[redacted]

Chapter [redacted] 5: Exponential & Logarithmic Equations Practice

1. Solve each of the following algebraically. (hint: change to the common base)

a. $16^{2x-3} = 32^{x+2}$

Common base $\quad (2^4)^{2x-3} = (2^5)^{x+2} \quad$ expand

Cancel bases $\quad \cancel{2}^{8x-12} = \cancel{2}^{5x+10}$

solve: $\quad 8x - 12 = 5x + 10$

$-5x \qquad +12$

$3x = \frac{22}{3} \rightarrow \boxed{x = \frac{22}{3}}$

b. $9(27)^{x-1} = \left(\frac{1}{3}\right)^{4-2x}$

Common base $\quad (3^2)(3^3)^{x-1} = (3^{-1})^{4-2x}$

expand $\quad 3^2 \cdot 3^{3x-3} = 3^{-4+2x}$

combine terms (add exponents) $\quad \cancel{3}^{2+3x-3} = \cancel{3}^{-4+2x}$

cancel base $\quad 3x - 1 = -4 + 2x$

$-2x \qquad +1$

$\boxed{x = -3}$

c. $\left(\frac{1}{25}\right)^{x} = 5^x (125)^{x+1}$

Common base $\quad (5^{-2})^x = (5)^x (5^3)^{x+1}$

expand add exponents $\quad 5^{-2x} = 5^x \cdot 5^{3x+3}$

$\cancel{5}^{-2x} = \cancel{5}^{4x+3}$

cancel base $\quad -2x = 4x + 3$

$-4x$

solve: $\quad \frac{-6x}{-6} = \frac{3}{-6}$

$\boxed{x = -\frac{1}{2}}$

2. The following function represents the growth rate of bacteria in the intestinal tract of mice after eating a probiotic where P(t) represents the total population of bacteria and t represents time in hours. Use it to answer the following questions:

$$P(t) = 2(2)^{\frac{t}{0.68}}$$

a. Determine the time it took for the bacteria to double.

since base is '2' → doubling time is $n = 0.68$ hr.

b. Determine the total number of bacteria present after 3.2 hours.

$t = 3.2$

$P(t) = 2(2)^{\frac{3.2}{0.68}}$

$P(t) = \boxed{52 \text{ bacteria}}$

* round to whole # for living things

c. Determine the time it took for the bacteria population to reach 4,096.

$P(t) = 4096$

$\frac{4096}{2} = \frac{2(2)^{\frac{t}{0.68}}}{2}$

$2048 = 2^{t/0.68}$

common base → $2^{11} = 2^{\frac{t}{0.68}}$

cancel base

$11 = \frac{t}{0.68}$

$\boxed{t = 7.48 \text{ hours}}$

$A = A_0(b)^{\frac{t}{n}}$

3. Given the following information, determine an exponential equation that represents the growth or decay.

a. A 5.3 gram sample of Carbon-14 that decays and has a half-life of 5740 years.

$A = 5.3\left(\frac{1}{2}\right)^{\frac{t}{5740}}$

b. The growth rate of mold cells that triple every 5 days. Assume you start with one cell.

$A = (3)^{\frac{t}{5}}$

↑ $1+r$

c. The population of Langley increases by 0.5% per year with a current population of 127,000 people.

$A = 127000(1.005)^t$

↓ $1-r$

d. The percent of light that is lost with every 2 meter depth in a lake is 25%.

$A = (0.75)^{\frac{d}{2}}$

HIGH - LOW

$I = 10$

4. Determine how much more intense and earthquake of Magnitude 9.4 on the Richter scale would be in comparison to an earthquake of Magnitude 3.7 on the Richter scale.

$I = 10^{9.4-3.7}$

$I = 10^{5.7}$

$\boxed{I = 501,187.23 \text{ times more intense}}$

5. Write the following in exponential form:
   a. $\log_9 h = 2p$

Boot the base

$9^{2p} = h$

   b. $\log_t 5 = r^2$

BOOT

$t^{r^2} = 5$

6. Write the following in logarithmic form:
   a. $2x = y^{1.5}$

base is base
log = exponent

$\log_y 2x = 1.5$

   b. $99^{0.034} = w$

"

$\log_{99} w = 0.034$

7. Expand the following logarithms:

a. $\log_7\left(\frac{x^2\sqrt[3]{y}}{z^4}\right) = \log_7 x^2 + \log_7 \sqrt[3]{y} - \log_7 z^4$

product, quotient law

$= 2\log_7 x + \log_7 y^{\frac{1}{3}} - 4\log_7 z$

power law

$= \boxed{2\log_7 x + \frac{1}{3}\log_7 y - 4\log_7 z}$

b. $\log_2\left(\frac{8r^3}{t^2\sqrt{w}}\right) = \log_2 8 + \log_2 r^3 - (\log_2 t^2 + \log_2 \sqrt{w})$

power, quotient law

$= \cancel{\log_2} \cancel{2}^3 + 3\log_2 r - 2\log_2 t - \log_2 w^{\frac{1}{2}}$

expand power law

$= \boxed{3 + 3\log_2 r - 2\log_2 t - \frac{1}{2}\log_2 w}$

simplify (common base)

8. Write as a single logarithm and simplify:

a. $\frac{1}{3}(\underbrace{\log_a x - \log_a \sqrt{x}}) + \log_a 3x^2$

quotient law

subtract exponents

$1 - \frac{1}{2} \to \frac{2}{2} - \frac{1}{2}$
$= \frac{1}{2}$

$\frac{1}{3}\log_a\left(\frac{x}{x^{\frac{1}{2}}}\right) + \log_a 3x^2$

$\frac{1}{3}\log_a x^{\frac{1}{2}} + \log_a 3x^2$

product law

$\frac{1}{3}\log_a\left(x^{\frac{1}{2}} \cdot 3x^2\right)$ $\to \boxed{\frac{1}{3}\log_a\left(3x^{\frac{5}{2}}\right)}$

add exponents $\to \frac{1}{2} + 2 \to \frac{1}{2} + \frac{4}{2} \to \frac{5}{2}$

b. $2\log 8x - 3\log x - \log 5 + 2\log x$

power law

$\log(8x)^2 - \log x^3 - \log 5 + \log x^2$

product, quotient law

$= \log\left(\frac{64x^2 \cdot x^2}{x^3 \cdot 5}\right)$

$= \boxed{\log\left(\frac{64x}{5}\right)}$

9. Solve the following (hint: change to exponential form):

a. $\log_3(x-5)=2$

Restrictions/NPVs

BOOT

$x-5=3^2$

$x-5>0$

$\xrightarrow{+5}$

$x-5=9$

$x>5$

$\xrightarrow{+5}$

$\boxed{x=14}$ ✓

b. $\log_x\left(\dfrac{49}{169}\right)=2$

NPVs

$\sqrt{x^2}=\sqrt{\dfrac{49}{169}}$

$x>0,\ x\neq 1$

$x=\pm\dfrac{7}{13}$

$\boxed{x=\dfrac{7}{13}}$ $x\neq -\dfrac{7}{13}$ extraneous

10. Solve the following (algebraically using log laws):

a. $\log_2(x-4)=4-\log_2(x+2)$

collect logs + product law

$+\log_2(x+2)$

BOOT

$\log_2(x-4)(x+2)=4$

expand + solve

$(x-4)(x+2)=2^4$

$x^2-2x-8=16$

-16

$x^2-2x-24=0$

$(x-6)(x+4)=0$

$\boxed{x=6}$ $x\neq -4$ extraneous

NPV $x-4>0$ $x+2>0$

$x>4$ $x>-2$

b. $\log_3(x-4)-\log_3(x+2)=\log_3 5$

quotient law

$\cancel{\log_3}\left(\dfrac{x-4}{x+2}\right)=\cancel{\log_3}5$

cancel logs

$\dfrac{x-4}{x+2}=5\,(x+2)$

NPV

$x-4>0$ $x+2>0$

$x>4$ $x>-2$

solve

$x-4=5x+10$

$-5x$ $+4$

$-4x=14$

$\div -4$ $\div -4$

$\rightarrow$ $x\neq -\dfrac{7}{2}$ extraneous $\boxed{x=\text{no solution}}$

11. Solve algebraically (use logs):

- log both sides
- expand
- solve for x

a. $5^{2x+1} = 11^{x-2}$

$(2x+1)\log 5 = (x-2)\log 11$

$2x\log 5 + \log 5 = x\log 11 - 2\log 11$

$-x\log 11 \qquad -\log 5$

$2x\log 5 - x\log 11 = -2\log 11 - \log 5$

factor x

$x(2\log 5 - \log 11) = -2\log 11 - \log 5$

$\boxed{x = \dfrac{-2\log 11 - \log 5}{2\log 5 - \log 11}}$

b. $4^{2x+1} = 9\left(4^{1-x}\right)$

$\dfrac{4^{2x+1}}{4^{1-x}} = 9$

subtract exponents

$(2x+1)-(1-x)$

$2x+1-1+x$

$3x$

$\log 4^{3x} = \log 9$

$\dfrac{3x\log 4}{3\log 4} = \dfrac{\log 9}{3\log 4}$

$\boxed{x = \dfrac{\log 9}{3\log 4}}$

12. Simplify and solve (use log laws):

a. $7^{\log_7 x + 2\log_7 x} = 216$

$\cancel{7}^{\log_7 x\cdot x^2} = 216$

$\sqrt[3]{x^3} = \sqrt[3]{216}$

$\boxed{x = 6}$ ✓

NPV

$x > 0$

$x^2 > 0$

$x \neq 0$

b. $\log_2\left(\log_{9x^2}\left(\log_4 16\right)\right) = -1$

$\log_{9x^2}\left(\log_4 4^2\right) = 2^{-1}$

$2 = (9x^2)^{\frac{1}{2}}$

$\dfrac{2}{3} = \dfrac{\sqrt{9}\,x}{3}$

$\boxed{x = \frac{2}{3}}$ ✓

NPV

$9x^2 > 0,\ 9x^2 \neq 1$

$x \neq 0,\ x \neq \pm\frac{1}{3}$

c. $x\log_{25} 5 = 8 - \log_6 36$

common base · common base

$x\log_{25} 25^{\frac{1}{2}} = 8 - \log_6 6^2$

$x\left(\frac{1}{2}\right) = 8 - 2$

$\dfrac{\frac{1}{2}x}{\frac{1}{2}} = \dfrac{6}{\frac{1}{2}}$

$\boxed{x = 12}$ ✓

NPV

$x \in \mathbb{R}$

13. The following function represents the blood pressure, in mmHg, of young adults related to the blood vessel volume, in microlitres.

$$V(p) = 0.23 + 0.35\log(p - 56.1)$$

a. Determine the blood vessel volume if the blood pressure is 120mmHg.

$$V(p) = 0.23 + 0.35\log(120 - 56.1)$$
$$= 0.23 + 0.35\log 63.9$$
$$V(p) = 0.86\,\mu l$$

b. Determine the blood pressure if the vessel volume is 0.9 microlitres.

$$0.9 = 0.23 + 0.35\log(p - 56.1)$$
$$-0.23$$
$$\frac{0.67}{0.35} = \frac{0.35\log(p-56.1)}{0.35}$$
$$\frac{0.67}{0.35} = \log_{10}(p-56.1)$$
BOOT
$$10^{\frac{0.67}{0.35}} = p - 56.1$$
$$+56.1$$
$$p = 10^{\frac{0.67}{0.35}} + 56.1$$
$$\boxed{p = 138.19 \text{ mmHg}}$$

14. Given the following information, determine an exponential equation that represents the growth or decay, then answer the question algebraically:

a. Carbon-14 has a half-life of 5740 years. How long would it take a 5.3 gram sample to decay to 1.9 grams.

$$\frac{1.9}{5.3} = \frac{5.3(0.5)^{\frac{t}{5740}}}{5.3}$$
$$\log\frac{1.9}{5.3} = \log 0.5^{t/5740}$$
$$5740 \times \frac{\log\left(\frac{1.9}{5.3}\right)}{\log 0.5} = \frac{\frac{t}{5740}\log 0.5}{\log 0.5}$$
$$t = \frac{5740\log\left(\frac{1.9}{5.3}\right)}{\log 0.5}$$
"exact"
$$\boxed{t = 8495.16 \text{ years old}}$$

b. A certain kind of mold triples every 7 days. How long would it take a sample of 125 cells to grow to 100,000 cells?

$$\frac{100000}{125} = \frac{125(3)^{\frac{t}{7}}}{125}$$

$$\log 800 = \log 3^{t/7}$$

$$7 \times \frac{\log 800}{\log 3} = \frac{t}{7}\frac{\log 3}{\log 3}$$

$$t = \frac{7\log 800}{\log 3} \quad \text{exact}$$

$$\boxed{t = 42.59 \text{ days}}$$

(43 days)

c. The population of Langley increases by 0.5% per year with a current population of 127,000 people. What would the populations be in 10 years?

$$A = 127{,}000(1.005)^{10}$$

$$\boxed{A = 133494 \text{ people}}$$ OR 133495 ppl. OK too.

d. How long would it take for the populations to reach 200,000 people?

$$\frac{200{,}000}{127000} = \frac{127000(1.005)^t}{127000}$$

$$\log \frac{200}{127} = \log(1.005)^t$$

$$\frac{\log\left(\frac{200}{127}\right)}{\log 1.005} = \frac{t \log 1.005}{\log 1.005}$$

$$t = \frac{\log\left(\frac{200}{127}\right)}{\log 1.005}$$

$$\boxed{t = 91.05 \text{ yr.}}$$

91yr

e. If an earthquake in Town A is 3500 times more intense than an earthquake of magnitude 4.2 in Town B, what is the magnitude of the earthquake in Town A?

$$\log 3500 = \log 10^{A-4.2} \qquad \log 10 = 1$$

$$\log 3500 = A - 4.2$$

+4.2

$$A = \log 3500 + 4.2 \rightarrow \boxed{A = 7.4}$$ Magnitude Earthquake Town A

f. If the pH of stomach acid at 2.2 is 50,000 times more acidic than urine, what is the pH of urine?

$$\log 50000 = \log 10^{u-2.2}$$

$$\log 50000 = u - 2.2$$

+2.2

$$u = \log 50000 + 2.2$$

$$\boxed{u = 6.9 \text{ pH for urine}}$$