

KEY Jan 2023

Pre-Calculus 12: Chapter 3 Practice Questions: Polynomial Functions

1. Complete the following table for each function: (2 marks)

Characteristic	$f(x) = 3x^4 - x^3 - 13x^2 + 7x + 9$	$f(x) = -5x^3 + 6x^2 - 11$
Leading coefficient (+ or -)	+ (+3)	- (-5)
Degree (odd/even)	even (4)	odd (3)
End Behavior (ex. Up into QII and down into QIV; Down into QIII and QIV)	up into QII + up into QI	up into QII + down into QIII
Possible number of x-intercepts	0 - 4	1 - 3
Possible number of turns	1 or 3	0 or 2
Coordinate of y-intercept	(0, 9)	(0, -11)
Domain	$\{x \mid x \in \mathbb{R}\}$	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq MN, y \in \mathbb{R}\}$	$\{y \mid y \in \mathbb{R}\}$

2. Use long division to find the quotient of the following. Write the answer as a division statement. (1 mark)

$$(2x^3 + 3x^2 - 5x + 2) \div (x + 3)$$

$$\begin{array}{r}
 2x^2 - 3x + 4 \\
 x + 3 \overline{) 2x^3 + 3x^2 - 5x + 2} \\
 \underline{-(2x^3 + 6x^2)} \\
 -3x^2 - 5x \\
 \underline{-(-3x^2 - 9x)} \\
 4x + 2 \\
 \underline{-(4x + 12)} \\
 -10
 \end{array}$$

Quotient + Remainder
 $2x^2 - 3x + 4 - \frac{10}{x+3}$

Division Statement

$$\frac{2x^3 + 3x^2 - 5x + 2}{x + 3} = 2x^2 - 3x + 4 - \frac{10}{x + 3}$$

3. Use synthetic division to determine the quotient of the following. Write the answer as a division statement. (1 mark)

$$\frac{x^3 + 2x^2 + 9}{x + 3}$$

$$\begin{array}{r|rrrrr}
 x+3 & & x^3 & +2x^2 & +0x & +9 \\
 3 & & 1 & 2 & 0 & 9 \\
 - & & & 3 & -3 & 9 \\
 \hline
 x & & 1 & -1 & 3 & 0 \\
 & & x^2 & -x & +3 &
 \end{array}$$

no remainder

Division Statement

$$\frac{x^3 + 2x^2 + 9}{x + 3} = x^2 - x + 3$$

4. Using the remainder theorem, determine the remainder of the following: (1 mark each)

a. $(3x^4 - x^3 + 2x^2 + 7x - 11) \div (x - 2)$

$x=2$

$$R = 3(2)^4 - (2)^3 + 2(2)^2 + 7(2) - 11$$

$$R = 48 - 8 + 8 + 14 - 11 \longrightarrow \boxed{R = 51}$$

b. $(-4x^2 - 2x + 17) \div (x + 3)$

$x=-3$

$$R = -4(-3)^2 - 2(-3) + 17$$

$$= -36 + 6 + 17 \longrightarrow \boxed{R = -13}$$

5. Determine the value of k in the following if the remainder is 7: (1 mark)

$(kx^3 + 3x + 1) \div (x + 2)$ $\rightarrow x = -2$

$$k(-2)^3 + 3(-2) + 1 = 7$$

$$-8k - 6 + 1 = 7$$

$$-8k - 5 = 7$$

$$\begin{array}{r} -8k = 12 \\ \underline{-8} \quad \underline{-8} \end{array}$$

$$\longrightarrow \boxed{k = -\frac{3}{2}}$$

6. Determine the value of k if the binomial $x - 1$ is a factor of $x^3 + kx^2 + 4x - 2$. (1 mark)

$x=1$ $R=0$

$$(1)^3 + k(1)^2 + 4(1) - 2 = 0$$

$$1 + k + 4 - 2 = 0$$

$$k + 3 = 0$$

$$\boxed{k = -3}$$

7. What are all of the possible integral zeros of the following: (1 mark)

$$f(x) = x^3 - 8x + 14 \rightarrow \pm 1, \pm 2, \pm 7, \pm 14$$

8. Fully factor the following: (3 marks each)

a. $10x^3 + 37x^2 + 37x + 6$

$$P(-2) = 10(-2)^3 + 37(-2)^2 + 37(-2) + 6 = 0 \checkmark$$

$x+2$	$10x^3 + 37x^2 + 37x + 6$
2	10 37 37 6
-	20 34 6
x	10 17 3 0

$10x^2 + 17x + 3$

$$\begin{aligned} 30 & \swarrow \searrow \\ 15, 2 & = 17 \\ (10x^2 + 15)(10x^2 + 2) & \\ \underline{\quad \quad \quad} & \\ (2x + 3)(5x + 1) & \end{aligned}$$

$$(x+2)(2x+3)(5x+1)$$

b. $x^4 + 4x^3 - 7x^2 - 34x - 24$

$$P(-1) = (-1)^4 + 4(-1)^3 - 7(-1)^2 - 34(-1) - 24 = 0 \checkmark$$

$$P(-2) = (-2)^4 + 4(-2)^3 - 7(-2)^2 - 34(-2) - 24 = 0 \checkmark$$

$x+1$	$x^4 + 4x^3 - 7x^2 - 34x - 24$
1	1 4 -7 -34 -24
-	1 3 -10 -24
x	1 3 -10 -24 0

$x+2$	$x^3 + 3x^2 - 10x - 24$
2	1 3 -10 -24
-	2 2 -24
x	1 1 -12 0

$$x^2 + x - 12 \quad \begin{matrix} -12 \\ +4, -3 = 1 \end{matrix}$$

$$(x+4)(x-3)$$

$$(x+1)(x+2)(x+4)(x-3)$$

c. $x^5 - 10x^4 + 22x^3 + 28x^2 - 55x - 50$

$$P(-1) = 0 \rightarrow x+1$$

$$P(2) = 0 \rightarrow x-2$$

$$P(5) = 0 \rightarrow x-5$$

①

$x+1$	$x^5 - 10x^4 + 22x^3 + 28x^2 - 55x - 50$
1	1 -10 22 28 -55 -50
-	1 -11 33 -5 -50
x	1 -11 33 -5 -50 0

②

$x-2$	$x^3 - 11x^2 + 33x - 5$
-2	1 -11 33 -5 -50
-	-2 16 -30 -50
x	1 -9 15 25 0

③

$x-5$	$x^2 - 4x - 5$
-5	1 -9 15 25
-	-5 20 25
x	1 -4 -5 0

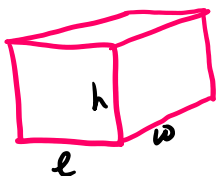
$$x^2 - 4x - 5 \quad \begin{matrix} -5 \\ -5, +1 = -4 \end{matrix}$$

$$(x+1)(x-5)$$

$$= (x+1)(x-2)(x-5)(x+1)(x-5)$$

$$= (x-2)(x+1)^2(x-5)^2$$

9. If a box has a volume represented by $x^3 + 5x^2 - 2x - 24$, determine the dimensions of the box. (2 marks)



$V = l \cdot w \cdot h$ fully factor polynomial

$P(x) = 0$

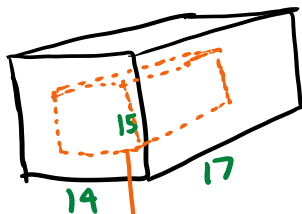
$$\begin{array}{r|rrrr} x-2 & x^3 & +5x^2 & -2x & -24 \\ -2 & 1 & 5 & -2 & -24 \\ - & & -2 & -14 & -24 \\ \hline x & 1 & 7 & 12 & 0 \end{array}$$

$x^2 + 7x + 12$ $\begin{matrix} 12 \\ \swarrow \searrow \\ 4, 3 = 7 \end{matrix}$

$(x+4)(x+3)$

dimensions = $(x-2), (x+4), (x+3)$

10. A rectangular prism of marble stone is used to create a sculpture. Its dimensions are 14 in by 17 in by 15 in. If the carver wants to reduce the volume by 1230 in^3 by removing the same amount from each side, how much should be removed and what are the new dimensions of the prism? (2 marks)



original volume $\rightarrow V = 15 \times 14 \times 17$
 $V = 3570 \text{ in}^3$

subtract

new volume $\rightarrow V = 3570 - 1230$

$V = 2340 \text{ in}^3$

$P(x) = 0$

$$\begin{array}{r|rrrr} x-2 & x^3 & -46x^2 & +703x & -1230 \\ -2 & 1 & -46 & 703 & -1230 \\ - & & -2 & 88 & -1230 \\ \hline x & 1 & -44 & 615 & 0 \end{array}$$

$(x-2)(x^2 - 44x + 615)$

$x = 2$

not factorable \therefore quadratic formula

$x = \frac{-(-44) \pm \sqrt{(-44)^2 - 4(1)(615)}}{2(1)}$

$= \frac{44 \pm \sqrt{-524}}{2}$

$x = \text{no solution}$

① $(14-x)(17-x)(15-x) = 2340$
 $(14-x)(255 - 17x - 15x + x^2) = 2340$
 $(14-x)(255 - 32x + x^2) = 2340$

$3570 - 448x + 14x^2 - 255x + 32x^2 - x^3 = 2340$

$3570 - 703x + 46x^2 - x^3 = 2340$

$0 = x^3 - 46x^2 + 703x - 1230$

③ new dimensions
 $14-2, 15-2, 17-2$
 $12 \times 13 \times 15 \text{ in}$

11. Graph the function showing the steps in the table of values beginning with the base function. (2 marks)

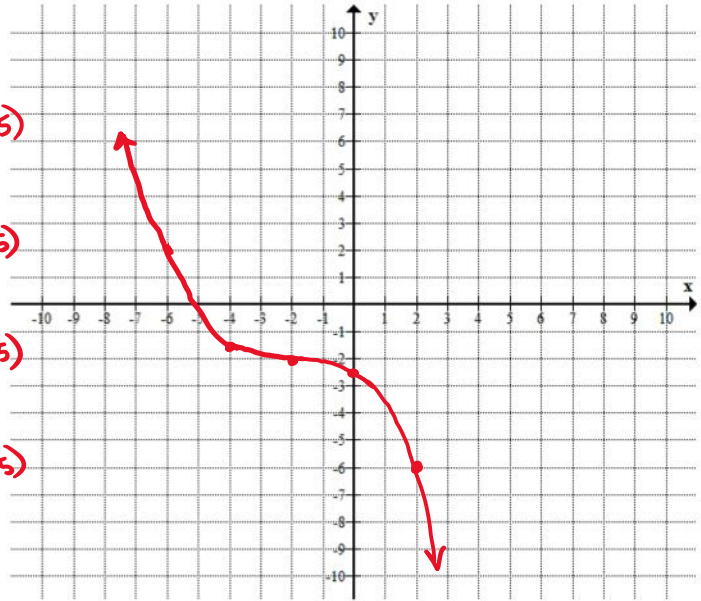
$$y = \frac{1}{2} \left(-\frac{1}{2}x - 1 \right)^3 - 2$$

$$y = x^3$$

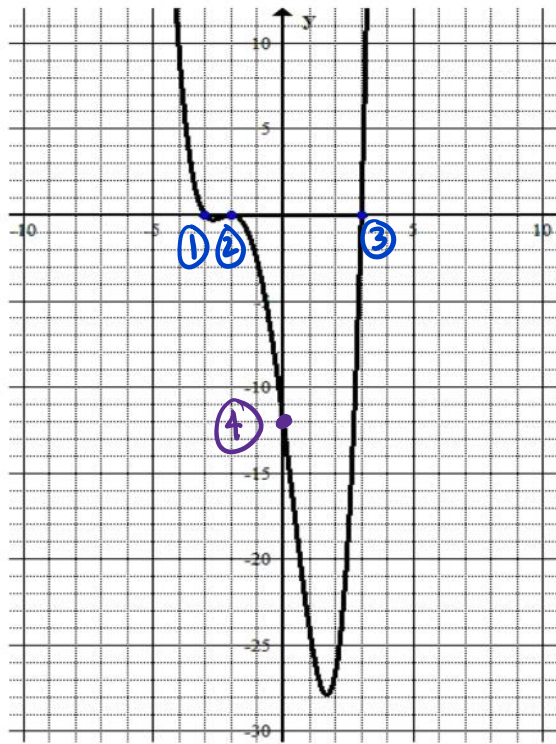
x	y
-3	-27
-2	-8
-1	-1
0	0
1	1
2	8
3	27

$-\frac{1}{2}x$	$\frac{1}{2}y$
6	$-\frac{27}{2}$
4	-4
2	$-\frac{1}{2}$
0	0
-2	$\frac{1}{2}$
-4	4
-6	$\frac{27}{2}$

$-2x-2$	$\frac{1}{2}y-2$
4	$-\frac{31}{2}$ (-15.5)
2	-6
0	$-\frac{5}{2}$ (-2.5)
-2	-2
-4	$-\frac{3}{2}$ (-1.5)
-6	2
-8	$\frac{93}{2}$ (46.5)



12. Given the graph below, determine the equation of the function. (2 marks)



① $x = -3 \rightarrow (x + 3)$

② $x = -2 \text{ M2} \rightarrow (x + 2)^2$

③ $x = 3 \rightarrow (x - 3)$

④ $y\text{-int} = (0, -12)$

$$y = a(x + 3)(x - 3)(x + 2)^2$$

$$-12 = a(0 + 3)(0 - 3)(0 + 2)^2$$

$$-12 = a(3)(-3)(4)$$

$$-12 = -36a$$

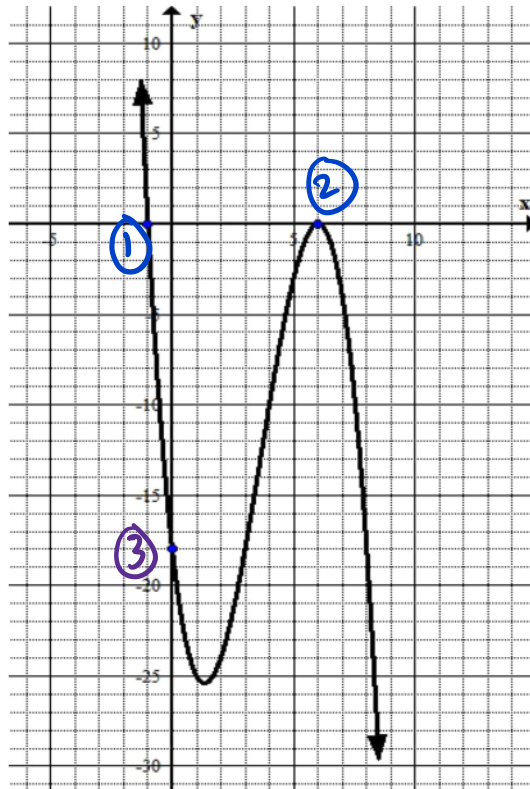
$$\frac{-12}{-36} = \frac{-36a}{-36}$$

$$a = \frac{1}{3}$$

Final Equation =

$$y = \frac{1}{3}(x + 3)(x - 3)(x + 2)^2$$

13. Given the graph below, determine the equation of the function. (2 marks)



① $x = -1 \rightarrow (x+1)$

② $x = 6 \text{ M2} \rightarrow (x-6)^2$

③ $y\text{-int} = (0, -18)$

$$y = a(x+1)(x-6)^2$$

$$-18 = a(0+1)(0-6)^2$$

$$-18 = a(1)(36)$$

$$\frac{-18}{36} = \frac{36a}{36}$$

$$a = -\frac{1}{2}$$

Final equation =

$$y = -\frac{1}{2}(x+1)(x-6)^2$$

14. Determine the equation of the function given the following information: (2 marks)

Roots at -3, -2 (multiplicity 2), and 1 (multiplicity 3) and a y-intercept at (0, 8)

$$y = a(x+3)(x+2)^2(x-1)^3$$

$$8 = a(0+3)(0+2)^2(0-1)^3$$

$$8 = a(3)(4)(-1)$$

$$\frac{8}{-12} = \frac{-12a}{-12}$$

$$a = -\frac{2}{3}$$

→ Final Equation =

$$y = -\frac{2}{3}(x+3)(x+2)^2(x-1)^3$$

15. Determine the x-intercept(s) and y-intercept algebraically (fully factor). Sketch the graph the equation below without the graphing calculator. SHOW WORK. (3 marks each)

a. $y = -x^3 + 5x^2 - 7x + 3$ \rightarrow y-int (0,3)

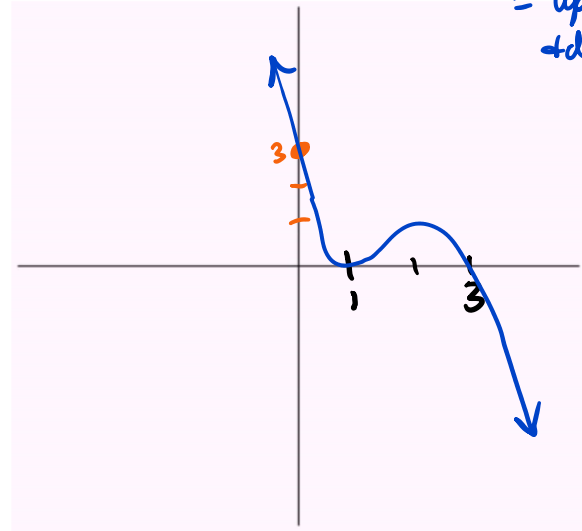
odd degree -LC
= up into QII
& down into QIV

$P(1) = 0 \checkmark$
 \downarrow

$x-1$	$-x^3 + 5x^2 - 7x + 3$
-1	$-1 \quad 5 \quad -7 \quad 3$
$-$	$1 \quad -4 \quad 3$
x	$-1 \quad 4 \quad -3 \quad 0 \checkmark$

$-x^2 + 4x - 3$
 $-(x^2 - 4x + 3)$
 $y = -(x-3)(x-1)(x-1)$

$y = -(x-3)(x-1)^2 \rightarrow x=3, x=1 \text{ M2}$



b. $y = x^4 + 3x^3 - 14x^2 - 12x + 40$

\rightarrow deg = 4 +LC \therefore up into QII + QI

$P(2) = 0 \rightarrow x-2$

$P(-2) = 0 \rightarrow x+2$

$x-2$

-2	$1 \quad 3 \quad -14 \quad -12 \quad 40$
$-$	$-2 \quad -10 \quad 8 \quad 40$
x	$1 \quad 5 \quad -4 \quad -20 \quad 0 \checkmark$

$x+2$

2	$1 \quad 5 \quad -4 \quad -20$
$-$	$2 \quad 6 \quad -20$
x	$1 \quad 3 \quad -10 \quad 0 \checkmark$

$x^2 + 3x - 10$
 $(x+5)(x-2)$

$y = (x-2)^2 (x+2)(x+5)$
 $x=2, x=-2, x=-5$
M2

y-int
(0,40)

