Ch5.4 Notes and Practice of Application Questions

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Chapter 5: Section 5.4 - Applications of Trigonometric Functions

Sometime you will use trigonometric functions to represent periodic applications; such as, the tide depths, sunrise, sunset, ferris wheels, wheels and anything that is periodical (cyclical). Here the period will be in integers (without π). The functions will be written in the form:

$$y = a\cos\frac{2\pi}{p}(x-c)+d \text{ and } y = a\sin\frac{2\pi}{p}(x-c)+d$$

Notice that the *b* value is now written as $\frac{2\pi}{p}$. This means that when you find the period using the formula we've already learned $\left(period = \frac{2\pi}{b}\right)$, that the period is the denominator of the *b* value if it is written in the this form with 2π in the numerator. The period of these functions will be rational numbers and the units will be related to the question. As long as you have π in the formula, the period will be in rational numbers

Examples: Find the period of the following functions:

1) $y = \cos\pi\theta$

Answer:

$b = \pi$

$period = p = \frac{2\pi}{b} = \frac{2\pi}{\pi} = \boxed{2}$

2) $y = \sin 2\pi x$

Answer:

$b = 2\pi$

$p = \frac{2\pi}{b} = \frac{2\pi}{2\pi} = \boxed{1}$

3) $y = \cos\frac{2\pi}{17}(x-3)$

Answer:

$b = \frac{2\pi}{17}$

$p = \frac{2\pi}{b} = \frac{2\pi}{\frac{2\pi}{17}} = 2\pi \times \frac{17}{2\pi} = \boxed{17}$

4) $y = \sin\frac{2\pi}{22.89}(x+4.61)+4.7$

$p = 2\pi \times \frac{1}{b}$ OR $\frac{2\pi}{b}$

$p = 2\pi \times \frac{22.89}{2\pi} = 22.89$

5) $y = -2\cos\frac{\pi}{12}(x+3)-1$

$p = 2\pi \times \frac{12}{\pi} = 24$

To graph these functions with a calculator see the following example:

<u>Example 1:</u> Given the equation: $y = 13.2\cos\frac{2\pi}{342}(t-101)+6.5$ find appropriate units for your window and graph with the TI-83.

1) First you must figure out the period so you know how long a cycle is.

$$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{\frac{2\pi}{342}} = 2\pi \times \frac{342}{2\pi} = 342$$

The length of one complete cosine cycle is 342 units.
Also, since the pattern starts at 101 due to the phase shift, we want to see all the way to 443 (phase shift + period) for the complete picture.

2) Next you need to know where the minimum and maximum values are using the following formulas:

Minimum = d – a
Minimum = 6.5 – 13.2
Minimum = -6.7

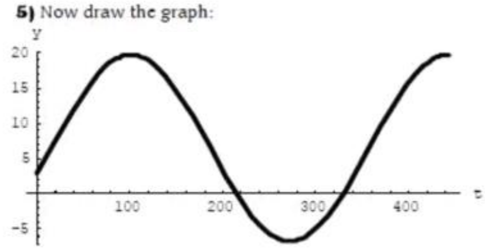
Maximum = d + a
Maximum = 6.5 + 13.2
Maximum = 19.7

3) Now choose a scale. The best way to do this is by picking numbers large enough that you won't have too many ticks on either axis. For the x-axis, we're going from 0 to about 450, so use a scale of 100. For the y-axis, since we want to see from -7 to +20, a scaling of 5 would be good. Of course, these are just guidelines and you could use several different scales and still obtain a good graph.

4) Use the following window settings:

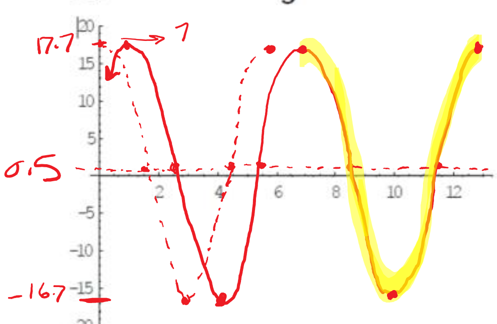
X_{min}: 0
X_{max}: 443
X_{scl}: 20
Y_{min}: -6.7
Y_{max}: 19.7
Y_{scl}: 5

5) Now draw the graph:



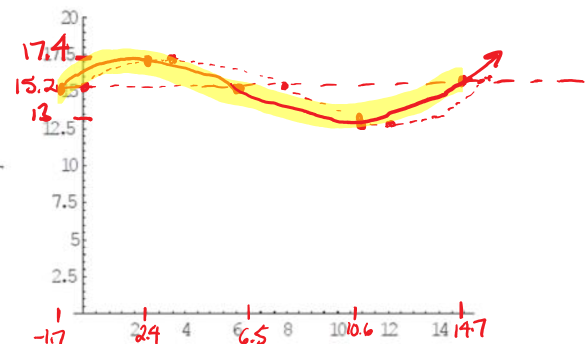
Practice Examples: Graph these on the grid provided using your graphing calculator:

1. $y = 17.2\cos\frac{\pi}{3}(t-7)+0.5$



$a = 17.2$
$d = 0.5$
$p = \frac{2\pi}{} \times \frac{3}{\pi} = 6$
$c = \text{p.s.} = 7$

2. $y = 2.2\sin 0.123\pi(t+1.7)+15.2$



$a = 2.2$
$d = 15.2$
$p = \frac{2\pi}{0.123\pi} = 16.3$ ($\div 4$ is 4.1 per quarter)
$c = -1.7$

With application questions, you may be asked to evaluate the function for a particular value.

For example: The height of an object in metres is given by the equation $h(t) = 2.2\sin 0.123\pi(t+1.7)+15.2$ where time is in seconds.

a) At what height with the object be after 2.5 seconds?

$h(t) = 2.2 \sin 0.123\pi(2.5+1.7) + 15.2$

$\boxed{h(t) = 17.40\text{ m}}$

b) When is the first time that the object reaches a height of 16.3 metres?

MI algebraically: $16.3 = 2.2 \sin 0.123\pi(t+1.7) + 15.2$

$-15.2 \qquad -15.2$

$\dfrac{1.1}{2.2} = \dfrac{2.2 \sin 0.123\pi(t+1.7)}{2.2}$

$0.5 = \sin 0.123\pi(t+1.7)$

$\dfrac{\sin^{-1}(0.5)}{0.123\pi} = \dfrac{0.123\pi(t+1.7)}{0.123\pi}$

$\dfrac{\sin^{-1}(0.5)}{0.123\pi} = t+1.7$

$-1.7 \qquad -1.7$

$t = \dfrac{\sin^{-1}(0.5)}{(0.123\pi)} - 1.7$

$t = -0.34$

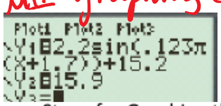
subtract from half period

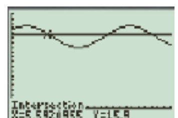
$8.15 - 0.34$

$\boxed{t_1 = 7.81}$

c) How long is the object above 15.9 metres?

MII graphing calc





$t = 5.59$

Steps for Graphing these functions without the Graphing Calculator:

1. Determine the amplitude, vertical displacement, period and phase shift
2. Adjust the scale on the *x*-axis according to the period – the scale is in integers
3. Adjust the scale on the *y*-axis according to the amplitude and vertical displacement (use the Max and Min)
4. Determine your new midline on the graph.
5. Determine your starting point on the graph from the sine or cosine and phaseshift and vertical displacement
6. Divide the period by 4 and graph the 4 sections of the curve ending at the phaseshift plus the period considering your max and min.

Examples: Graph the following functions.

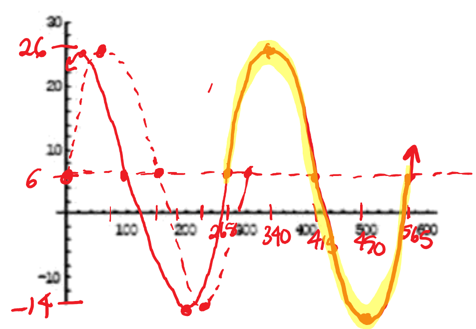
a) $y = 20\sin\dfrac{2\pi}{300}(t-265)+6$

$a = 20$

$d = 6$

$p = 2\pi \times \dfrac{300}{2\pi} = 300 \rightarrow \div 4 = 75$

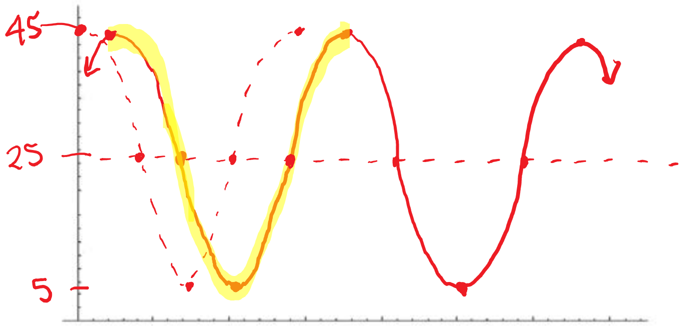
$c = P.S = 265.$



Practice Examples:
Graph the following on a separate sheet of graph paper:

1) $y = 20\cos\frac{2\pi}{15}(x-3)+25$

$a = 20$
$d = 25$
$p = 15$
$c = 3$



More Application Problems:

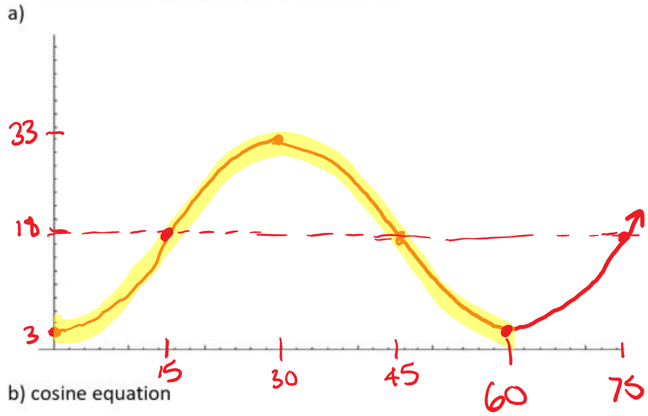
The classic Ferris Wheel problem

One of the most common application questions for graphing trigonometric functions involves Ferris wheels, since the up and down motion of a rider follows the shape of a sine or cosine graph.

Example: A Ferris wheel has a diameter of 30 m, with the centre 18 m above the ground. It makes one complete rotation every 60 s.

a) Draw the graph of one complete cycle, assuming the rider starts at the lowest point.
b) Find the cosine equation of the graph.
c) What is the height of the rider at 52 seconds?
d) At what time(s) is the rider at 20 m?

a)



$a = 15$
$d = 18$
$p = 60 \rightarrow b = \frac{2\pi}{60}$ or $\frac{\pi}{30}$
$c = 0$ for $-\cos$.

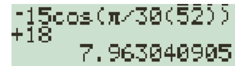
b) cosine equation

$$h(t) = -15\cos\frac{\pi}{30}t + 18$$

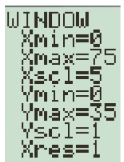
Alternatively, you can write a sine equation adjusting the phaseshift: $h(t) = -15\sin\frac{\pi}{30}(t+15)+18$

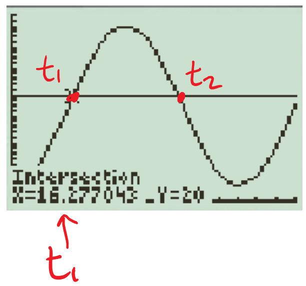
c) t = 52 seconds. Substitute this into your equation and solve for the height

$h(t) = -15\cos\frac{\pi}{30}(52) + 18 \rightarrow h(t) = 7.96m$



d) Graph your equation in $y_1 =$ and a horizontal line where $y_2 = 20$ and find the intercepts





$t_1 = 16.28s$

$t_2 = 43.72s$.

Practice Example:

answer is below

1. A Ferris wheel has a radius of 35 m and starts 2 m above the ground. It rotates once every 53 seconds.

a) Determine the cosine equation of the graph, if the rider gets on at the lowest point.

b) Sketch two complete cycles of a graph representing the height of a rider above the ground, assuming the rider gets on the Ferris wheel at the lowest point.

c) What is the height of the rider at 81 seconds?

d) At what time does the rider first reach a height of 51 m?

Answer:

1. a) ***a-value:*** The amplitude is 35 (upside down since the rider gets on at the bottom, so use a negative in the equation)

b-value: $b = \frac{2\pi}{P} = \frac{2\pi}{53}$

c-value: None for cosine

d-value: The lowest point is 2 m off the ground and the radius is 35 m, so the midline will be at 37 m.

$$h(t) = -35\cos\frac{2\pi}{53}t + 37$$

c) To find the height at 81 seconds, plug the time into the equation.

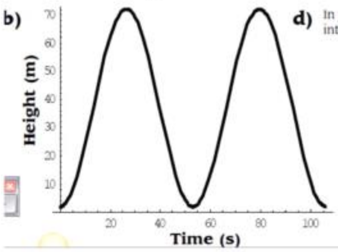
$$h(t) = -35\cos\frac{2\pi}{53}t + 37$$

$$h(81) = -35\cos\frac{2\pi}{53}(81) + 37$$

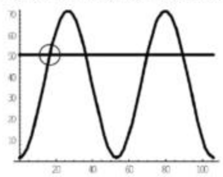
$$h(81) = 71.4$$

Or use the simpler method of 2^{nd} → Trace → Value → x = 81

b)



d) In your TI-83, graph the equation and also the line y = 51. The first intersection point will give the time the height first reaches 51 m.



51 m is first reached at 16.7 s

Other periodic application questions:

Events that are cyclic, such as seasonal variations in temperature, can be modeled with trigonometric functions.

Need Calendar for dates + # day in year.

Example: The average temperature for Regina is hottest at 27 °C on July 28, and coolest at -16 °C on January 10.

a) Write the cosine equation for the graph.

b) Draw the graph that approximates the temperature curve for the year.

c) What is the average temperature expected for October 4?

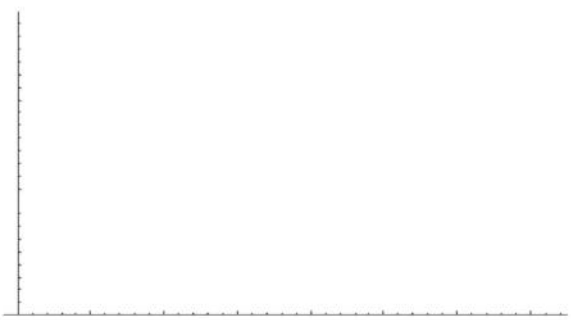
d) The average temperature is higher than 23 °C for how many days?

a) cosine equation

Alternatively, sine equation

Skip - not on test

b)



c) Using a calendar, October 4th is the 277th day of the year.

d) graph the function and the line y=23 and find the 2 intersection points; subtract the last intersection day and first intersection day to get the total number days.

Example 2: The earliest sunset occurs at 5:34 PM on Dec. 21, and the latest at 11:45 PM on June 21.
a) Write the cosine equation of the graph.
b) Draw the graph approximating the sunrise times throughout the year.
c) What is the sunset time on April. 6?
d) The sunset time is earlier than 8 PM for what percentage of the year?

Same here

a) cosine equation

Conversion Reminder:

To get the hours, think of the 24-hour clock. Then divide the number of minutes by 60 to finish off the decimal.

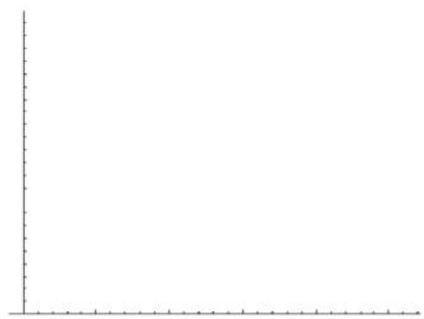
5:34 PM → 5 PM is 17 hours on the 24-hour clock. Then, 11/60 = 0.57. So, the time we use is 17.57

11:45 PM → 11 PM is 23 hours on the 24-hour clock. Then, 45/60 = 0.75. So, the time we use is 23.75

More Examples: 12:00 AM = 0 hours
12:30 AM = 0.5 hours
7:15 PM = 19.25 hours
11: 49 PM = 23.82 hours

Alternatively, sine equation

b)



c) Using a calendar, April 6th is the 96th day of the year.

d) graph the function and the line y=20 and find the 2 intersection points; add the first intersection of days with the difference between 365 and the last intersection to get the total number days then divide by 365 and convert to percentage.

Practice Example:

1. The highest average temperature for the Edmonton region is 24 °C, and occurs on July 20. The coldest average temperature is -16 °C, and occurs on January 14.

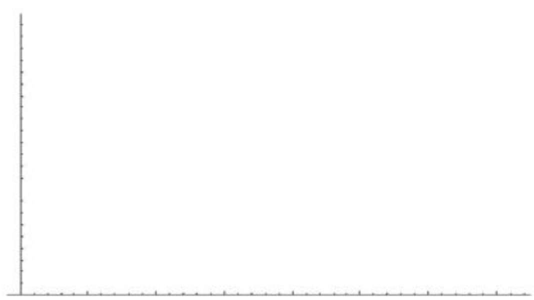
a) Write a cosine equation for the graph.

answer is below

b) Draw the graph that approximates the temperature curve for the year.

c) What is the average temperature expected for November 4?

d) The average temperature is below 0 °C for how many days?



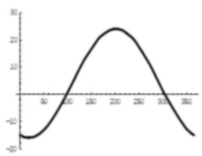
Answer:

1.

a)

$$T(d) = 20\cos\frac{2\pi}{365}(d-201)+4$$

b)



c) -1.36 °C

d) 159 days