Tuesday, Nov. 28th

Plan For Today:

1. Any questions from 9.2 Graphing Rational Functions and Analysing Characteristics
   - Do some 9.2 Review
   - Do 9.2 Check-in Quiz
2. Finish Chapter 9: Rational Functions
   - ✓ 9.1: Rational Function Transformations
   - ✓ 9.2: Analysing Rational Functions (Characteristics of Graphs)
   - **9.3: Graphs and Solving Rational Functions**
3. Work on Practice Questions from Workbook

$3x\cdot\left(\frac{5}{x}-\frac{1}{3}\right)=3x\cdot\left(\frac{1}{x}\right)$	*Multiply both sides by the LCD.*
$3x\cdot\frac{5}{x}-3x\cdot\frac{1}{3}=3x\cdot\frac{1}{x}$	*Distribute.*
$15-x=3$	*Simplify, and then solve.*
$-x=-12$	
$x=12$	

- **UNIT 3 EXAM REWRITE**
  - Start time is 12:30pm

Plan Going Forward:

1. Go through 9.3 questions in textbook and finish working on all Ch9 workbook practice and practice review handout.

   - **9.3 CHECK-IN QUIZ ON THURSDAY, NOV. 30TH**

   - **CHAPTER 9 PROJECT DUE TUESDAY, DEC. 5TH**
   - **CHAPTER 9 TEST ON TUESDAY, DEC. 5TH**

2. We will start Topic 10 (Geometric Sequences & Series) on Thursday after finishing and reviewing Ch9. This topic is not in the workbook so you will receive a notes package.

Please let me know if you have any questions or concerns about your progress in this course.
The notes from today will be posted at anurita.weebly.com after class.
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Tuesday, Nov. 28th In-Class Notes

9.2 Review

c) $f(x) = \dfrac{x^2 + 3x - 4}{x^2 + 2x - 8}$

-4: $-1, +4 = 3$

-8: $-2, +4 = 2$

$f(x) = \dfrac{(x-1)\cancel{(x+4)}}{(x-2)\cancel{(x+4)}}$

$\boxed{f(x) = \dfrac{x-1}{x-2}}$

$y = \dfrac{-4-1}{-4-2}$

$y = \dfrac{5}{7}$

HA $y=1$

x-int

$0 = \dfrac{x-1}{x-2}$

$0 = x-1$

$x=1$

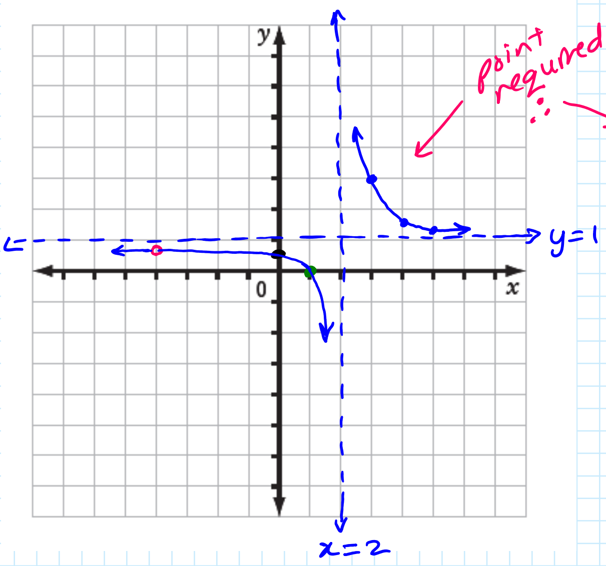
y-int

$y = \dfrac{0-1}{0-2}$

$y = \dfrac{1}{2}$

Characteristic	Answer
Restrictions NPV	$x \neq 2$, $x \neq -4$ OR $x = 2$, $x = -4$
Asymptote(s)	VA $x=2$ HA $y=1$
Point(s) of Discontinuity	$x=-4$ $\left(-4, \frac{5}{7}\right)$
x-intercept	$(1,0)$
y-intercept	$\left(0, \frac{1}{2}\right)$
Domain	$\{x \mid x \neq -4, 2, x \in R\}$. or $\{x \mid x \neq -4, x \neq 2, x \in R\}$.
Range	$\{y \mid y \neq \frac{5}{7}, 1, y \in R\}$.

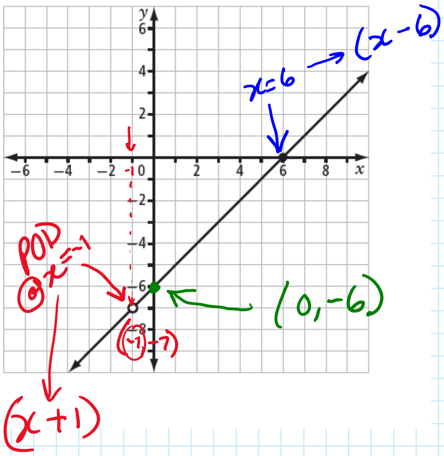
or $\{y \mid y \neq \frac{5}{7}, y \neq 1, y \in R\}$.



$y = \dfrac{x-1}{x-2}$

x	y	
3	2	$\frac{3-1}{3-2}$
4	$\frac{3}{2}$	$\frac{4-1}{4-2}$
5	$\frac{4}{3}$	$\frac{5-1}{5-2}$

b)



Recall:

$$y = \overset{?}{a}\frac{(x-6)(x+1)}{(x+1)}$$

$$-6 = \frac{a(0-6)(0+1)}{(0+1)}$$

$$\frac{-6}{-6} = \frac{-6a}{-6}$$

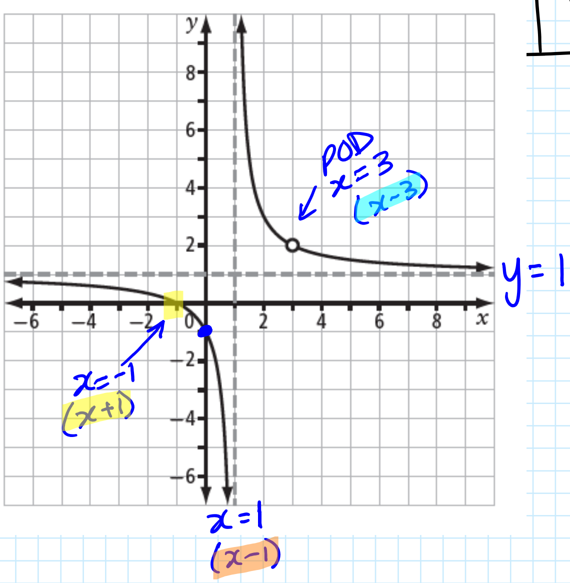
$$a = 1$$

$$\boxed{y = \frac{(x-6)(x+1)}{x+1}}$$

OR

$$\boxed{y = \frac{x^2-5x-6}{x+1}}$$

c)



$$y = \frac{1(x+1)(x-3)}{(x-1)(x-3)}$$

$$\boxed{y = \frac{(x+1)(x-3)}{(x-1)(x-3)}}$$

or

$$\boxed{y = \frac{x^2-2x-3}{x^2-4+3}}$$

d) vertical asymptote: $x = 7$ → $(x-7)$
x-intercepts: -1 and -3

$(x+1)$ $(x+3)$

$$\boxed{y = \frac{(x+1)(x+3)}{(x-7)}}$$

possible equation

$$y = a(x+1)(x+3)$$

$(x+1) \quad (x+3)$

$$y = \frac{a(x+1)(x+3)}{x-7}$$

to get accurate equation, use another point on graph ex. y-int to solve for 'a'

4.3 practice

p. 318 #1

b) $\frac{6}{x} - \frac{9}{x-1} = \frac{1}{4}$

LCD $4x(x-1)$ $\left[\frac{6}{x} - \frac{9}{x-1} = \frac{1}{4}\right]$

NPV $x \neq 0, x=1$

$24(x-1) - 36x = x(x-1)$

$24x - 24 - 36x = x^2 - x$

$-24x + 36x + 24$

$0 = x^2 + 11x + 24$

$0 = (x+3)(x+8)$

✓ $\boxed{x=-3 \quad x=-8}$

c) $\frac{2x}{x+3} + \frac{x}{x-3} = \frac{18}{x^2-9}$

Factor. $(x+3)(x-3)$

LCD $(x+3)(x-3)$ $\left[\frac{2x}{\cancel{x+3}} + \frac{x}{\cancel{x-3}} = \frac{18}{\cancel{(x+3)(x-3)}}\right]$

NPV $x \neq \pm 3$

$2x(x-3) + x(x+3) = 18$

$2x^2 - 6x + x^2 + 3x = 18$

-18

GCF 3

$3x^2 - 3x - 18 = 0$

$3(x^2 - x - 6) = 0$

$3(x-3)(x+2) = 0$

$x \neq 3$ extraneous. $\boxed{x=-2}$ $x \neq \pm 3$

p. 320 #4

c) $\frac{x}{x-2} + \frac{2}{x+3} = \frac{10}{x^2+x-6}$

LCD $(x+3)(x-2)$ $\left[\frac{x}{x-2} + \frac{2}{x+3} = \frac{10}{(x+3)(x-2)}\right]$

NPV $x \neq -3, 2$

#3b p. 319 ✱Quadric Formula.

Next

$$D = ST$$

$$S = \frac{D}{T}$$

$$\boxed{T = \frac{D}{S}}$$

9.3 Workbook

KEY IDEAS

Solving Rational Equations

You can solve rational equations algebraically or graphically.

- **Algebraically**
  Solving algebraically determines the exact solution and any extraneous roots. To solve algebraically,
  – Equate to zero and list the restrictions.
  – Factor the numerator and denominator fully (if possible).
  – Multiply each term by the lowest common denominator to eliminate the fractions.
  – Solve for x.
  – Check the solution(s) against the restrictions.
  – Check the solution(s) in the original equation.

Example:

$$\frac{16}{x+6} = 4 - x$$

$$x + \frac{16}{x+6} - 4 = 0, x \neq -6$$

$$(x+6)\left(x + \frac{16}{x+6} - 4\right) = (x+6)(0)$$

$$(x+6)(x) + \left(\cancel{x+6}^{\,1}\right)\left(\frac{16}{\cancel{x+6}_{\,1}}\right) - (x+6)(4) = 0$$

$$x^2 + 6x + 16 - 4x - 24 = 0$$

$$x^2 + 2x - 8 = 0$$

$$(x+4)(x-2) = 0$$

roots: $x = -4$ and $x = 2$

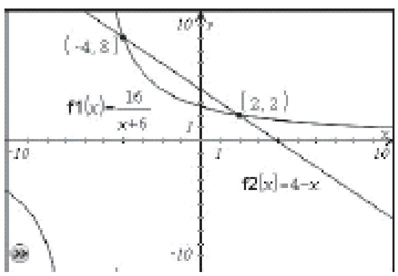
- **Graphically**
  There are two methods for solving equations graphically.

  Method 1: Use a System of Two Functions
  – Graph each side of the equation on the same set of axes.
  – The solution(s) will be the x-coordinate(s) of any point(s) of intersection.

  Method 2: Use a Single Function
  – Rearrange the equation so that one side is equal to zero.
  – Graph the corresponding function.
  – The solution(s) will be the x-intercept(s).

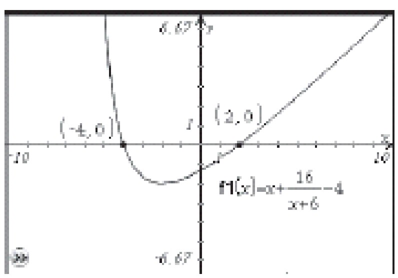
Example: $\frac{16}{x+6} = 4 - x$

Graph $y = \frac{16}{x+6}$ and $y = 4$ on the same axes.



The points of intersection are (−4, 8) and (2, 2), so the roots are $x = -4$ and $x = 2$.

Graph $y = x + \frac{16}{x+6} - 4$.



x-intercepts: $x = -4$ and $x = 2$

Working Example 1: Relate Roots and *x*-Intercepts

• roots = solution determined algebraically

• x-int on the graph are the same solutions/roots.

a) Find the solution to $\frac{3}{x} = 1 + \frac{x-13}{6}$ algebraically.

b) Verify your solutions graphically.

Solution

$\frac{3}{x} = 1 + \frac{x-13}{6}$

$0 = 1 + \frac{x-13}{6} - \frac{\boxed{3}}{\boxed{x}}$, $x \neq 0$ State restrictions and equate to 0. R or NPV $x \neq 0$

$0 = (\underline{6x})(1) + (\underline{\cancel{6}x})\left(\frac{x-13}{\cancel{6}}\right) - (\underline{6\cancel{x}})\left(\frac{\boxed{3}}{\boxed{\cancel{x}}}\right)$ Multiply by LCD. LCD = Lowest Common Denominator (usu. one of each diff. denom.)

$0 = (\underline{6x}) + (\underline{x^2 - 13x}) - (\underline{18})$ → $6x + x^2 - 13x - 18$

$0 = \underline{x^2 - 7x - 18}$ Combine like terms. -18; $-9, +2 = -7$

$0 = (\underline{x-9})(\underline{x+2})$ Factor.

$(\underline{x-9}) = 0$ or $(\underline{x+2}) = 0$

$x = \underline{9}$ $x = \underline{-2}$ Solve.

Is either value a non-permissible value? NO

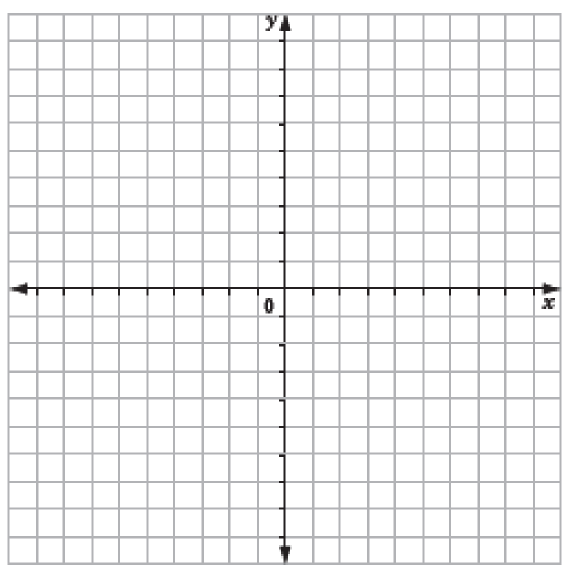
Check by substitution: Skip → check NPV (R) ∴ if $x \neq 0$ then $\boxed{x = -2, 9}$

SKIP

b) Use technology to graph the original function using a single function or a system of two functions. Sketch the graph on the grid. Label the asymptote(s). If you used two functions, label the point(s) of intersection. If you used one function, label the *x*-intercept(s).

$x =$ ____________

Compare the solution(s) obtained algebraically and graphically. What can you conclude?



To see a similar example, see Example 1 on pages 459–460 of *Pre-Calculus 12*.

To see a similar example, see Example 1 on pages 459–460 of *Pre-Calculus 12*.

Working Example 2: Compare the Accuracy of Graphical and Algebraic Solutions

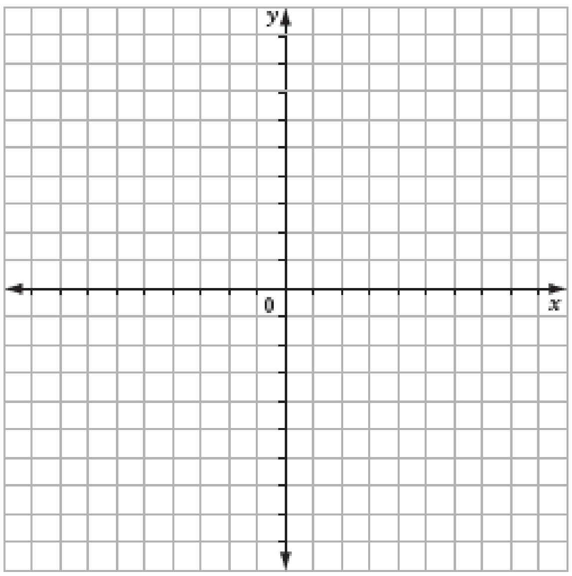
a) Solve $\frac{3}{2x} - \frac{2x}{x+1} = -2$ graphically, using either one or two functions. (Use a different graphical method than the one you used in Working Example 1.)

b) Verify your solution(s) algebraically.

Solution

Use technology to graph the function, using either a single function or a system of two functions. Sketch the graph below. Label the asymptote(s) and the point(s) of intersection or the x-intercept(s).

SKIP



b) Solve algebraically. Describe your solution strategy to the right of each step.

LCD $2x(x+1)$

$$\frac{3}{2x} - \frac{2x}{x+1} = -2$$

① factor any denominators if possible

② Determine NPV + LCD

③ Mutiply all terms by LCD

④ solve for x + check Restrictions.

NPV $x \neq 0, -1$

$\cancel{2x}(x+1)\left[\frac{3}{\cancel{2x}}\right] - (2x\cancel{(x+1)})\left[\frac{2x}{\cancel{x+1}}\right] = (2x(x+1))[-2]$

OR $2x(x+1)\left[\frac{3}{\cancel{2x}} - \frac{2x}{\cancel{x+1}} = -2\right]$

$3x + 3 - 4x^2 = -4x(x+1)$

$3x + 3 \cancel{-4x^2} = -4x^2 - 4x$

$+4x$ $\cancel{+4x^2}$ -3

Compare the solutions you obtained graphically and algebraically. Which is more accurate?

$7x = -3$

$\boxed{x = -\frac{3}{7}}$ ✓ $x \neq -1, 0$

To see a similar example, see Example 2 on pages 460–461 of *Pre-Calculus 12*.

Working Example 3: Solve a Rational Equation With an Extraneous Root

Solve $\frac{x}{x-1} - 2x = \frac{x+1}{2x-2}$ algebraically and graphically. Compare the solutions.

Solution

List the restriction(s) of the function.

① Factor Denom 1st
② LCD, NPV (R)
③ Multiply by LCD ⊢
④ solve
⑤ check R

Solve algebraically.

$$\frac{x}{x-1} - 2x = \frac{x+1}{2x-2} \quad ①$$

$$2(x-1)\left[\frac{x}{\cancel{x-1}} - 2x = \frac{x+1}{2(x-1)}\right] \quad ② \; 2(x-1) \to x \neq 1$$

$$2x - 4x(x-1) = x+1$$

$$2x - 4x^2 + 4x = x + 1$$

$$-4x^2 + 5x - 1 = 0$$

$$-(4x^2 - 5x + 1) = 0$$

$$-(4x-1)(x-1) = 0$$

$$x = \frac{1}{4} \qquad x \neq 1 \text{ extraneous}$$

AC = 4, $-4, -1$ = -5

Decomposition

$4x^2 - 4x / - x + 1$

$4x(x-1) - (x-1)$

$(4x-1)(x-1)$

SHORT-CUT

$\left(\frac{4x}{4} - \frac{4}{4}\right)(4x-1)$

$(x-1)(4x-1)$

Check the solution(s) against the restriction(s).

Check your solution in the original equation.

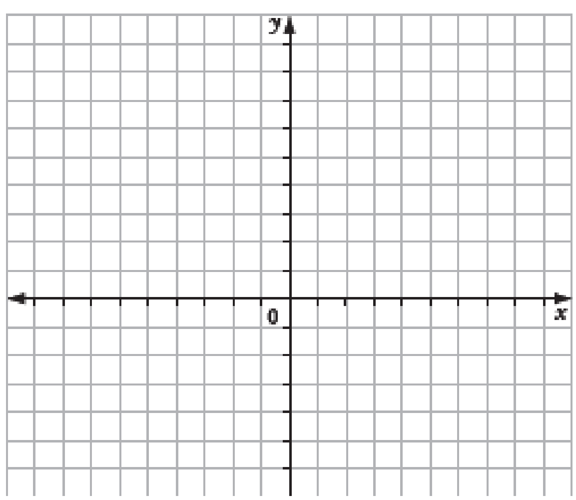
only solution

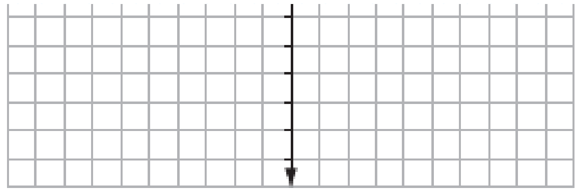
$$\boxed{x = \frac{1}{4}}$$

Use technology to graph the function using either graphical method. Sketch the graph on the grid. Label the asymptote(s) and point(s) of intersection or x-intercept(s).

$x =$ ________________

Compare the solution you obtained algebraically to the one you obtained graphically. What do you notice?





To see a similar example, see Example 3 on pages 462–463 of *Pre-Calculus 12*.

Solving Rational Equations - Always do restrictions and/or checks (asymptotes or PODs)

How to solve Rational Equations?

1. Factor all the denominators.
2. Find the least common denominator (LCD).
3. Multiply each side of the equation by the LCD to cancel out the denominator.
4. Solve the equation.
5. Check solution(s). Reject any solution that makes the denominator = 0.

Example:

$x \neq 0$

$x \neq 8$

$$\frac{1}{x}+\frac{1}{x-8}=\frac{1}{3}$$

LCD = 3x(x-8)

$$\frac{1}{\cancel{x}}\cdot 3\cancel{x}(x-8)+\frac{1}{\cancel{x-8}}\cdot 3x\cancel{(x-8)}=\frac{1}{\cancel{3}}\cdot \cancel{3}x(x-8)$$

$$3(x-8)+3x=x(x-8)$$

$$3x-24+3x=x^2-8x$$

$$x^2-14x+24=0$$

$$(x-12)(x-2)=0$$

$$x=12 \text{ or } x=2$$

Check the solutions.

Rational Equations Restrictions

Consider this equation: $\frac{2x+2}{x-5}=x-7$

Let's multiply both sides by x–5: $\frac{(x-5)}{}\cdot\frac{2x+2}{(x-5)}=(x-5)(x-7)$

Cross-out and multiply: $\cancel{(x-5)}\cdot\frac{2x+2}{\cancel{(x-5)}}=x^2-12x+35$

Hey, now it's quadratic!

: $2x+2=x^2-12x+35$

Subtract 2x and 2 from both sides: $0=x^2-14x+33$

Factor: $0=(x-11)(x-3)$

Apply the zero-factor property: *Either* $(x-11)=0$ *or* $(x-3)=0$

Solve *Either* $x=11$ *or* $x=3$ $x \neq 5$

$$3x\cdot\left(\frac{5}{x}-\frac{1}{3}\right)=3x\cdot\left(\frac{1}{x}\right)$$ *Multiply both sides by the LCD.*

$$3x\cdot\frac{5}{x}-3x\cdot\frac{1}{3}=3x\cdot\frac{1}{x}$$ *Distribute.*

$$15-x=3$$ *Simplify, and then solve.*

$$-x=-12$$
$$x=12$$

a) $\dfrac{5}{6x^2-9x}=\dfrac{1}{2x-3}-\dfrac{2}{3x}$

$$\frac{5}{3x(2x-3)}=\frac{1}{2x-3}-\frac{2}{3x}\Bigg|\cdot 3x(2x-3)$$
$$5=3x-4x+6$$
$$x=1$$

$x\neq 0 \quad x\neq \frac{3}{2}$

b) $\dfrac{24}{x^2-x-6}-\dfrac{x-1}{x+2}=\dfrac{x+3}{3-x}$

$$\frac{24}{(x+2)(x-3)}-\frac{x-1}{x+2}=-\frac{x+3}{x-3}\Bigg|\cdot(x+2)(x-3)$$
$$24-(x-1)(x-3)=-(x+3)(x+2)$$
$$24-x^2+4x-3=-x^2-5x-6$$
$$9x=-27,$$
$$x=-3.$$

$$\frac{t-2}{t+2}+\frac{1}{t-2}=\frac{4}{t^2-4}$$
$$\frac{t-2}{t+2}+\frac{1}{t-2}=\frac{4}{(t+2)(t-2)}$$
$$(t-2)(t+2)\left[\frac{t-2}{t+2}+\frac{1}{t-2}=\frac{4}{(t+2)(t-2)}\right]$$
$$(t-2)(\cancel{t+2})\left(\frac{t-2}{\cancel{t+2}}\right)+(t-2)(t+2)\left(\frac{1}{t-2}\right)=(t-2)(\cancel{t+2})\left(\frac{4}{(\cancel{t+2})(t-2)}\right)$$
$$(t-2)(t-2)+(t+2)=4$$
$$t^2-3t+6=4$$
$$t^2-3t+2=0$$
$$(t-2)(t-1)=0$$

Does not satisfy original equation

$$\cancel{t=2}\ \ or\ \ t=1$$
$$t=1$$

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$$\frac{1}{x}-1=\frac{3}{4x}\qquad \text{LCD}=4x$$
$$4x\bullet\left(\frac{1}{x}-1\right)=\left(\frac{3}{4x}\right)\bullet 4x$$
$$4\cancel{x}\bullet\frac{1}{\cancel{x}}-4x\bullet 1=\left(\frac{3}{\cancel{4x}}\right)\bullet\cancel{4x}$$
$$4-4x=3$$
$$-4x=-1$$
$$x=\frac{1}{4}$$

$$(x-2)(x+2)\cdot\left(\frac{8}{x+2}+\frac{8}{x-2}\right)=3(x-2)(x+2)$$
$$(x-2)(x+2)\cdot\frac{8}{x+2}+(x-2)(x+2)\cdot\frac{8}{x-2}=3(x-2)(x+2)$$
$$8(x-2)+8(x+2)=3(x^2-4)$$
$$8x-16+8x+16=3x^2-12$$
$$16x=3x^2-12$$
$$0=3x^2-16x-12$$

SOLVE FOR t:

$\frac{1}{t}+\frac{2}{t}=\frac{3}{5}$ $\quad t\neq 0$ | $\frac{1}{t}+\frac{2}{3t}=\frac{3}{5}\longrightarrow t\neq 0$

LCD: $3t$

SOLVE FOR t:

$\frac{1}{t}+\frac{2}{t}=\frac{3}{5}$ $\quad t\neq 0$ LCD: $5t$ | $\frac{1}{t}+\frac{2}{3t}=\frac{3}{5}$ $\quad t\neq 0$ LCD: $15t$

SOLVE FOR t:

LCD: $t(t+3)$ $\quad \frac{1}{t+3}+\frac{2}{t}=\frac{-3}{t(t+3)}$ $\quad t+3=0$ $\boxed{t\neq -3 \;\; t\neq 0}$

SOLVE FOR t:

$$\frac{1}{t}+\frac{2}{t}=\frac{3}{5}$$

$$t \neq 0$$

$$\frac{1+2}{t}=\frac{3}{5}$$

$$\frac{3}{t}=\frac{3}{5}$$

$$\boxed{t=5}$$

$$\frac{1}{t}+\frac{2}{3t}=\frac{3}{5} \longrightarrow t \neq 0$$

$$\frac{1}{t}\cdot\frac{3}{3}+\frac{2}{3t}=\frac{3}{5} \qquad \text{LCD: } 3t$$

$$\frac{3}{3t}+\frac{2}{3t}=\frac{3}{5}$$

$$\frac{5}{3t} \times \frac{3}{5} \longrightarrow \frac{25}{9}=\frac{\cancel{9}t}{\cancel{9}} \qquad \boxed{\frac{25}{9}=t}$$

SOLVE FOR t:

$$\frac{1}{t}+\frac{2}{t}=\frac{3}{5} \qquad t \neq 0 \quad \text{LCD: } 5t$$

$$5t\left(\frac{1}{t}+\frac{2}{t}\right)=5t\cdot\frac{3}{5}$$

$$5\cancel{t}\cdot\frac{1}{\cancel{t}}+5\cancel{t}\cdot\frac{2}{\cancel{t}}=\cancel{5}t\cdot\frac{3}{\cancel{5}}$$

$$\frac{15}{3}=\frac{\cancel{3}t}{\cancel{3}} \qquad \boxed{5=t}$$

$$\frac{1}{t}+\frac{2}{3t}=\frac{3}{5} \qquad t \neq 0 \quad \text{LCD: } 15t$$

$$15t\left(\frac{1}{t}+\frac{2}{3t}\right)=15t\cdot\frac{3}{5}$$

$$15\cancel{t}\cdot\frac{1}{\cancel{t}}+\overset{5}{\cancel{15}}\cancel{t}\cdot\frac{2}{\cancel{3}\cancel{t}}=\overset{3}{\cancel{15}}t\cdot\frac{3}{\cancel{5}}$$

$$\frac{25}{9}=\frac{\cancel{9}t}{\cancel{9}} \qquad \boxed{\frac{25}{9}=t}$$

SOLVE FOR t:

$$\text{LCD: } t(t+3) \qquad \frac{1}{t+3}+\frac{2}{t}=\frac{-3}{t(t+3)} \qquad t+3=0$$

$$\boxed{\begin{matrix} t \neq -3 \\ t \neq 0 \end{matrix}}$$

$$\frac{1}{t+3}\cdot\frac{t}{t}+\frac{2}{t}\cdot\frac{t+3}{t+3}=\frac{-3}{t(t+3)}$$

$$\frac{t}{t(t+3)}+\frac{2(t+3)}{t(t+3)}=\frac{-3}{t(t+3)}$$

$$\frac{t+2(t+3)}{t(t+3)}=\frac{-3}{t(t+3)}$$

$$t+2t+6=-3$$

$$3t+\cancel{6}=-3$$

$$\quad\ \cancel{-6}\quad -6$$

$$\frac{\cancel{3}t}{\cancel{3}}=\frac{-9}{3}$$

$$t \neq -3$$

Megan and her friends are organizing a fundraiser for the local children's hospital. They are asking local businesses to each donate a door prize. So far, they have asked nine businesses, but only one has donated a prize. Their goal was to have three quarters of the businesses donate. If they succeed in getting every business to donate a prize from now on, how many more businesses do they need to ask to reach their goal?