quad)$	$x^2 - 19x - 42$ Place the two factors (-21 & +2) in the brackets to form the binomials	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><math>3x^2</math></td> <td></td> </tr> <tr> <td></td> <td>-14</td> </tr> </table>	$3x^2$			-14					
$3x^2$												
	-14											
$3x^2 - 21x + 2x - 14$ $3x(x-7) + 2(x-7)$	Place the two factors (-21 & +2) in the brackets to form the binomials $(3x-21)(3x+2)$	Divide and reduce the constant in each bracket by the A from original trinomial $(x-21)(x+2)$	Place the two factors with x in the second and third box <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><math>3x^2</math></td> <td>-21x</td> </tr> <tr> <td>2x</td> <td>-14</td> </tr> </table>	$3x^2$	-21x	2x	-14					
$3x^2$	-21x											
2x	-14											
Place each term in front of the brackets in its own bracket and write the other common binomial in one bracket.	Reduce the terms in the binomials like you would fractions	If there is still a denominator, write that number in front of the x in the same brackets to get final factored form.	Common factor each row and each column and collect the factors in two brackets for final factored form.									
$(3x+2)(x-7)$ <b>Done!</b>	$(\cancel{1}3x - \cancel{2}14)(3x+2)$ $= (x-7)(3x+2)$ <b>Done!</b>	$(x-\frac{21}{3})(x+\frac{2}{3}) = (x-7)(x+\frac{2}{3})$ $(x-7)(x+\frac{2}{3}) = (x-7)(3x+2)$	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><math>3x^2</math></td> <td>-21x</td> <td>3x</td> </tr> <tr> <td>2x</td> <td>-14</td> <td>2</td> </tr> <tr> <td>x</td> <td>-7</td> <td></td> </tr> </table>	$3x^2$	-21x	3x	2x	-14	2	x	-7	
$3x^2$	-21x	3x										
2x	-14	2										
x	-7											
			$(3x+2)(x-7)$									

Complete the following using a method of your choice:

a)  $6x^2 - 5x - 4$

b)  $2x^2 + 11x + 5$

c)  $2x^2 + x - 1$

d)  $2x^2 - 3x - 2$

To solve, you make each binomial bracket equal zero and solve for  $x$ .

Solving the example from above:

$$(3x+2)(x-7)$$

$$3x+2=0 \quad x-7=0$$

$$3x=-2 \quad \boxed{x=7}$$

$$\boxed{x=-\frac{2}{3}}$$

## Factoring Trinomials (a = 1)

**Factor each completely.**

1)  $b^2 + 8b + 7$

2)  $n^2 - 11n + 10$

3)  $m^2 + m - 90$

4)  $n^2 + 4n - 12$

5)  $n^2 - 10n + 9$

6)  $b^2 + 16b + 64$

7)  $m^2 + 2m - 24$

8)  $x^2 - 4x + 24$

9)  $k^2 - 13k + 40$

10)  $a^2 + 11a + 18$

11)  $n^2 - n - 56$

12)  $n^2 - 5n + 6$

13)  $b^2 - 6b + 8$

14)  $n^2 + 6n + 8$

15)  $2n^2 + 6n - 108$

16)  $5n^2 + 10n + 20$

17)  $2k^2 + 22k + 60$

18)  $a^2 - a - 90$

19)  $p^2 + 11p + 10$

20)  $5v^2 - 30v + 40$

21)  $2p^2 + 2p - 4$

22)  $4v^2 - 4v - 8$

23)  $x^2 - 15x + 50$

24)  $v^2 - 7v + 10$

25)  $p^2 + 3p - 18$

26)  $6v^2 + 66v + 60$

## Factoring Trinomials (a = 1)

Factor each completely.

1)  $b^2 + 8b + 7$

$(b + 7)(b + 1)$

2)  $n^2 - 11n + 10$

$(n - 10)(n - 1)$

3)  $m^2 + m - 90$

$(m - 9)(m + 10)$

4)  $n^2 + 4n - 12$

$(n - 2)(n + 6)$

5)  $n^2 - 10n + 9$

$(n - 1)(n - 9)$

6)  $b^2 + 16b + 64$

$(b + 8)^2$

7)  $m^2 + 2m - 24$

$(m + 6)(m - 4)$

8)  $x^2 - 4x + 24$

Not factorable

9)  $k^2 - 13k + 40$

$(k - 5)(k - 8)$

10)  $a^2 + 11a + 18$

$(a + 2)(a + 9)$

11)  $n^2 - n - 56$

$(n + 7)(n - 8)$

12)  $n^2 - 5n + 6$

$(n - 2)(n - 3)$

$$13) b^2 - 6b + 8$$
$$(b - 4)(b - 2)$$

$$14) n^2 + 6n + 8$$
$$(n + 2)(n + 4)$$

$$15) 2n^2 + 6n - 108$$
$$2(n + 9)(n - 6)$$

$$16) 5n^2 + 10n + 20$$
$$5(n^2 + 2n + 4)$$

$$17) 2k^2 + 22k + 60$$
$$2(k + 5)(k + 6)$$

$$18) a^2 - a - 90$$
$$(a - 10)(a + 9)$$

$$19) p^2 + 11p + 10$$
$$(p + 10)(p + 1)$$

$$20) 5v^2 - 30v + 40$$
$$5(v - 2)(v - 4)$$

$$21) 2p^2 + 2p - 4$$
$$2(p - 1)(p + 2)$$

$$22) 4v^2 - 4v - 8$$
$$4(v + 1)(v - 2)$$

$$23) x^2 - 15x + 50$$
$$(x - 10)(x - 5)$$

$$24) v^2 - 7v + 10$$
$$(v - 5)(v - 2)$$

$$25) p^2 + 3p - 18$$
$$(p - 3)(p + 6)$$

$$26) 6v^2 + 66v + 60$$
$$6(v + 10)(v + 1)$$

Create your own worksheets like this one with **Infinite Algebra 1**. Free trial available at [KutaSoftware.com](https://www.kutaSoftware.com)

Factoring Trinomials ( $a > 1$ )

**Factor each completely.**

1)  $3p^2 - 2p - 5$

2)  $2n^2 + 3n - 9$

3)  $3n^2 - 8n + 4$

4)  $5n^2 + 19n + 12$

5)  $2v^2 + 11v + 5$

6)  $2n^2 + 5n + 2$

7)  $7a^2 + 53a + 28$

8)  $9k^2 + 66k + 21$

9)  $15n^2 - 27n - 6$

10)  $5x^2 - 18x + 9$

11)  $4n^2 - 15n - 25$

12)  $4x^2 - 35x + 49$

13)  $4n^2 - 17n + 4$

14)  $6x^2 + 7x - 49$

15)  $6x^2 + 37x + 6$

16)  $-6a^2 - 25a - 25$

17)  $6n^2 + 5n - 6$

18)  $16b^2 + 60b - 100$

Factoring Trinomials ( $a > 1$ )**Factor each completely.**

1)  $3p^2 - 2p - 5$

$(3p - 5)(p + 1)$

2)  $2n^2 + 3n - 9$

$(2n - 3)(n + 3)$

3)  $3n^2 - 8n + 4$

$(3n - 2)(n - 2)$

4)  $5n^2 + 19n + 12$

$(5n + 4)(n + 3)$

5)  $2v^2 + 11v + 5$

$(2v + 1)(v + 5)$

6)  $2n^2 + 5n + 2$

$(2n + 1)(n + 2)$

7)  $7a^2 + 53a + 28$

$(7a + 4)(a + 7)$

8)  $9k^2 + 66k + 21$

$3(3k + 1)(k + 7)$

$$9) 15n^2 - 27n - 6$$
$$3(5n + 1)(n - 2)$$

$$10) 5x^2 - 18x + 9$$
$$(5x - 3)(x - 3)$$

$$11) 4n^2 - 15n - 25$$
$$(n - 5)(4n + 5)$$

$$12) 4x^2 - 35x + 49$$
$$(x - 7)(4x - 7)$$

$$13) 4n^2 - 17n + 4$$
$$(n - 4)(4n - 1)$$

$$14) 6x^2 + 7x - 49$$
$$(3x - 7)(2x + 7)$$

$$15) 6x^2 + 37x + 6$$
$$(x + 6)(6x + 1)$$

$$16) -6a^2 - 25a - 25$$
$$-(2a + 5)(3a + 5)$$

$$17) 6n^2 + 5n - 6$$
$$(2n + 3)(3n - 2)$$

$$18) 16b^2 + 60b - 100$$
$$4(b + 5)(4b - 5)$$

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# Factoring & Simplifying Rationals Practice

Factor and Simplify Each Rational Polynomial Function

★ include NPVs (restrictions)

$$1. f(x) = \frac{x^2 + x - 2}{x^2 - x - 6}$$

$\begin{matrix} -2 \\ +3, -1 = 1 \end{matrix}$       $\begin{matrix} -6 \\ +2, -3 = -1 \end{matrix}$

$$f(x) = \frac{(x+2)(x-1)}{(x+2)(x-3)}$$

$$f(x) = \frac{x-1}{x-3}$$

NPV  
 $x \neq -2, 3$

$$2. f(x) = \frac{2x^2}{x^2 - 1}$$

← diff. of sq

$$f(x) = \frac{2x^2}{(x+1)(x-1)}$$

NPV  
 $x \neq -1, 1$

$$3. f(x) = \frac{3}{x-2}$$

done.

NPV  
 $x \neq 2$

$$4. f(x) = \frac{2x-1}{x}$$

done

NPV  $x \neq 0$

$$5. f(x) = \frac{x^2 + x - 12}{x^2 - 9}$$

$$f(x) = \frac{(x+4)(x-3)}{(x+3)(x-3)}$$

$$f(x) = \frac{x+4}{x+3}$$

NPV  
 $x \neq \pm 3$

$$6. f(x) = \frac{x^2 - 4}{x + 3}$$

$$f(x) = \frac{(x+2)(x-2)}{x+3}$$

$x \neq -3$

$$7. f(x) = \frac{x^2 - x}{x + 1}$$

$$f(x) = \frac{x(x-1)}{x+1}$$

$x \neq -1$

$$8. f(x) = \frac{x^2 - x - 2}{x - 1}$$

$$f(x) = \frac{(x-2)(x+1)}{x-1}$$

$x \neq 1$

$$9. f(x) = \frac{x+1}{x^2 + 3x + 2}$$

$$f(x) = \frac{x+1}{(x+1)(x+2)}$$

$$f(x) = \frac{1}{x+2}$$

$x \neq -2, -1$

$$10. f(x) = \frac{x^2 - 9}{x^2 - 2x - 3}$$

$$f(x) = \frac{(x+3)(x-3)}{(x-3)(x+1)}$$

$$f(x) = \frac{x+3}{x+1}$$

$x \neq -1, 3$

# 9.1 Intro

## Section 9.1: Intro to Graphing Rational Functions with Transformations

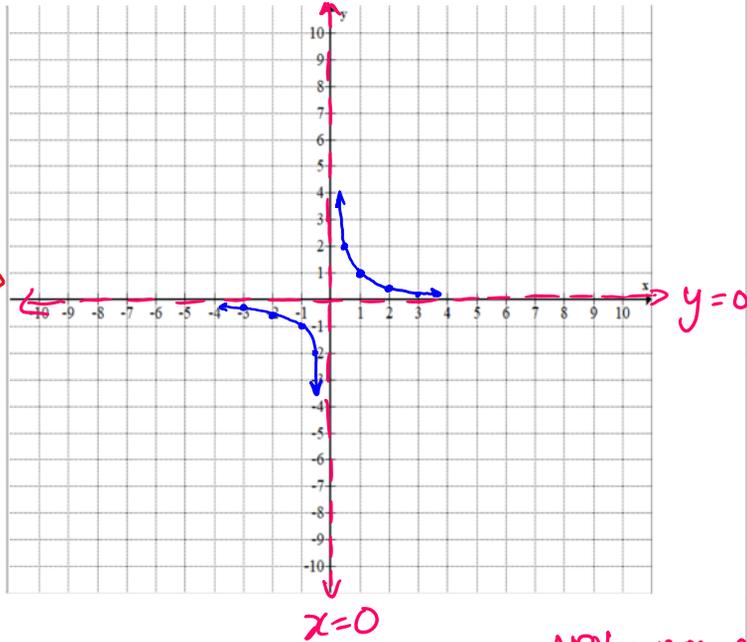
Graph the following functions on the grid provided. Show your base function table of values and the transformed function's table of values. Also include the domain and range.

Rationals = Fractions  
ex:  $\frac{3}{2}, \frac{12}{7}$

Rational Equation  
= variable in denominator  
ex:  $y = \frac{1}{x}, \frac{x^2-4}{x+2}$

1.  $y = \frac{1}{x}$  This is the base function of a rational function before transformations

x	y
-3	$-\frac{1}{3}$
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
0	$\emptyset$
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$



$y = \frac{1}{x}$   
 $y = 1x - 2$   
 $y = -2$   
 $y = \frac{1}{0}$   
 =  $\emptyset$  undefined  
 = asymptote  
 $\therefore x=0$  is a NPV (Restriction)  
 \* TWO Asymptotes

Domain:  $\{x \mid x \neq 0, x \in \mathbb{R}\}$

$x < 0, x > 0$

NPV = non-permissible value

Range:  $\{y \mid y \neq 0, y \in \mathbb{R}\}$

Equations of the asymptotes: VA:  $x=0$  HA:  $y=0$

This is the standard form of a rational function for graphing transformations:

$y = af(x)$   
 $y = a \cdot \frac{1}{x}$   
 $y = \frac{a}{x}$

$y = \frac{a}{b(x-h)} + k$

$h$  and  $k$  determine the location of the asymptotes.  
 VA:  $x=h$  HA:  $y=k$

Vert. Stretch by  $a$   
 up/down by  $k$   
 right/left by  $h$   
 horiz stretch by  $\frac{1}{b}$

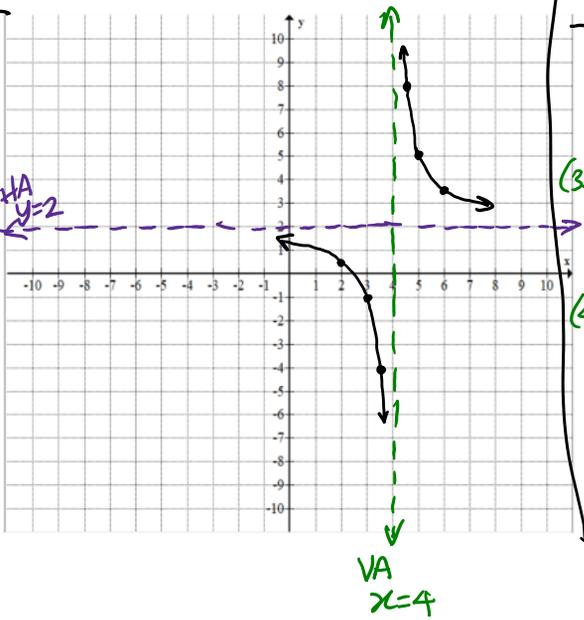
2.  $y = \frac{3}{x-4} + 2$

Annotations:  
 - Red arrow:  $\rightarrow$  VE of 3  
 - Purple arrow:  $\rightarrow$  4 right  $\rightarrow$  2 up  
 - Green arrow:  $\rightarrow$  4 right

Vertical asymptote:  $x = 4$   
 Horizontal asymptote:  $y = 2$

x	$y = \frac{1}{x}$
-3	$-\frac{1}{3}$
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
0	$\emptyset$
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$
3	

x	3y
-2	$-\frac{3}{2}$
-1	-3
$-\frac{1}{2}$	-6
0	$\emptyset$
$\frac{1}{2}$	6
1	3
2	$\frac{3}{2}$



FINAL

x+4	3y+2
2	$\frac{1}{2}$
3	-1
$(35)\frac{1}{2}$	-4
4	$\emptyset$
$(45)\frac{9}{2}$	8
5	5
6	$\frac{7}{2} (35)$

Annotations:  
 -  $-\frac{3}{2} + \frac{4}{2}$  (next to 1/2)  
 -  $\frac{3}{2} + \frac{4}{2}$  (next to 7/2)

Domain:  $\{x \mid x \neq 4, x \in \mathbb{R}\}$

Range:  $\{y \mid y \neq 2, y \in \mathbb{R}\}$

Asymptotes: HA:  $y = 2$     VA:  $x = 4$

Simplify the following by factoring and state the non-permissible values (NPVs or Restrictions):

← b/c denominators cannot equal zero

1.  $y = \frac{x^2 - 3x}{x}$

GCF  $y = \frac{x(x-3)}{x}$

$y = x-3$

NPV  $x \neq 0$

2.  $y = \frac{x+4}{3x^2+10x-8}$

AC  $-24$

Decomposition  $3x^2+12x-2x-8$

$3x(x+4)-2(x+4)$

$(3x-2)(x+4)$

short-cut  $12, -2 = 10$

$(\frac{3x}{3} + \frac{12}{3})(\frac{3x}{3} - \frac{2}{3})$

$(x+4)(3x-2)$

3.  $y = \frac{2x^2 - 4x}{x^2 + 5x - 14}$

or  $\frac{1}{x}$

$y = \frac{2x(x-2)}{(x+7)(x-2)}$

$y = \frac{2x}{x+7}$

NPV:  $x+7 \neq 0$   $x-2 \neq 0$

$x \neq -7, x \neq 2$

$y = \frac{x+4}{(x+4)(3x-2)}$

$y = \frac{1}{3x-2}$

NPVs from all denominators in original question

$x+4 \neq 0$   $3x-2 \neq 0$

→ -4      → +2/3

NPVs  $x \neq -4$   $x \neq \frac{2}{3}$

## 9.1: Rational Function Transformations

### Graphing the basic rational function with transformations

#### 8-4 Rational Functions

The rational function  $f(x) = \frac{1}{x}$  can be transformed by using methods similar to those used to transform other types of functions.

$|a|$  → vertical stretch or compression factor  
 $a < 0$  → reflection across the  $x$ -axis

$k$  → vertical translation  
down for  $k < 0$ ; up for  $k > 0$

$$f(x) = \frac{a}{x - h} + k$$

$h$  → horizontal translation  
left for  $h < 0$ ; right for  $h > 0$

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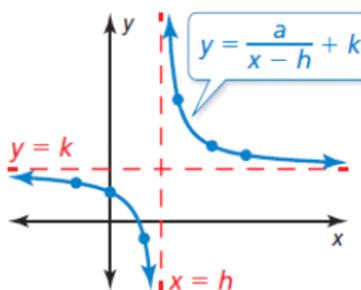
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### Core Concept

#### Graphing Translations of Simple Rational Functions

To graph a rational function of the form  $y = \frac{a}{x - h} + k$ , follow these steps:

- Step 1 Draw the asymptotes  $x = h$  and  $y = k$ .
- Step 2 Plot points to the left and to the right of the vertical asymptote.
- Step 3 Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



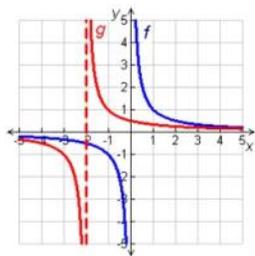
# Rational Functions

## Example 1: Transforming Rational Functions

Using the graph of  $f(x) = \frac{1}{x}$  as a guide, describe the transformation and graph each function.

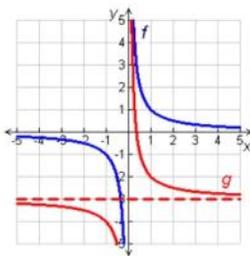
A.  $g(x) = \frac{1}{x+2}$

Because  $h = -2$ ,  
translate  $f$  2 units left.



B.  $g(x) = \frac{1}{x} - 3$

Because  $k = -3$ ,  
translate  $f$  3 units down.



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## Rational Functions and Transformations

**Graph**  $f(x) = \frac{5}{x-2}$ .

$$f(x) = \frac{a}{x-h} + k$$

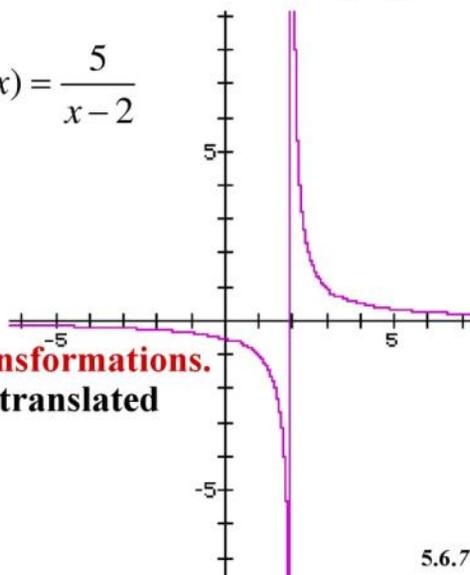
**Compare the graph of**  $f(x) = \frac{5}{x-2}$   
**with**  $f(x) = \frac{1}{x}$ .

Vertical stretch by a  
factor of 5, horizontal  
translation 2 units right.

**Sketch the graph using transformations.**  
The horizontal asymptote is translated  
2 units right as well.

Domain:  $x \neq 2, x \in R$ .

Range:  $y \neq 0, y \in R$



5.6.7

1. Match the rational equation with the correct graph. Explain your rationale for how you matched the equation with the graph.

a)  $y = -\frac{1}{x}$

b)  $y = -\frac{1}{x-3} + 4$

c)  $y = -\frac{1}{x+1} - 2$

