CHAPTER 1

[redacted] Practice Review Questions

KEY

Geometric Sequences & Series

*Note: Arithmetic 1.1-1.2 is missing - please refer to Workbook + check-in quiz

1. Determine the common ratio of the geometric sequence 8, 12, 18, 27, . . .

$$r = \frac{12}{8} = \boxed{\frac{3}{2}}$$

2. The general term of a geometric sequence is $t_n = 8(-3)^{n-1}$. Determine the common ratio.

n = term number

$a = 8$

$\boxed{r = -3}$

3. Calculate the 12th term of the geometric sequence: 5, 15, 45, . . .

$a = 5$

$r = \frac{15}{5} = 3$

$$t_{12} = ar^{n-1}$$

$$n = 12 \quad = 5(3)^{12-1}$$

$$= 5(3)^{11} \rightarrow 5(177147)$$

$$\boxed{t_{12} = 885{,}735}$$

4. Which term of the geometric sequence 5, 15, 45, . . . is 885 735?

$n = ?$

$$t_n = ar^{n-1}$$

$$\frac{885735}{5} = \frac{5(3)^{n-1}}{5}$$

$$177147 = 3^{n-1}$$

Common base method

$$3^{11} = 3^{n-1}$$

$$11 = n-1$$

+1

$$n = 12 \rightarrow \boxed{t_{12} \text{ is } 12^{th} \text{ term}}$$

log method

$$\frac{\log 177147}{\log 3} = \frac{(n-1)\log 3}{\log 3}$$

$$\frac{\log 177147}{\log 3} = n-1$$

+1

$$n = \frac{\log 177147}{\log 3} + 1$$

$\boxed{n = 12}$ t_{12}

5. Determine the number of terms in the geometric sequence:

$$\frac{1}{128}, \frac{1}{32}, \frac{1}{8}, \ldots, 2048$$

$a = \frac{1}{128}$

$n = ?$

$2048 = ar^{n-1}$

$\frac{2048}{\frac{1}{128}} = \frac{\frac{1}{128}(4)^{n-1}}{\frac{1}{128}}$

$2048 \cdot \frac{128}{1} = 4^{n-1}$

$262144 = 4^{n-1}$

$\cancel{4}^{9} = \cancel{4}^{n-1}$

$9 = n-1$

$+1$

$n = 10$

$\boxed{10 \text{ terms}}$

$r = \frac{\frac{1}{32}}{\frac{1}{128}}$

$= \frac{1}{\cancel{32}_{1}} \times \frac{\cancel{128}^{4}}{1}$

$r = 4$

6. The second term of a geometric series is -16 and the seventh term is 512. Determine the first term.

$t_2 = -16$

$t_7 = 512$

$\frac{t_7}{t_2} \longrightarrow \frac{\cancel{a}r^6}{\cancel{a}r} = \frac{512}{-16}$

$\sqrt[5]{r^5} = \sqrt[5]{-32}$

$r = -2 \longrightarrow t_2 \Rightarrow ar = -16$

$\frac{a(-2)}{-2} = \frac{-16}{-2}$

$\boxed{a = 8}$

7. The 3^{rd} term of a geometric sequence is 48 and the 6^{th} term is $\frac{81}{4}$. Find the 1^{st} term of the sequence.

$t_3 = 48 \quad t_6 = \frac{81}{4}$

$$\frac{ar^5}{ar^2} = \frac{\frac{81}{4}}{48}$$

reciprocal

$$= \frac{81}{4} \times \frac{1}{48}$$

$$\sqrt[3]{r^3} = \sqrt[3]{\frac{27}{64}}$$

$\sqrt[3]{27} = 3$

$\sqrt[3]{64} = 4$

$$r = \frac{3}{4}$$

t_3

$ar^2 = 48$

$a\left(\frac{3}{4}\right)^2 = 48$

$\frac{9}{16}a = 48 \div \frac{9}{16}$

$a = \cancel{48}^{16} \times \frac{16}{\cancel{9}_3}$

$\boxed{a = \frac{256}{3}}$

$t_1 = \frac{256}{3}$

8. A geometric sequence of positive terms has $t_1 = 320$ and $t_7 = 78\ 125$. Find t_4.

$a = 320 \qquad t_7 = ar^6 = 78125$

$$\frac{320r^6}{320} = \frac{78125}{320}$$

$\frac{78125 \div 5}{320 \div 5} = \frac{15625}{64}$ no factors

$4 \cdot 4 \cdot 4$

$$\sqrt[6]{r^6} = \sqrt[6]{\frac{15625}{64}}$$

$$r = \pm\frac{5}{2} \rightarrow t_4 = ar^3$$

$$= 320\left(\pm\frac{5}{2}\right)^3$$

$$t_4 = \pm \cancel{320}^{40} \cdot \frac{125}{\cancel{8}_1}$$

$$t_4 = \pm 5000$$

only positive as stated in question

$\therefore$ final answer $\boxed{t_4 = 5000}$

omit

9. For a geometric sequence, $t_7 = 5x + 2$ and $t_{10} = x - 23$. If the common ratio, r, is 2, find the value of t_{10}.

$r=2$

$$\frac{t_{10}}{t_7} \rightarrow \frac{a(2)^9}{a(2)^7} = \frac{x-23}{5x+2}$$

$$(5x+2)\ 2^2 = \frac{x-23}{5x+2}$$

$$20x+8 = x-23$$

$$-x \qquad -8$$

$$\frac{19x}{19} = \frac{-31}{19}$$

$$x = -\frac{31}{19}$$

$$t_{10} = x-23$$

$$t_{10} = -\frac{31}{19} - 23$$

$$= -\frac{31}{19} - \frac{437}{19}$$

$$\boxed{t_{10} = \frac{468}{19}}$$

10. If $x, 4, 8x$ are three consecutive terms in a geometric sequence, determine the values of x.

$$r = \frac{t_2}{t_1} = \frac{t_3}{t_2}$$

$$\frac{4}{x} = \frac{8x}{4}$$

solve

$$\frac{16}{8} = \frac{8x^2}{8}$$

$$\sqrt{2} = \sqrt{x^2}$$

$$\boxed{x = \pm\sqrt{2}}$$

10.2

11. If the sum of the first 5 terms of a geometric series is -328 and the common ratio is -4, determine the first term.

$n=5$ $\quad S_5=-328$ $\quad r=-4$

S formula. $S_n=\dfrac{a(1-r^n)}{1-r}$

$$-328=\frac{a(1-(-4)^5)}{1-(-4)}$$

$$5(-328)=\frac{a(1+1024)}{5}$$

$$\frac{-1640}{1025}=\frac{1025a}{1025}$$

$$a=-\frac{1640}{1025}\begin{matrix}\div 5\\ \div 5\end{matrix}$$

$$a=-\frac{328}{205}\begin{matrix}\div 2\div 2\div 2\div 41\\ \div 41\end{matrix}$$

$\boxed{a=-\frac{8}{5}}$ 1st term

12. Determine the sum of the first 10 terms of the geometric sequence -4, 6, -9, . . ., to the nearest tenth.

$a=-4$ $\quad r=\frac{6}{-4}$ $\quad r=-\frac{3}{2}$

$$S_{10}=\frac{a(1-r^n)}{1-r}$$

$$=\frac{-4\left(1-\left(-\frac{3}{2}\right)^{10}\right)}{1-\left(-\frac{3}{2}\right)}$$

$$=\frac{-4\left(1-\frac{59049}{1024}\right)}{\frac{2}{3}+\frac{3}{2}}$$

$$=\frac{-4\left(\frac{1024}{1024}-\frac{59049}{1024}\right)}{\frac{5}{2}}$$

$$S_{10}=\frac{-4\left(\frac{-58025}{1024}\right)}{\frac{5}{2}}$$

$$=\frac{\cancel{232100}^{46420}}{\cancel{1024}_{512}}\times\frac{\cancel{2}}{\cancel{5}}$$

$$=\frac{4640}{512}\quad \begin{matrix}\leftarrow 32\cdot 145\\ \leftarrow 32\cdot 16\end{matrix}$$

$$\boxed{S_{10}=\frac{145}{16}}$$

to nearest 10th $\boxed{S_{10}=9.1}$

13. A doctor prescribes medication to be taken for 7 days. The amount taken on the first day is 310 mg. On each successive day, the amount taken is one half the amount taken on the previous day. What is the total amount of medication taken? (Accurate to the nearest mg.)

$$\left.\begin{array}{l} a = 310 \\ r = 0.5 \\ n = 7 \end{array}\right\}$$

$$S_7 = \frac{310(1-0.5^7)}{1-0.5}$$

$$= \frac{310(0.9921875)}{0.5}$$

$$S_7 = 615.15625 \text{ mg}$$

$$\boxed{S_7 = 615 \text{ mg}} \text{ taken}$$

14. Jim worked for a company for 8 years. His starting annual salary was $32 000. Each year his salary increased by 2% over the previous year's salary. What is the total amount of money Jim earned with this company?

$r = 1 + 0.02$ $n = 8$ $a = 32000$

$r = 1.02$

$$S_8 = \frac{32000(1-(1.02)^8)}{1-1.02}$$

$$= \frac{32000(1-1.02^8)}{-0.02}$$

$$\boxed{S_8 = \$274,655.01} \text{ earned over the 8 years.}$$

a=30

$r = 1 - 0.06$

$r = 0.94$

15. An aquarium originally containing 30 liters of water loses 6% of its water to evaporation every day. Determine a geometric sequence which shows the number of liters of water in the aquarium on 5 consecutive days.

5 terms

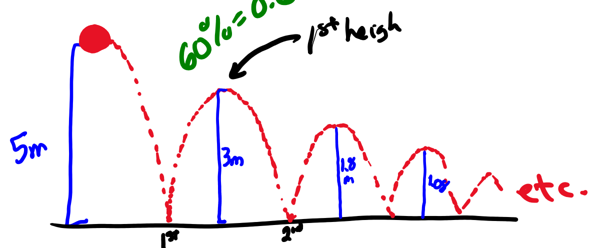
$t_1 = a$, $t_2 = ar^1$, $t_3 = ar^2$, $t_4 = ar^3$, $t_5 = ar^4$

$30, \; 30(0.94), \; 30(0.94)^2, \; 30(0.94)^3, \; 30(0.94)^4$

$\boxed{= 30\,L, \; 28.2\,L, \; 26.508\,L, \; 24.91752\,L, \; 23.4224688\,L}$

OK = 30L, 28.2 L, 26.5 L, 24.9 L, 23.4 L

16. A ball is dropped from a height of 5 m. After each bounce, it rises to 60% of its previous height. What is the total vertical distance the ball travels before it comes to rest?



$5 + \underbrace{6 + 3.6 + 2.16}_{\text{geometric}}$

$S_\infty = \dfrac{6}{1 - 0.6}$

$= 15\,m + 5$

$\boxed{S_\infty = 20\,m}$

IF: what hieght did the ball reach at 10th bounce

5, 3, 1.8, 1.08

a=3 n=10

r=0.6

$t_{10} = 3(0.6)^{10-1}$

$\boxed{= 0.03\,m}$

17. Determine the sum of the infinite geometric series: $3 - 1 + \frac{1}{3} - \frac{1}{9} + \ldots$

$a = 3 \quad r = \frac{-1}{3}$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{3}{1-\left(-\frac{1}{3}\right)}$$

$$= \frac{3}{\frac{3}{3}+\frac{1}{3}}$$

$$= \frac{3}{\frac{4}{3}}$$

$$S_\infty = 3 \times \frac{3}{4}$$

$$\boxed{S_\infty = \frac{9}{4}}$$

omit 18. For what values of x will the following infinite geometric series have a finite sum?

$(x + 1) + (x + 1)^2 + (x + 1)^3 + \ldots$

$$-1 < r < 1$$

$$-1 < \frac{(x+1)^2}{x+1} < 1$$

$$-1 < x+1 < 1$$

$-1 \quad -1$

$$\boxed{\begin{matrix} -2 < x < 0 \\ x \neq -1 \end{matrix}}$$

Check ex: $x = -0.5$

$(-0.5+1), (-0.5+1)^2, (-0.5+1)^3, \ldots$

$0.5, \ 0.25, \ 0.125, \ldots$

infinite series ✓

omit

19. For what values of x will the following infinite geometric series have a finite sum?

$(x-4)+(x-4)^2+(x-4)^3+\ldots$

$-1 < r < 1$

$-1 < \frac{(x-4)^2}{x-4} < 1$

$-1 < x-4 < 1$

+4 +4

$\boxed{3 < x < 5 \quad x \neq 4}$

check. $x-4$, $(x-4)^2$, $(x-4)^3$, ...

ex: 4.5 4.5 4.5

$(4.5-4)$, $(4.5-4)^2$, $(4.5-4)^3$

0.5, $(0.5)^2$, $(0.5)^3$

0.5, 0.25, 0.125, ...

Infinite series. ✓

20. Determine the sum of geometric series: $2-1+\frac{1}{2}-\frac{1}{4}+\ldots$

$a=2$ $r=-\frac{1}{2}$ $n=\infty$

$S_\infty = \frac{a}{1-r}$

$= \frac{2}{1-(-\frac{1}{2})}$

$= \frac{2}{\frac{2}{2}+\frac{1}{2}}$

$= \frac{2}{\frac{3}{2}}$

$= 2 \times \frac{2}{3}$

$\boxed{S_\infty = \frac{4}{3}}$

21. If the sum of an infinite geometric series is 90 and the common ratio is $-\frac{1}{5}$, determine the value of the first term.

$S_\infty = 90$ $\quad r = -\frac{1}{5}$

$a = ?$

$$S_\infty = \frac{a}{1-r}$$

$$90 = \frac{a}{1-\left(-\frac{1}{5}\right)}$$

$$90 = \frac{a}{\frac{5}{5}+\frac{1}{5}}$$

$$90 = \frac{a}{\frac{6}{5}}$$

$$\cancel{90}^{18}\left(\frac{6}{\cancel{5}_{1}}\right) = a$$

$$\boxed{a = 108}$$

22. Evaluate the following: ← means determine sum

$$\sum_{k=2}^{6} 4(3)^k$$

1st term $\quad$ 2nd term $\quad$ 3rd term

$a =$

$r =$

$n =$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$k=2$ $\quad$ 5 $\quad$ 6

$4(3)^2 + 4(3)^3 + 4(3)^4$

$4(9) + 4(27) + 4(81)$

$36 + 108 + 324 + \ldots \_\_\_ \; \_\_\_ = 4356$

$a = 36$ $\quad r = \frac{108}{36} = 3$ $\quad$ check $r = \frac{324}{108} = 3$

$\underline{n = 5}$

$$S_5 = \frac{36(1-3^5)}{1-3}$$

$$= \frac{36(-242)}{-2} \Rightarrow \boxed{4356}$$

$$\sum_{k=2}^{6} 4(3)^k = \boxed{4356}$$

23. Evaluate the following:

$$\sum_{k=1}^{\infty} 50\left(\frac{1}{4}\right)^{k}$$

$a = 50\left(\frac{1}{4}\right)^{1}$ $\quad t_2 = 50\left(\frac{1}{4}\right)^{2}$

$a = \frac{50}{4}$ $\quad = \frac{50}{16}$

$\underbrace{a = \frac{25}{2} \quad t_2 = \frac{25}{8}}$

$r = \dfrac{\frac{25}{8}}{\frac{25}{2}} \rightarrow r = \frac{\cancel{25}}{\cancel{8}_4} \times \frac{\cancel{2}}{\cancel{25}}$

$r = \frac{1}{4}$

$S_\infty = \dfrac{\frac{25}{2}}{\left(\frac{4}{4}\right)1 - \frac{1}{4}} \rightarrow S_\infty = \frac{25}{\cancel{2}} \times \frac{\cancel{4}^{2}}{3}$

$= \dfrac{\frac{25}{2}}{\frac{3}{4}}$ $\quad \boxed{S_\infty = \frac{50}{3}}$

24. Write an expression to represent the sum of the series given by

$$\sum_{k=0}^{15} 16(2)^{k+1}$$

① $n = 15 - 0 + 1$

$n = 16$

② $t_1 = 16(2)^{0+1}$ ③ $t_2 = 16(2)^{1+1}$

$= 16(2)$ $\quad = 16(2)^2$

$a = t_1 = 32$ $\quad t_2 = 64$

④ $r = \frac{64}{32}$

$r = 2$

⑤ $S_{16} = \dfrac{32(1 - 2^{16})}{1 - 2}$

$S_{16} = \dfrac{32(1 - 65536)}{-1}$

$= -32(-65535)$

$\boxed{S_{16} = 2{,}097{,}120}$

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25. Write using sigma notation: $\frac{3}{16} + \frac{3}{8} + \frac{3}{4} \ldots 1536$
& evaluate

① $a = \frac{3}{16}$

② $r = \dfrac{\frac{3}{8}}{\frac{3}{16}}$

$= \cancel{\frac{3}{8}} \times \frac{\cancel{16}^{2}}{\cancel{3}}$

$r = 2$

determine 'n' to get number of terms in series.

③ $t_n = ar^{n-1}$

$\dfrac{1536}{\frac{3}{16}} = \dfrac{\frac{3}{16}(2)^{n-1}}{\frac{3}{16}}$

$1536 \times \frac{16}{3} = 2^{n-1}$

$8192 = 2^{n-1}$

$\cancel{2}^{13} = \cancel{2}^{n-1}$

$13 = n-1$

$+1$

$n = 14$

④ $\sum_{k=1}^{14} ar^{k-1}$

$= \boxed{\sum_{k=1}^{14} \frac{3}{16}(2)^{k-1}}$

⑤ evaluate means determine sum

$S_{14} = \dfrac{\frac{3}{16}(1-2^{14})}{1-2}$

$= \dfrac{\frac{3}{16}(-16383)}{-1}$

$\boxed{S_{14} = \frac{49149}{16}}$

26. $-\frac{10}{3}+\frac{5}{3}-\frac{5}{6}+\frac{5}{12}+\dots$

write in sigma notation

+evaluate

① $a=-\frac{10}{3}$

② $r=\dfrac{\frac{5}{3}}{-\frac{10}{3}}$

$r=\frac{5}{3}\times-\frac{3}{10}$

$r=-\frac{1}{2}$

③ $n=\infty$

④ $\boxed{\sum_{k=1}^{\infty}-\frac{10}{3}\left(-\frac{1}{2}\right)^{k-1}}$

⑤ sum

$S_\infty=\dfrac{-\frac{10}{3}}{1-\left(-\frac{1}{2}\right)}$

$=\dfrac{-\frac{10}{3}}{\frac{2}{2}+\frac{1}{2}}$

$=\dfrac{-\frac{10}{3}}{\frac{3}{2}}$

$=-\frac{10}{3}\times\frac{2}{3}$

$\boxed{S_\infty=-\frac{20}{9}}$