Chapter 4 Homework to show for completion marks (5/5)

Due KEY

1. Convert each of the following to degrees:

a. $\frac{3\pi}{7} \times \frac{180}{\pi} = \frac{540}{7} = 77.14^\circ$

b. $-\frac{5\pi}{3} \times \frac{180}{\pi} = -300^\circ$

c. 2.47 *radians* $\times \frac{180^\circ}{\pi} = 141.52^\circ$

2. Covert each of the following to radians (leave in exact form):

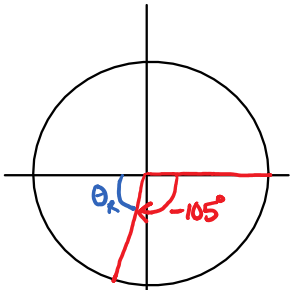
a. $140^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{9}$ (divide by 20)

b. $-625^\circ \times \frac{\pi}{180} = -\frac{125\pi}{36}$ (divide by 5)

c. $540^\circ \times \frac{\pi}{180} = 3\pi$ (divide by 180°)

3. For each of the following angles, graph in standard position, find two coterminal angles and determine the reference angle:

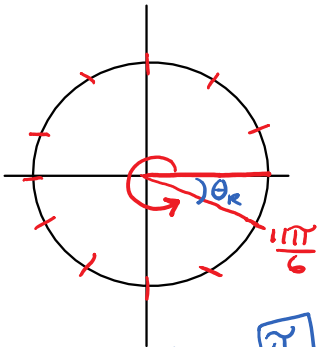
a. -105°



Reference Angle-$\theta_R = 180 - 105 = 75^\circ$

Coterminal Angles $= -105^\circ + 360^\circ = 225^\circ$

b. $\frac{11\pi}{6}$



$\theta_R = \frac{\pi}{6}$ $\left(\frac{12\pi}{6} - \frac{11\pi}{6}\right)$

Reference Angle: $\theta_R = 180 - 105 = 75°$

Coterminal Angles: $\theta_1 = -105° + 360° = 225°$

$\theta_2 = -105 + 360°(2) = 615°$

or $\theta_3 = -105 - 360° = -465°$

$\theta_R = \frac{\pi}{6}$ $\left(\frac{12\pi}{6} - \frac{11\pi}{6}\right)$

$\theta_1 = \frac{11\pi}{6} + \frac{12\pi}{6} = \frac{23\pi}{6}$

$\theta_2 = \frac{11\pi}{6} + \frac{12\pi}{6}(2) = \frac{35\pi}{6}$

or $\theta_3 = \frac{11\pi}{6} - \frac{12\pi}{6} = \frac{-\pi}{6}$

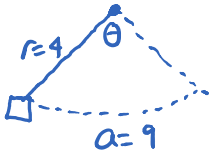
4. Determine the arc length subtended by the angle, $85°$, of a circle of radius 11cm.

$a = r\theta$ ..... θ = must be in radians

$= (11)\left(85° \times \frac{\pi}{180°}\right)$

$a = 16.32\text{cm}$

5. If a swing that is 4 meters long produces an arc of 9 meters, determine the angle (in degrees) that is produced by each swing.



$a = r\theta$

$\frac{9}{4} = \frac{4\theta}{4}$

$\theta = \frac{9}{4}$ → $\theta = \frac{9}{4} \times \frac{180°}{\pi}$ (45)

$= \frac{405}{\pi}$

$\theta = 128.92°$

6.

a. Determine the coordinates of all the points in the unit circle if the x-coordinate is $-\frac{3}{4}$.

$x^2 + y^2 = 1$

$\left(-\frac{3}{4}\right)^2 + y^2 = 1$

$y^2 = 1 - \frac{9}{16}$

$y^2 = \frac{16}{16} - \frac{9}{16}$

$\sqrt{y^2} = \sqrt{\frac{7}{16}}$

$y = \pm \frac{\sqrt{7}}{4}$

∴ coordinates are $\left(-\frac{3}{4}, -\frac{\sqrt{7}}{4}\right) + \left(-\frac{3}{4}, \frac{\sqrt{7}}{4}\right)$

b. Determine the coordinates of all the points in the unit circle if the y-coordinate is $-\frac{\sqrt{3}}{2}$.

$x^2 + y^2 = 1$

$x^2 + \left(-\frac{\sqrt{3}}{2}\right)^2 = 1$

$x^2 + \left(-\frac{\sqrt{3}}{2}\right)^2 = 1$

$x^2 = 1 - \frac{3}{4}$

$x^2 = \frac{4}{4} - \frac{3}{4}$

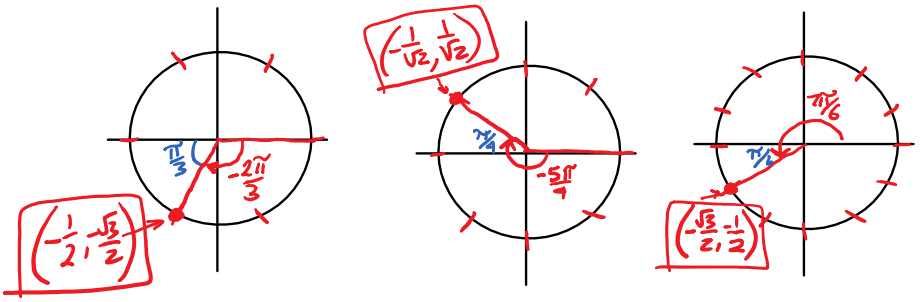
$\sqrt{x^2} = \sqrt{\frac{1}{4}}$

$x = \pm\frac{1}{2}$

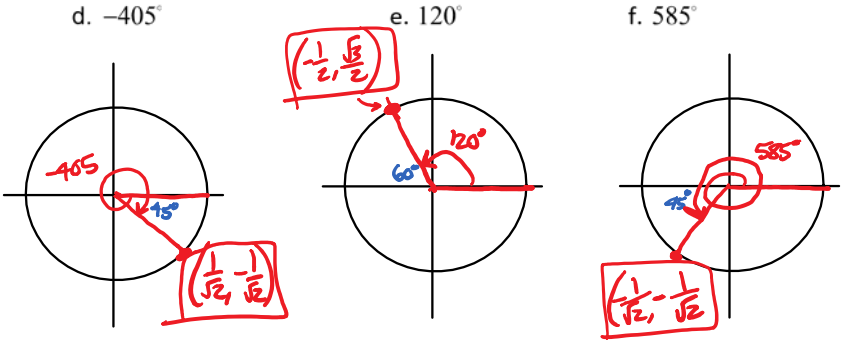
$\therefore$ coordinates are $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ & $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

7. Determine the exact point of the following angles in a unit circle:

a. $-\frac{2\pi}{3}$ b. $-\frac{5\pi}{4}$ c. $\frac{7\pi}{6}$



d. -405° e. 120° f. 585°



8. Determine the angle in radians and degrees for the following points in a unit circle:

a. $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ b. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ c. $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ d. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\frac{\pi}{4}$ or 45° in QIV

$\frac{\pi}{6}$ or 30° in QIII

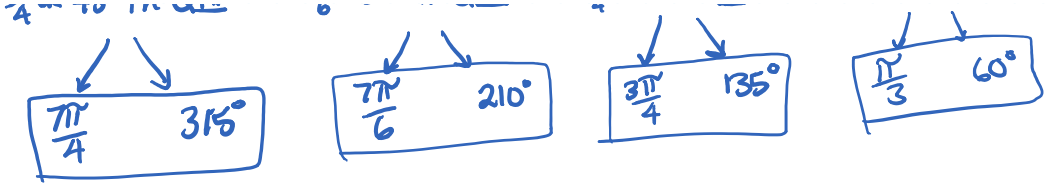
$\frac{\pi}{4}$ or 45° in QII

$\frac{\pi}{3}$ or 60° in QI

7π 210°

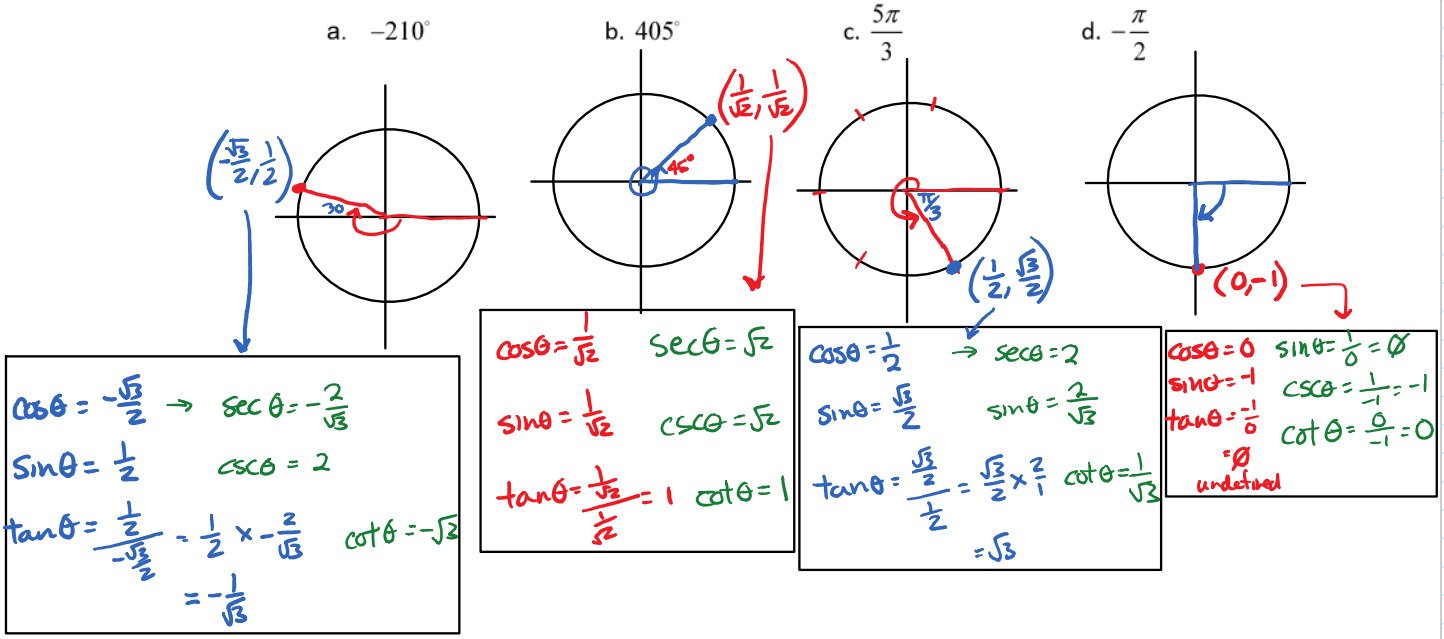
3π 135°

$\frac{\pi}{3}$ 60°



9. Give all 6 trigonometric ratios for the following angles: $\cos\theta, \sin\theta, \tan\theta, \sec\theta, \csc\theta, \cot\theta$

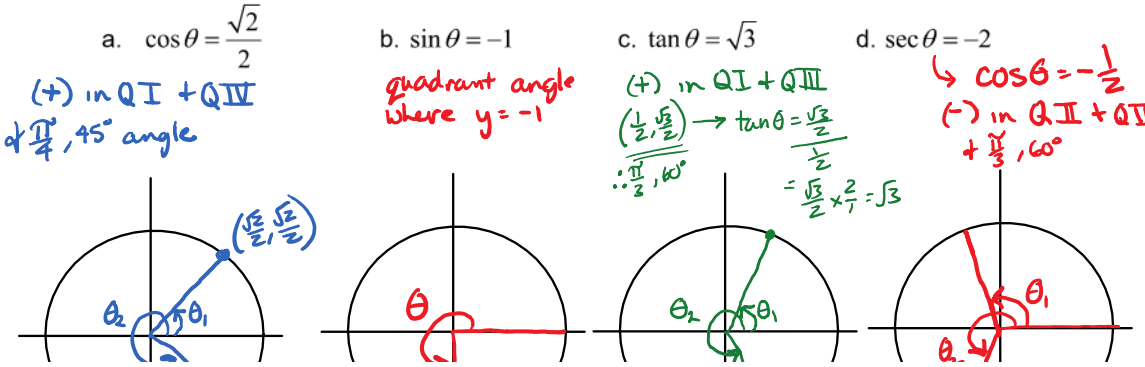
a. -210° b. 405° c. $\frac{5\pi}{3}$ d. $-\frac{\pi}{2}$

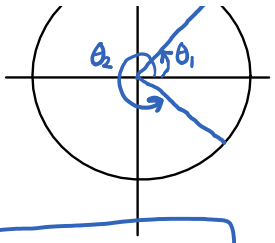


10. Determine all angles, θ, for each of the following and draw a diagram for each:

$0 \le \theta \le 2\pi$ and $0 \le \theta \le 360^\circ$

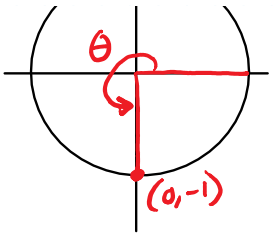
a. $\cos\theta = \frac{\sqrt{2}}{2}$ b. $\sin\theta = -1$ c. $\tan\theta = \sqrt{3}$ d. $\sec\theta = -2$



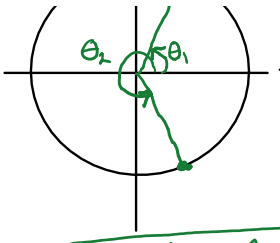


$\theta_1 = \frac{\pi}{4}$ or 45°

$\theta_2 = \frac{7\pi}{4}$ or 315°

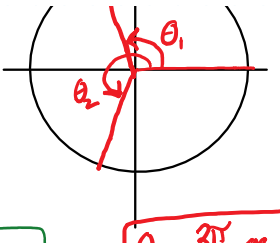


$\theta = \frac{3\pi}{2}$ or 270°



$\theta_1 = \frac{\pi}{3}$ or 60°

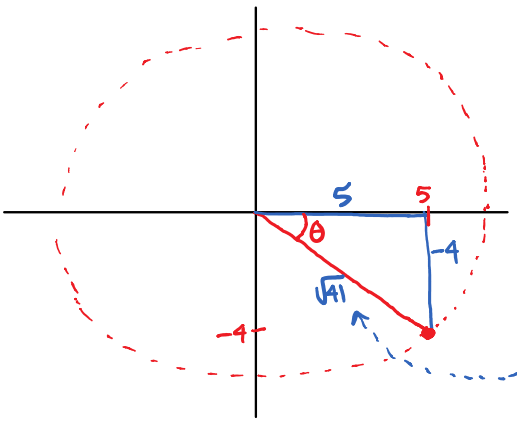
$\theta_2 = \frac{5\pi}{3}$ or 300°



$\theta_1 = \frac{2\pi}{3}$ or 120°

$\theta_2 = \frac{4\pi}{3}$ or 240°

11. Given the point $(5,-4)$ on the terminal arm of a circle, determine the 6 trigonometric ratios for angle θ.



Pythagorus

$5^2 + (-4)^2 = r^2$

$25 + 16 = r^2$

$41 = r^2$

$r = \pm\sqrt{41}$

$r = \sqrt{41}$

$\cos\theta = \frac{A}{H} = \frac{5}{\sqrt{41}}$

$\sin\theta = \frac{O}{H} = -\frac{4}{\sqrt{41}}$

$\tan\theta = \frac{O}{A} = -\frac{4}{5}$

$\sec\theta = \frac{\sqrt{41}}{5}$

$\csc\theta = -\frac{\sqrt{41}}{4}$

$\cot\theta = -\frac{5}{4}$