Chapter 4 Practice Questions

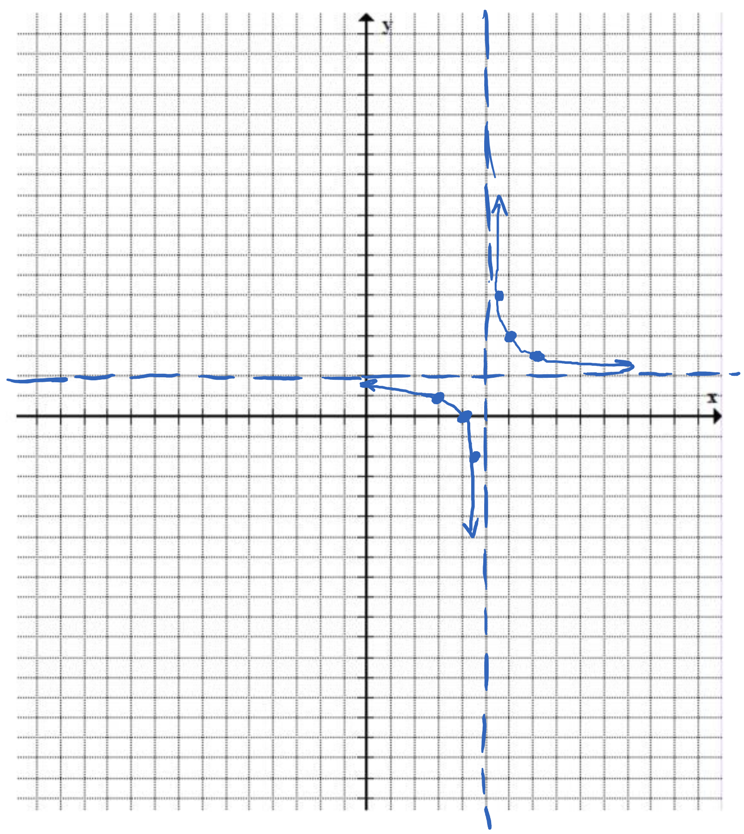
1. Describe the transformations and graph the base function AND the transformed function below – include mapping notation and the completed table of values.

   a. $y = \dfrac{2}{x-5} + 2$

   Describe the transformations: Vert. stretch by 2, 5 right, 2 up

   $y = \dfrac{1}{x}$

x	y
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
0	∅
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$



x	$2y$
-2	-1
-1	-2
$-\frac{1}{2}$	-4
0	∅
$\frac{1}{2}$	4
1	2
2	1

$x+5$	$2y+2$
3	1
4	0
$\frac{9}{2}$	-2
5	∅
$\frac{11}{2}$	6
6	4
7	3

State the domain and range: $\{x \mid x \neq 5, x \in \mathbb{R}\}$ $\{y \mid y \neq 2, y \in \mathbb{R}\}$

State the equation of the asymptote:

$x = 5$

$y = 2$

b. $y = \dfrac{1}{\frac{1}{2}(x+3)} - 4$

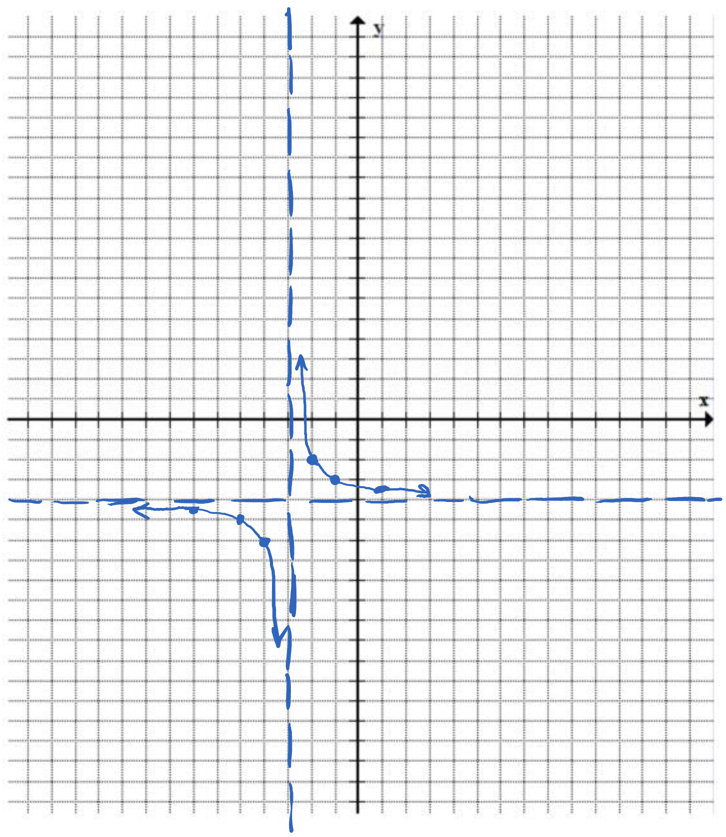
Describe the transformations: horiz. stretch by 2, 3 left, 4 down

$y = \dfrac{1}{x}$

x	y
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
0	∅
½	2
1	1
2	½

2x	y
-4	$-\frac{1}{2}$
-2	-1
-1	-2
0	∅
1	2
2	1
4	½

2x-3	y-4
-7	$-\frac{9}{2}$
-5	-5
-4	-6
-3	∅
-2	-2
-1	-3
1	$-\frac{7}{2}$



State the domain and range: $\{x \mid x \neq -3, x \in \mathbb{R}\}$

$\{y \mid y \neq -4, y \in \mathbb{R}\}$

State the equation of the asymptote:

$x = -3$

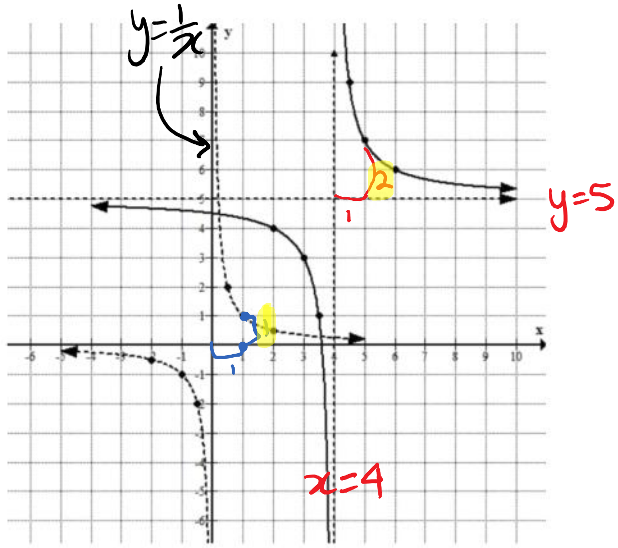
$y = -4$

2. Write an equation for each of the transformed graphs below. The base function is given.

a.

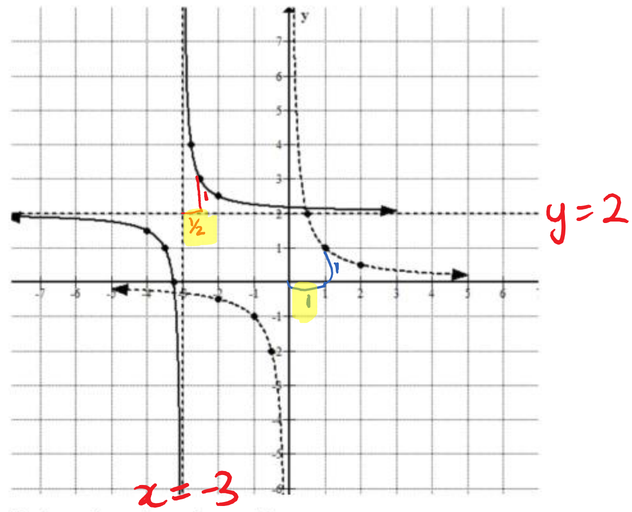
$$y = \frac{a}{b(x-h)} + k$$

$$\boxed{y = \frac{2}{x-4} + 5}$$



b.

$$\boxed{y = \frac{1}{2(x+3)} + 2}$$



3. Write the equation of each function with the given transformations:

a. $f(x) = \frac{1}{x}$ is stretched vertically by a factor of 2, stretched horizontal by a factor of 2, reflected in the y-axis, and translated 6 right and 3 down.

$$f(x) = \frac{2}{-\frac{1}{2}(x-6)} - 3$$

b. $y = \frac{1}{x}$ is stretched horizontally by a factor of ½, reflected in the x-axis, and translated 3 left and 1 down.

$$y = -\frac{1}{2(x+3)} - 1$$

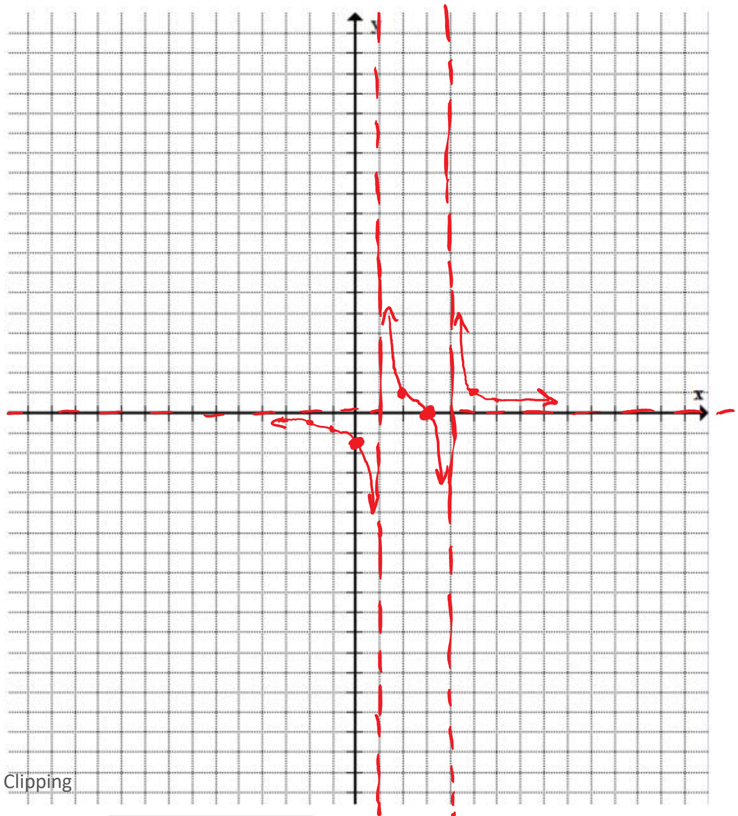
4. For each of the following functions, simplify (if possible), determine all of the characteristics of each and graph each function.

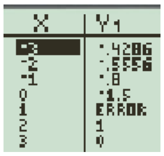
x- intercept
$x=3$

a. $y=\dfrac{2x-6}{x^2-5x+4}$

$$y=\frac{2(x-3)}{(x-4)(x-1)}$$

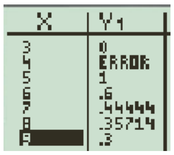
vertical asymptotes
$x=4$
$x=1$





X	Y_1
-3	-.4286
-2	-.5556
-1	-.8
0	-1.5
1	ERROR
2	1
3	0

4/25/2016 8:01 PM - Screen Clipping



X	Y_1
3	0
4	ERROR
5	1
6	.6
7	.44444
8	.35714
9	.3

4/25/2016 8:01 PM - Screen Clipping

Characteristic	Answer
Restrictions	$x\neq 4,\ x\neq 1\ (y\neq 0)$
Asymptote(s)	$x=4,\ x=1\ (y=0)$
Point(s) of Discontinuity	none
x-intercept	$(3,0)$
y-intercept	$\left(0,-\frac{3}{2}\right)$
Domain	$\{x \mid x\neq 1, x\neq 4, x\in\mathbb{R}\}$
Range	$\{y \mid y\in\mathbb{R}\}$

Show work here:

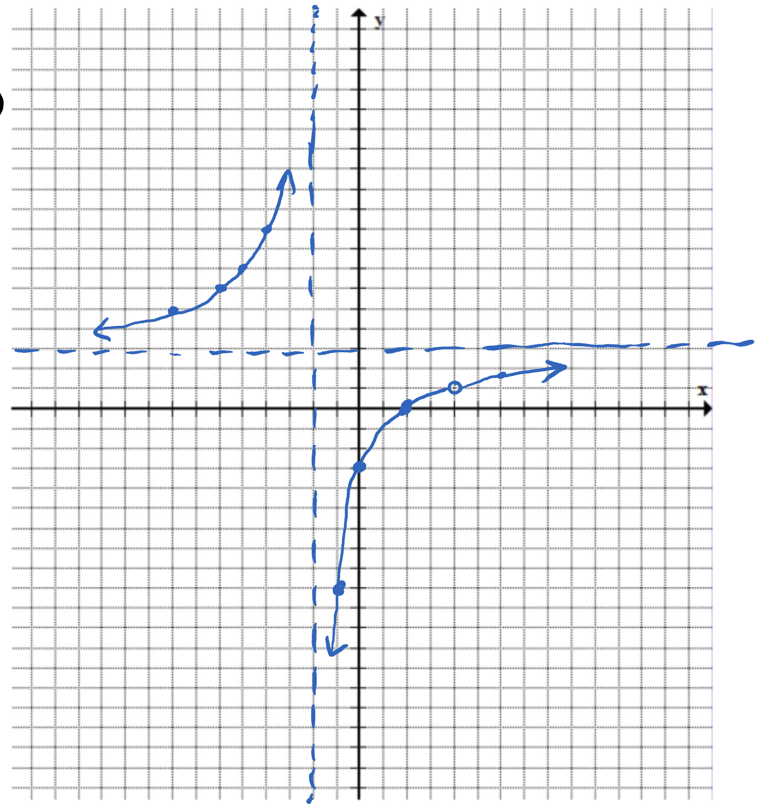
$$y=\frac{2(0)-6}{(0)^2-5(0)+4}=\frac{-6}{4}=-\frac{3}{2}$$

b. $y = \frac{3x^2 - 18x + 24}{x^2 - 2x - 8}$

$$y = \frac{3(x^2 - 6x + 8)}{x^2 - 2x - 8}$$

$$y = \frac{3(x-2)\cancel{(x-4)}}{\cancel{(x-4)}(x+2)}$$

$$\boxed{y = \frac{3(x-2)}{x+2}}$$



Characteristic	Answer
Restrictions	$x \neq -2$ $x \neq 4$ $y \neq 3$
Asymptote(s)	$x = -2$ $y = 3$
Point(s) of Discontinuity	$(4, 1)$
x-intercept	$x = 2.$ $(2, 0)$
y-intercept	$y = -3$ $(0, -3)$
Domain	$\{x \mid x \neq -2, x \neq 4, x \in \mathbb{R}\}$
Range	$\{y \mid y \neq 3, y \neq 1, y \in \mathbb{R}\}$

Show work here:

$$y = \frac{3(4-2)}{4+2} = 1$$

$$y = \frac{3(0-2)}{0+2} = -3$$

c. $y = \frac{2x^2 - 3x - 5}{2x - 5}$

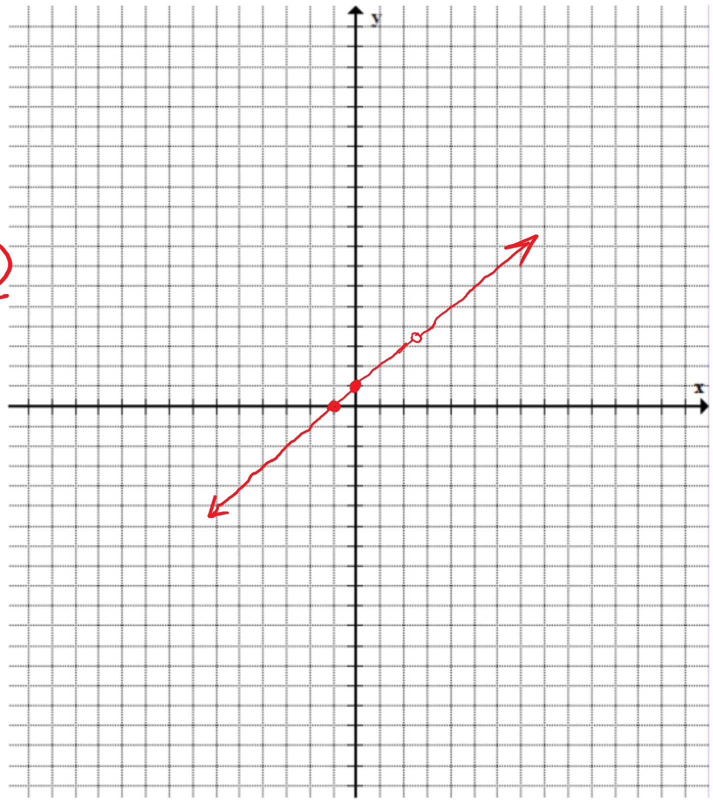
$2x - 5 = -10$

$-5, +2$

$(2x-5)(\cancel{2}x+\cancel{2})$

$(2x-5)(x+1) \rightarrow y = \frac{\cancel{(2x-5)}(x+1)}{\cancel{2x-5}}$

$y = x + 1$



Characteristic	Answer
Restrictions	$x \neq \frac{5}{2}$
Asymptote(s)	none
Point(s) of Discontinuity	$\left(\frac{5}{2}, \frac{7}{2}\right)$
x-intercept	$(-1, 0)$
y-intercept	$(0, 1)$
Domain	$\{x \mid x \neq \frac{5}{2}, x \in \mathbb{R}\}$
Range	$\{y \mid y \neq \frac{7}{2}, y \in \mathbb{R}\}$

Show work here:

$2x - 5 \neq 0$

$2x \neq 5$

$x \neq \frac{5}{2}$

$y = \frac{5}{2} + 1$

$= \frac{5}{2} + \frac{2}{2} = \frac{7}{2}$

$y = 0 + 1$

$= 1$

d. $y = \dfrac{2x^2+7x-4}{x^2+x-12}$

$y=2$ horiz. asymp.

$2x \cdot -4 = -8$

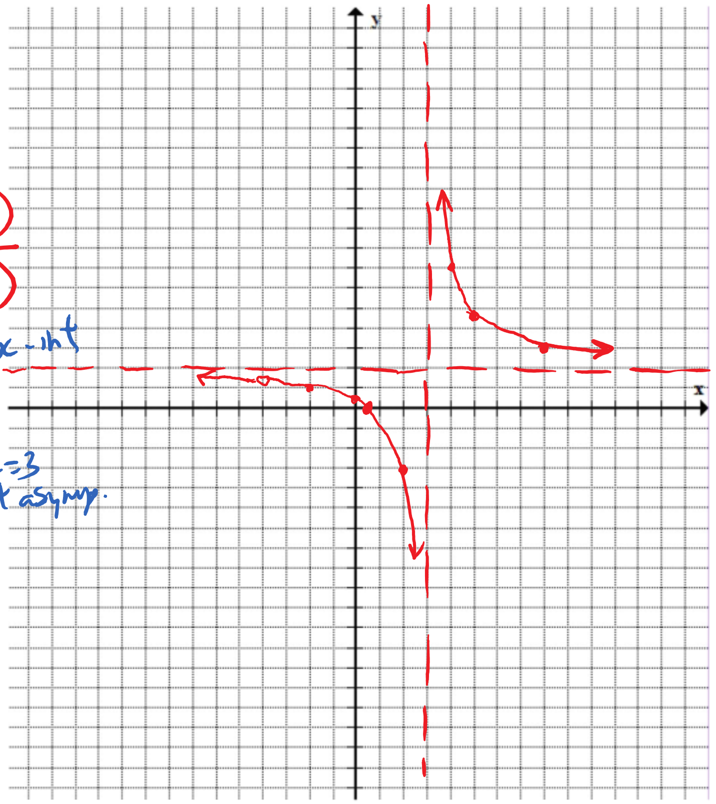
$8, -1$

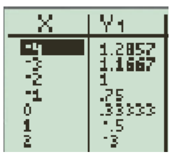
$(\cancel{2}x+\cancel{8})(2x-1)$

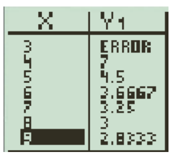
$(x+4)(2x-1) \rightarrow$ $y = \dfrac{\cancel{(x+4)}(2x-1)}{\cancel{(x+4)}(x-3)}$

$y = \dfrac{2x-1}{x-3}$ ← x-int

↖ $x=3$ vert asymp.







4/25/2016 8:15 PM - Screen Clipping 4/25/2016 8:17 PM - Screen Clipping

Characteristic	Answer
Restrictions	$x \neq -4, x \neq 3, y \neq 2$
Asymptote(s)	$x=3 \quad y=2$
Point(s) of Discontinuity	$\left(-4, \frac{9}{7}\right)$
x-intercept	$\left(\frac{1}{2}, 0\right)$
y-intercept	$\left(0, \frac{1}{3}\right)$
Domain	$\{x \mid x \neq -4, x \neq -3, x \in \mathbb{R}\}$
Range	$\{y \mid y \neq 2, y \neq \frac{9}{7}, y \in \mathbb{R}\}$

Show work here:

$\rightarrow y = \dfrac{2(-4)-1}{-4-3} = \dfrac{-9}{-7} = \dfrac{9}{7}$

$\rightarrow 2x-1=0$

$2x=1$

$x=\frac{1}{2}$

$y = \dfrac{2(0)-1}{0-3} = \dfrac{-1}{-3} = \dfrac{1}{3}$

5. Given the following information, determine a possible equation for the rational function: Show the final polynomial form of the rational function.
   a. Vertical asymptotes at $x=-4$ and $x=2$, an x-intercept at $x=5$ and $x=-2$

$$y=\frac{(x-5)(x+2)}{(x+4)(x-2)}$$

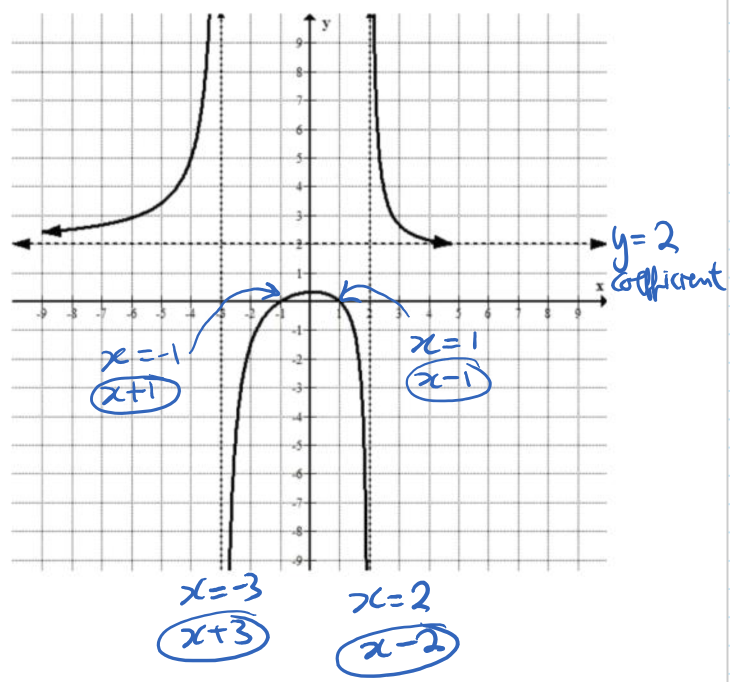
$$\boxed{y=\frac{x^2-3x-10}{x^2+2x-8}}$$

   b. A vertical asymptote at $x=-3$, x-intercept at $x=2$, a horizontal asymptote at $y=-4$, and a point of discontinuity at $x=1$

$$y=\frac{-4(x-2)(x-1)}{(x+3)(x-1)}=\frac{-4(x^2-3x+2)}{(x^2+2x-3)}$$

$$\boxed{y=\frac{-4x^2+12x-8}{x^2+2x-3}}$$

6. Given the graph of the rational function below, determine a possible equation of the function:
   Show the final polynomial form of the rational function.
   a.



$$y=\frac{2(x+1)(x-1)}{(x+3)(x-2)}$$

$$y=\frac{2(x^2-1)}{x^2+x-6}$$

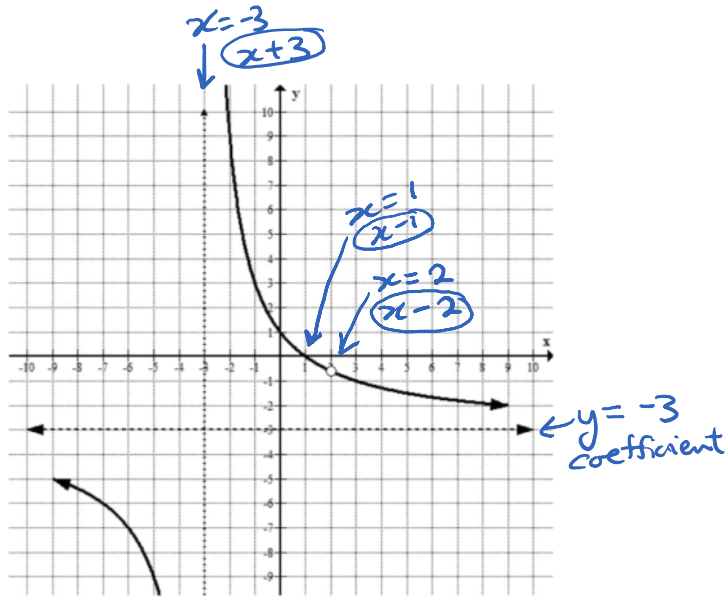
$$\boxed{y=\frac{2x^2-2}{x^2+x-6}}$$

b.

$$y = \frac{-3(x-1)(x-2)}{(x+3)(x-2)}$$

$$y = \frac{-3(x^2-3x+2)}{(x^2+x-6)}$$

$$\boxed{y = \frac{-3x^2+9x-6}{x^2+x-6}}$$



7. Solve the following functions graphically and two different ways of solving graphically:

a. $2x - x^2 = \frac{x-5}{x+9}$

$$\left.\begin{aligned} y_1 &= 2x - x^2 \\ y_2 &= \frac{x-5}{x+9} \end{aligned}\right\} 5\text{: intercepts}$$

$x = -0.2661$

$x = 2.1220$

or $\quad y = 2x - x^2 - \frac{x-5}{x+9} \; \} \; 2\text{: zeros.}$

b. $x = \frac{x^2}{x^2-9} + 6x$

$$\left.\begin{aligned} y_1 &= x \\ y_2 &= \frac{x^2}{x^2-9} + 6x \end{aligned}\right\} 5\text{: intercepts}$$

$x = -3.1017$

$x = 0$

$x = 2.9017$

or $\quad y_1 = \frac{x^2}{x^2-9} + 5x \; \} \; 2\text{: zeros.}$

8. Solve the following algebraically and show all restrictions:

a. $3x+2=\frac{5x+4}{x+1}$

$$(x+1)(3x+2) = \cancel{(x+1)}\left(\frac{5x+4}{\cancel{x+1}}\right)$$

$(x+1)3x + (x+1)2$

Restrictions

$x+1\neq 0$

$x\neq -1$

$$3x^2+5x+2 = 5x+4$$

$$3x^2-2=0$$

$$3x^2=2$$

$$x^2=\frac{2}{3}$$

$$\boxed{x=\pm\sqrt{\frac{2}{3}}}$$

b. $\left(\frac{3x}{4x+1}\right)-1=\frac{x}{2x-1}$

Restrictions

$x\neq -\frac{1}{4}$

$x\neq \frac{1}{2}$

$$\cancel{(4x+1)}(2x-1)\left(\frac{3x}{\cancel{4x+1}}\right)-1(4x+1)(2x-1) = (4x+1)\cancel{(2x-1)}\left(\frac{x}{\cancel{2x-1}}\right)$$

$$6x^2-3x \quad -8x^2+2x+1 = 4x^2+x$$

$$-(-6x^2-2x+1)=0$$

$$6x^2+2x-1=0$$

$$x=\frac{-2\pm\sqrt{2^2-4(6)(-1)}}{2(6)}$$

$$x=\frac{-2\pm\sqrt{28}}{12} = \frac{-\cancel{2}\pm\cancel{2}\sqrt{7}}{\cancel{12}}$$

$$\boxed{x=\frac{-1\pm\sqrt{7}}{6}}$$

c. $\frac{1}{x^2 - 9} + \frac{1}{x+3} = 0$

factor

$$\frac{1}{(x+3)(x-3)} + \frac{1}{x+3}\frac{(x-3)}{(x-3)} = 0$$

NPVs (Restrictions)

$x \neq 3,\ x \neq -3$

$$\frac{1 + x - 3}{(x+3)(x-3)} = 0$$

$$x - 2 = 0$$

$$\boxed{x = 2}$$

✓

d. $\cancel{\frac{2x}{x-1} - \frac{x}{x^2 - 4x + 3}} = \frac{x+1}{x-3} - 2$

$0 =$

LCD

$(x-3)$

$$0 = \frac{x+1}{x-3} - 2\left(\frac{x-3}{x-3}\right)$$

$$0 = \frac{x+1-2x+6}{x-3}$$

$$0 = 7 - x$$

$+x$

$$\boxed{x = 7}$$

NPV

$x \neq 3$

✓