

Chapter 4 Homework to show for completion marks (5/5)

Due KEY

1. Convert each of the following to degrees:

a. $\frac{3\pi}{7} \times \frac{180}{\pi} = \frac{540}{7} = \boxed{77.14^\circ}$

b. $-\frac{5\pi}{3} \times \frac{180}{\pi} = \boxed{-300^\circ}$

c. $2.47 \text{ radians} \times \frac{180}{\pi} = \boxed{141.52^\circ}$

2. Convert each of the following to radians (leave in exact form):

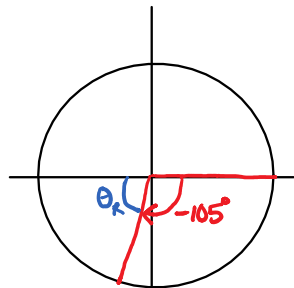
a. $140^\circ \times \frac{\pi}{180} = \boxed{\frac{7\pi}{9}}$
divide by 20

b. $-625^\circ \times \frac{\pi}{180} = \boxed{-\frac{125\pi}{36}}$
divide by 5

c. $540^\circ \times \frac{\pi}{180} = \boxed{3\pi}$
divide by 180

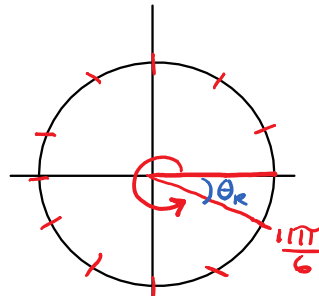
3. For each of the following angles, graph in standard position, find two coterminal angles and determine the reference angle:

a. -105°



Reference Angle: $\theta_r = 180 - 105 = \boxed{75^\circ}$
Coterminal angles: $\alpha = -105^\circ + 360^\circ = \boxed{255^\circ}$

b. $\frac{11\pi}{6}$



$\theta_r = \boxed{\frac{\pi}{6}}$ $\left(\frac{12\pi}{6} - \frac{11\pi}{6}\right)$

Reference Angle: $\theta_R = 180 - 105 = 75^\circ$

Coterminal Angles: $\theta_1 = -105^\circ + 360^\circ = 225^\circ$
 $\theta_2 = -105^\circ + 360^\circ(2) = 615^\circ$
 $\text{or } \theta_3 = -105^\circ - 360^\circ = -465^\circ$

$\theta_R = \frac{5\pi}{6}$ $(\frac{12\pi}{6} - \frac{11\pi}{6})$

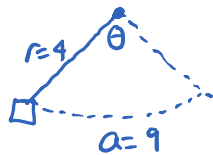
$\theta_1 = \frac{11\pi}{6} + \frac{12\pi}{6} = \frac{23\pi}{6}$
 $\theta_2 = \frac{11\pi}{6} + \frac{12\pi}{6}(2) = \frac{35\pi}{6}$
 $\text{or } \theta_3 = \frac{11\pi}{6} - \frac{12\pi}{6} = -\frac{\pi}{6}$

4. Determine the arc length subtended by the angle, 85° , of a circle of radius 11cm.

$a = r\theta$
 $= (11) \left(85^\circ \times \frac{\pi}{180^\circ} \right)$ $\theta = \text{must be in radians}$

$a = 16.32 \text{ cm}$

5. If a swing that is 4 meters long produces an arc of 9 meters, determine the angle (in degrees) that is produced by each swing.



$a = r\theta$
 $9 = 4\theta$
 $\frac{9}{4} = \frac{4\theta}{4}$
 $\theta = \frac{9}{4}$

$\theta = \frac{9}{4} \times \frac{180^\circ}{\pi} \times 45$
 $= \frac{405}{\pi}$

$\theta = 128.92^\circ$

6.

a. Determine the coordinates of all the points in the unit circle if the x-coordinate is

$-\frac{3}{4}$.

$x^2 + y^2 = 1$
 $\left(-\frac{3}{4}\right)^2 + y^2 = 1$
 $y^2 = 1 - \frac{9}{16}$
 $y^2 = \frac{16}{16} - \frac{9}{16}$
 $\sqrt{y^2} = \sqrt{\frac{7}{16}}$

$y = \pm \frac{\sqrt{7}}{4}$

\therefore coordinates are

$\left(-\frac{3}{4}, -\frac{\sqrt{7}}{4}\right) + \left(-\frac{3}{4}, \frac{\sqrt{7}}{4}\right)$

b. Determine the coordinates of all the points in the unit circle if the y-coordinate is

$-\frac{\sqrt{3}}{2}$.

$x^2 + y^2 = 1$
 $x^2 + \left(-\frac{\sqrt{3}}{2}\right)^2 = 1$

$$x^2 + \left(-\frac{\sqrt{3}}{2}\right)^2 = 1$$

$$x^2 = 1 - \frac{3}{4}$$

$$x^2 = \frac{4}{4} - \frac{3}{4}$$

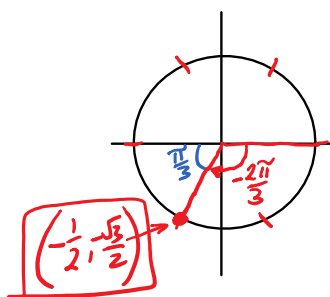
$$\sqrt{x^2} = \sqrt{\frac{1}{4}}$$

$$x = \pm \frac{1}{2}$$

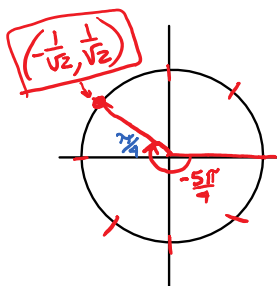
\therefore coordinates are $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

7. Determine the exact point of the following angles in a unit circle:

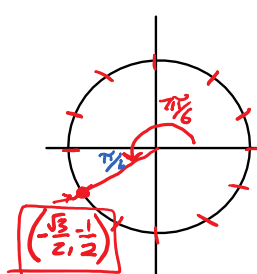
a. $-\frac{2\pi}{3}$



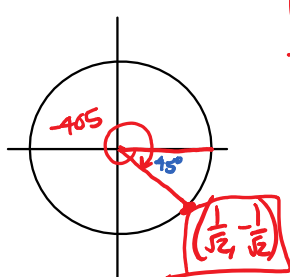
b. $-\frac{5\pi}{4}$



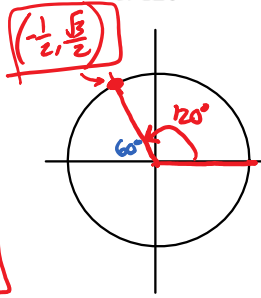
c. $\frac{7\pi}{6}$



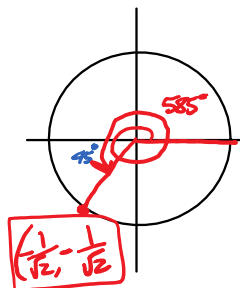
d. -405°



e. 120°



f. 585°



8. Determine the angle in radians and degrees for the following points in a unit circle:

a. $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$\frac{7\pi}{4}$ or 315° in QIV

b. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\frac{7\pi}{6}$ or 210° in QIII

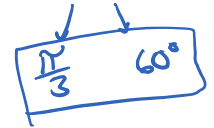
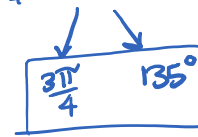
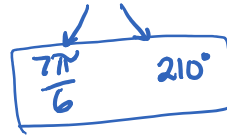
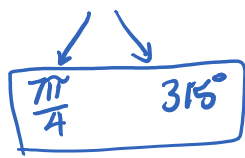
c. $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\frac{3\pi}{4}$ or 135° in QII

d. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

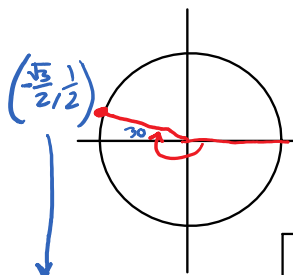
$\frac{\pi}{3}$ or 60° in QI

4 → 10 → 15 → 21 → 27 → 33 → 39 → 45 → 51 → 57 → 63 → 69 → 75 → 81 → 87 → 93 → 99 → 105 → 111 → 117 → 123 → 129 → 135 → 141 → 147 → 153 → 159 → 165 → 171 → 177 → 183 → 189 → 195 → 201 → 207 → 213 → 219 → 225 → 231 → 237 → 243 → 249 → 255 → 261 → 267 → 273 → 279 → 285 → 291 → 297 → 303 → 309 → 315 → 321 → 327 → 333 → 339 → 345 → 351 → 357 → 363 → 369 → 375 → 381 → 387 → 393 → 399 → 405 → 411 → 417 → 423 → 429 → 435 → 441 → 447 → 453 → 459 → 465 → 471 → 477 → 483 → 489 → 495 → 501 → 507 → 513 → 519 → 525 → 531 → 537 → 543 → 549 → 555 → 561 → 567 → 573 → 579 → 585 → 591 → 597 → 603 → 609 → 615 → 621 → 627 → 633 → 639 → 645 → 651 → 657 → 663 → 669 → 675 → 681 → 687 → 693 → 699 → 705 → 711 → 717 → 723 → 729 → 735 → 741 → 747 → 753 → 759 → 765 → 771 → 777 → 783 → 789 → 795 → 801 → 807 → 813 → 819 → 825 → 831 → 837 → 843 → 849 → 855 → 861 → 867 → 873 → 879 → 885 → 891 → 897 → 903 → 909 → 915 → 921 → 927 → 933 → 939 → 945 → 951 → 957 → 963 → 969 → 975 → 981 → 987 → 993 → 999



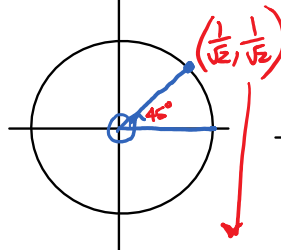
9. Give all 6 trigonometric ratios for the following angles: $\cos \theta, \sin \theta, \tan \theta, \sec \theta, \csc \theta, \cot \theta$

a. -210°



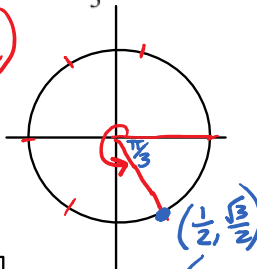
$$\begin{aligned} \cos \theta &= -\frac{\sqrt{3}}{2} & \rightarrow & \sec \theta = -\frac{2}{\sqrt{3}} \\ \sin \theta &= \frac{1}{2} & \csc \theta &= 2 \\ \tan \theta &= \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} & \cot \theta &= -\sqrt{3} \end{aligned}$$

b. 405°



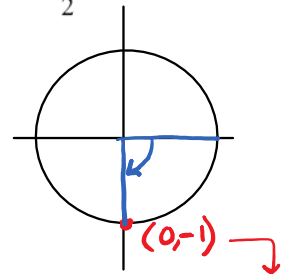
$$\begin{aligned} \cos \theta &= \frac{1}{\sqrt{2}} & \sec \theta &= \sqrt{2} \\ \sin \theta &= \frac{1}{\sqrt{2}} & \csc \theta &= \sqrt{2} \\ \tan \theta &= \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1 & \cot \theta &= 1 \end{aligned}$$

c. $\frac{5\pi}{3}$



$$\begin{aligned} \cos \theta &= \frac{1}{2} & \rightarrow & \sec \theta = 2 \\ \sin \theta &= -\frac{\sqrt{3}}{2} & \csc \theta &= -\frac{2}{\sqrt{3}} \\ \tan \theta &= \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3} & \cot \theta &= -\frac{1}{\sqrt{3}} \end{aligned}$$

d. $-\frac{\pi}{2}$



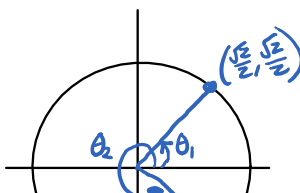
$$\begin{aligned} \cos \theta &= 0 & \sin \theta &= -1 \\ \sin \theta &= -1 & \csc \theta &= -1 \\ \tan \theta &= \frac{-1}{0} = \text{undefined} & \cot \theta &= \frac{0}{-1} = 0 \end{aligned}$$

10. Determine all angles, θ , for each of the following and draw a diagram for each:

$$0 \leq \theta < 2\pi \text{ and } 0 \leq \theta < 360^\circ$$

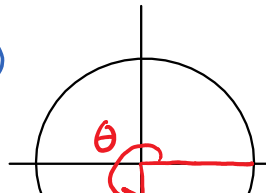
a. $\cos \theta = \frac{\sqrt{2}}{2}$

(+) in QI + QIV
at $\frac{\pi}{4}, 45^\circ$ angle



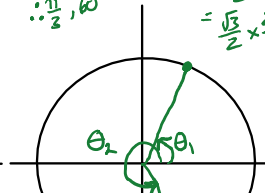
b. $\sin \theta = -1$

quadrant angle
where $y = -1$



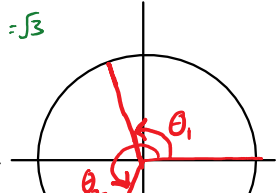
c. $\tan \theta = \sqrt{3}$

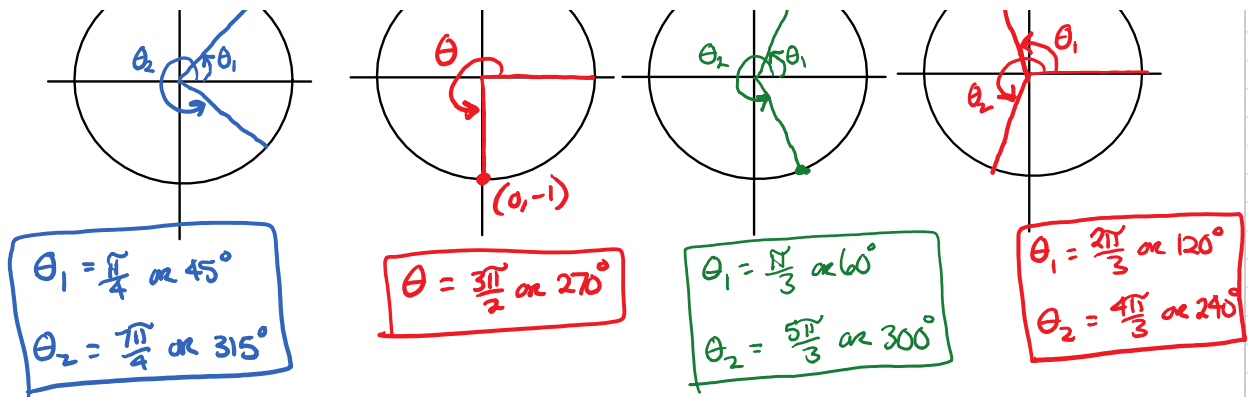
(+) in QI + QIII
 $(\frac{1}{2}, \frac{\sqrt{3}}{2}) \rightarrow \tan \theta = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$
 $\therefore \frac{\pi}{3}, 60^\circ$
 $= \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$



d. $\sec \theta = -2$

$\rightarrow \cos \theta = -\frac{1}{2}$
(-) in QII + QIII
at $\frac{2\pi}{3}, 120^\circ$





11. Given the point $(5, -4)$ on the terminal arm of a circle, determine the 6 trigonometric ratios for angle θ .

