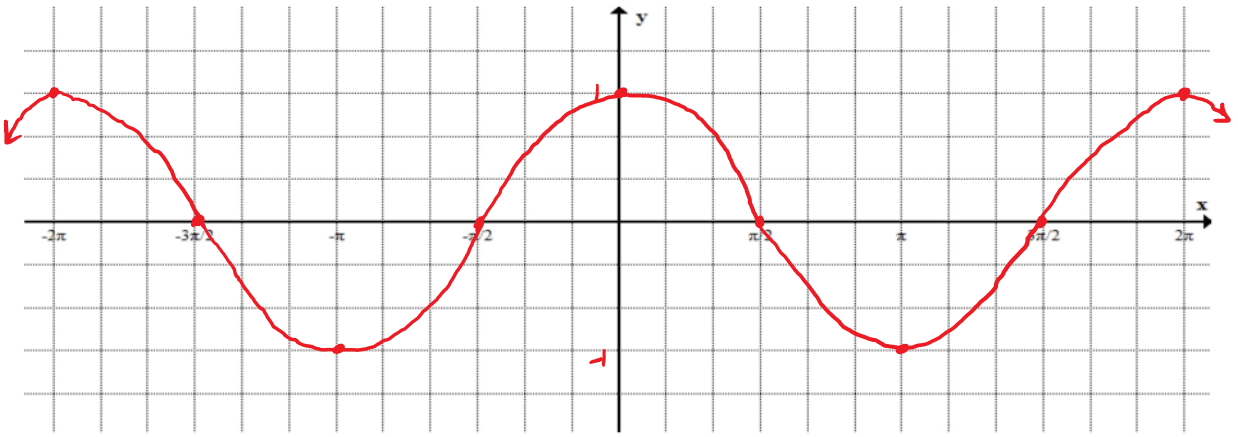
Chapter 5 Homework to show for completion marks (5/5)

Due KEY

1. a. Graph $y = \cos x$ for $-2\pi \le \theta \le 2\pi$



b. State the domain and range

$\{x \mid x \in \mathbb{R}\}$

$\{y \mid -1 \le y \le 1, y \in \mathbb{R}\}$.

c. State the period and amplitude.

$p = 2\pi$

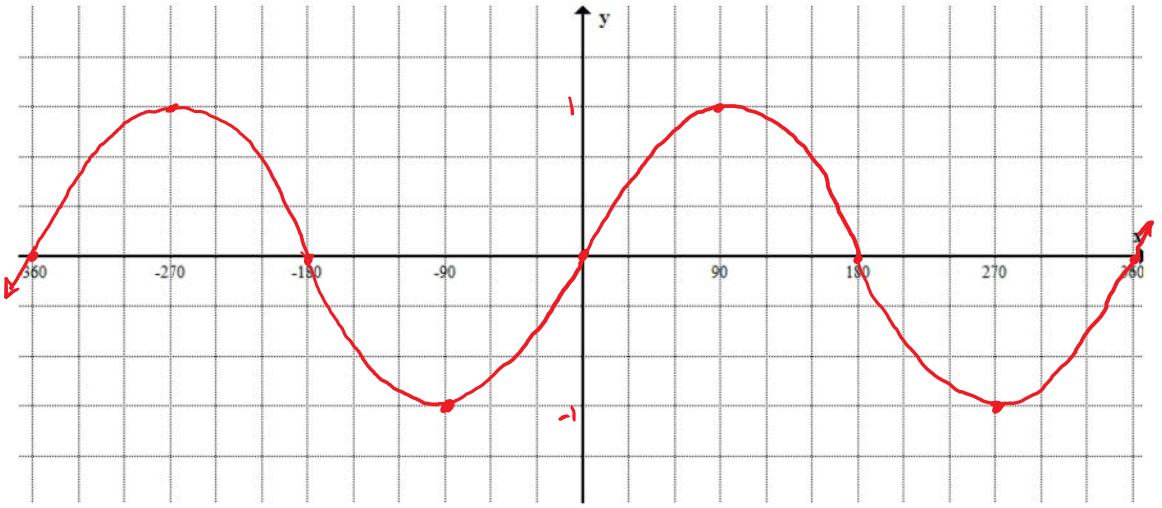
amp = 1

d. State the general equation for all x-intercepts.

1st x-intercept at $\frac{\pi}{2}$ + others are every π units away ∴

$x = \frac{\pi}{2} + \pi n, n \in I$

2. a. Graph $y = \sin x$ for $-360^\circ \le \theta \le 360^\circ$



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b. State the period and amplitude.

$p = 2\pi$

amp = 1

c. State the general equation for all x-intercepts.

1st x-intercept at 0 & next ones are every π units away ∴

$x = \pi n, n \in I$

3. Give the function $y = 5\sin\left(2\left(x - \frac{\pi}{4}\right)\right) - 3$, state the following characteristics:

a. Amplitude 5

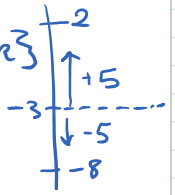
b. Period $= \frac{2\pi}{2} = \pi$

c. Phase shift $\frac{\pi}{4}$ (right)

d. Vertical displacement $= -3$ (down 3)

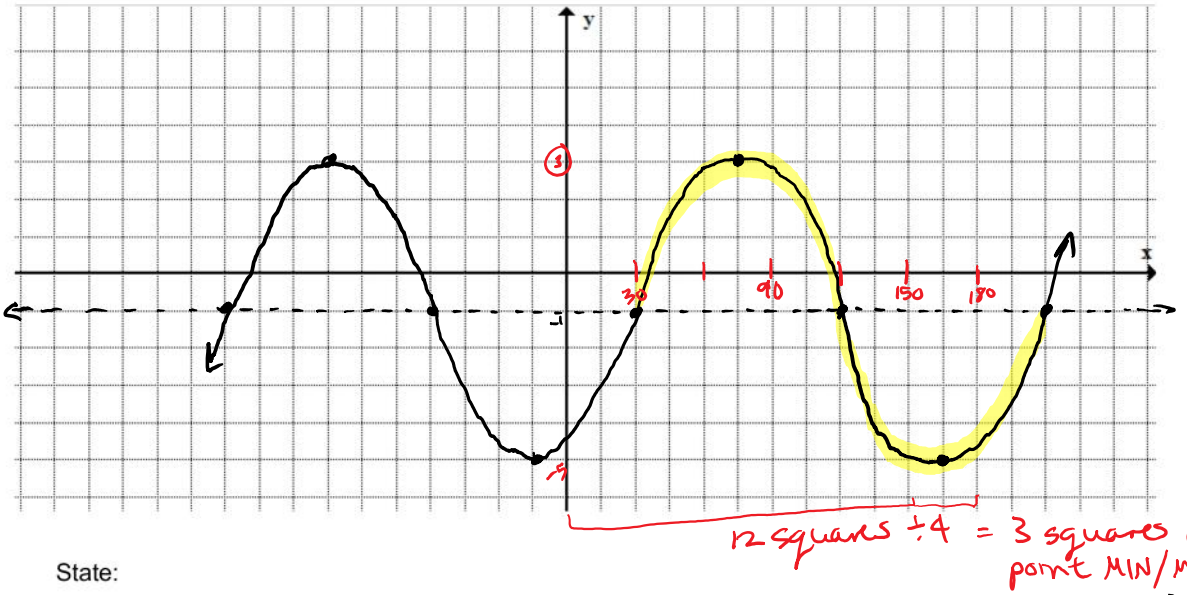
e. Domain $\{x \mid x \in \mathbb{R}\}$

f. Range $\{y \mid -8 \leq y \leq 2, y \in \mathbb{R}\}$



4. Graph the following function $y = 4\sin(2x - 60^\circ) - 1$ Graph 2 complete cycles.

$y = 4\sin(2(x - 30)) - 1$ $a = 4$ VD $= -1$ $P = \frac{360}{2} = 180^\circ$ PS $= 30$ (right)



12 squares ÷ 4 = 3 squares per point MIN/MAX/MID

State:

a. Amplitude $= 4$

b. Period $= \frac{360}{2} = 180^\circ$

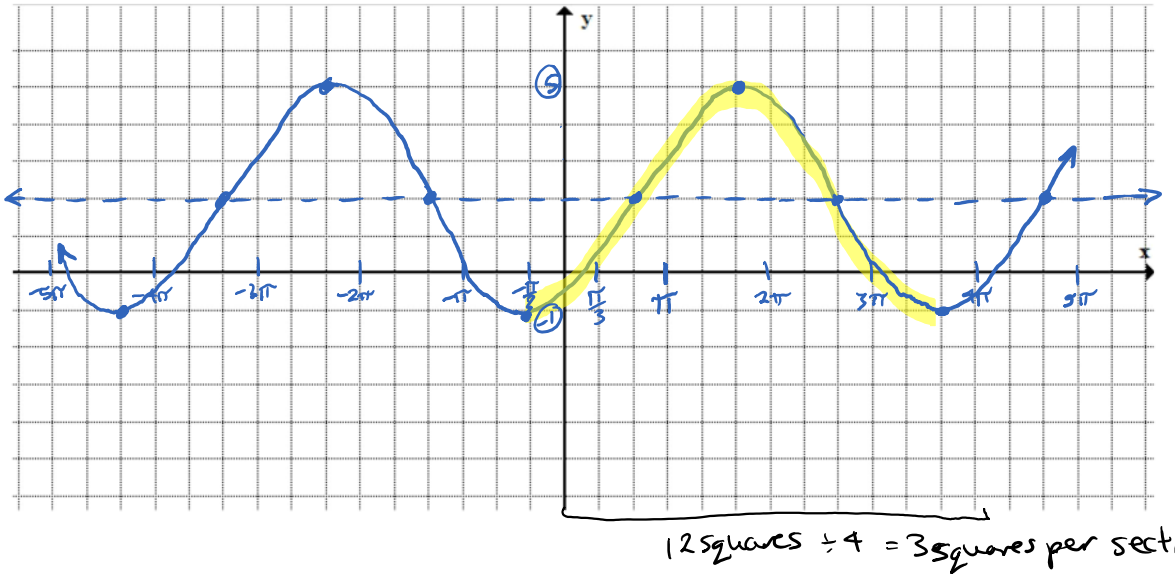
c. Phase shift $= 30$ (right)

d. Vertical displacement $= -1$ (down)

e. Domain $= \{x \mid x \in \mathbb{R}\}$

f. Range $= \{y \mid -5 \leq y \leq 3, y \in \mathbb{R}\}$

5. Graph the following function $y = -3\cos\left(\frac{1}{2}\left(x+\frac{\pi}{3}\right)\right)+2$ Graph 2 complete cycles.



State:

a. Amplitude = 3 (- is reflection over x-axis)

b. Period = $2\pi \times \frac{2}{1} = 4\pi$

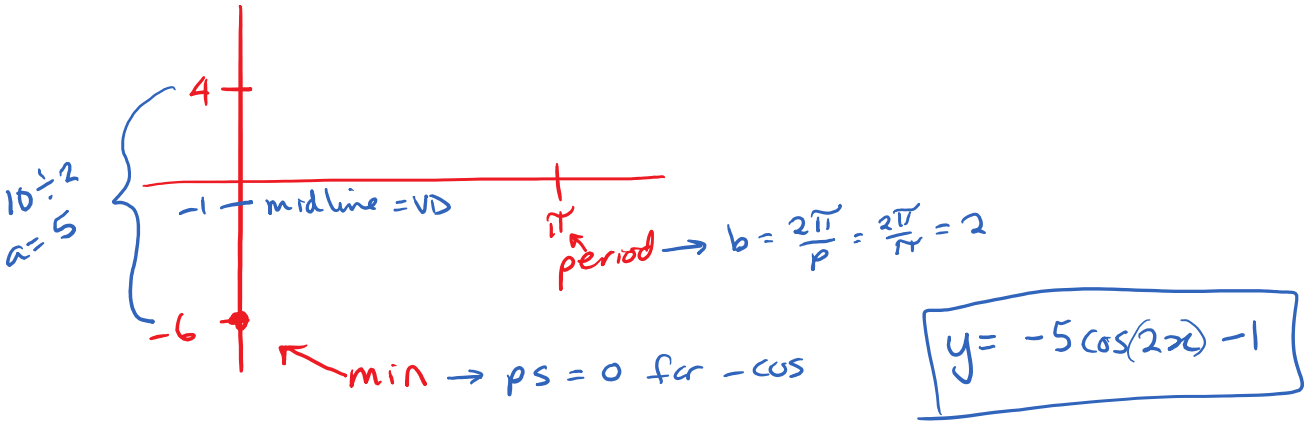
c. Phase shift = $-\frac{\pi}{3}$ (left)

d. Vertical displacement = 2 (up)

e. Domain = $\{x \mid x \in \mathbb{R}\}$

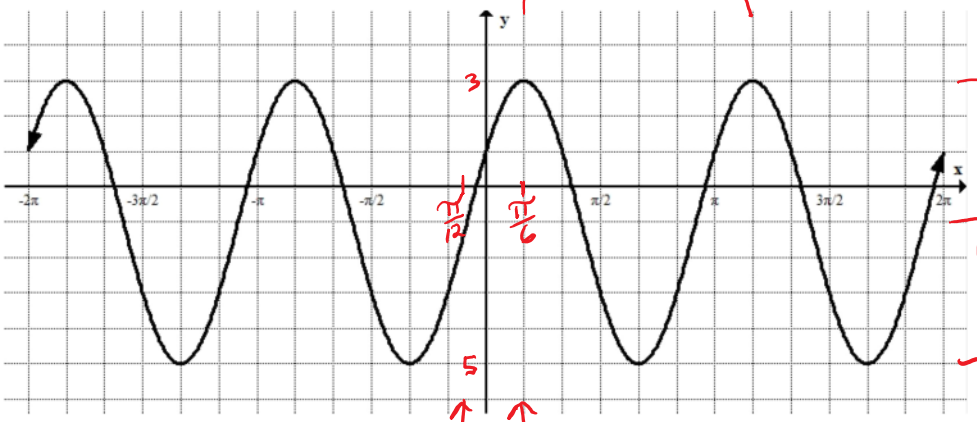
f. Range = $\{y \mid -1 \le y \le 5, y \in \mathbb{R}\}$

6. If the range of a function is $-6 \le y \le 4$ and the minimum occurs at $(0,-6)$ and the period of the function is π, determine the equation of the function.



7. Determine the equation of each of the following functions.

a.



$p = \pi \rightarrow b = \frac{2\pi}{\pi} = 2$

$8 \div 2 \rightarrow a = 4$

$VD = -1$

$ps = \frac{\pi}{6}$ for +cos

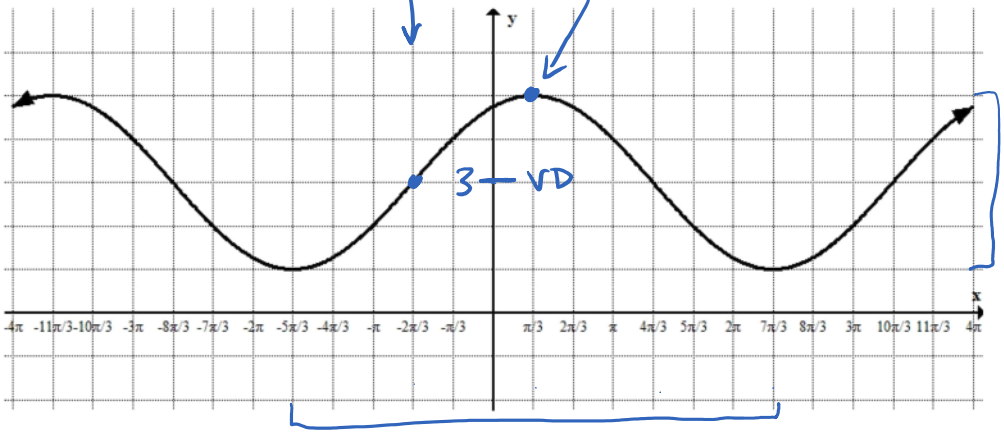
$ps = -\frac{\pi}{12}$ for +sin

$y = 4\cos\left(2\left(x - \frac{\pi}{6}\right)\right) - 1$

or

$y = 4\sin\left(2\left(x + \frac{\pi}{12}\right)\right) - 1$

b.



$ps = -\frac{2\pi}{3}$ for +sin

$ps = \frac{\pi}{3}$ for +cos

$4 \div 2$

$a = 2$

$p = 4\pi$

$\hookrightarrow b = \frac{2\pi}{4\pi} = \frac{1}{2}$

$y = 2\cos\left(\frac{1}{2}\left(x - \frac{\pi}{3}\right)\right) + 3$

or

$y = 2\sin\left(\frac{1}{2}\left(x + \frac{2\pi}{3}\right)\right) + 3$

9. The rotation of the wheel of a bicycle is can be modelled by the equation $h(t) = 22\sin\left(\frac{\pi}{1.04}t\right) + 22$ where h is the height in cm, and t is the time in seconds.

a. Determine the height of a mark on the tire's tread after 12.3 seconds.

$t = 12.3s$

$h(12.3) = 22\sin\left(\frac{\pi}{1.04}(12.3)\right) + 22$ (in calc., radian mode)

$= 10.62$ cm

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22sin(π/1.04(12.
3))+22
      10.61856809
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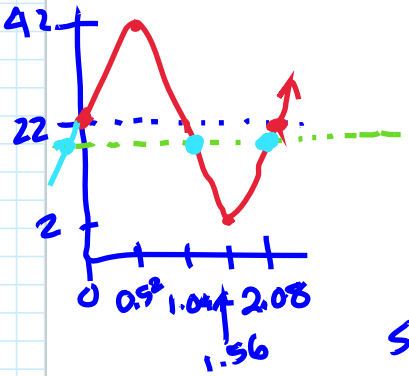
b. Determine the first 3 times that the height of the mark in the tread is 20 cm above the ground.



$20 = 22\sin\frac{\pi}{1.04}t + 22$

-22

$-2 = 22\sin\frac{\pi}{1.04}t$

$\frac{-2}{22} = \frac{22}{22}\sin\frac{\pi}{1.04}t$

$\sin^{-1}\left(-\frac{1}{11}\right) = \sin\frac{\pi}{1.04}t$

$\sin^{-1}\left(-\frac{1}{11}\right) = \frac{\pi}{1.04}t$

$\left(\frac{1.04}{\pi}\right)\times$

$t = \frac{1.04}{\pi}\sin^{-1}\left(-\frac{1}{11}\right)$

radian mode for calc

$t = -0.0301$

$+ 2.08$

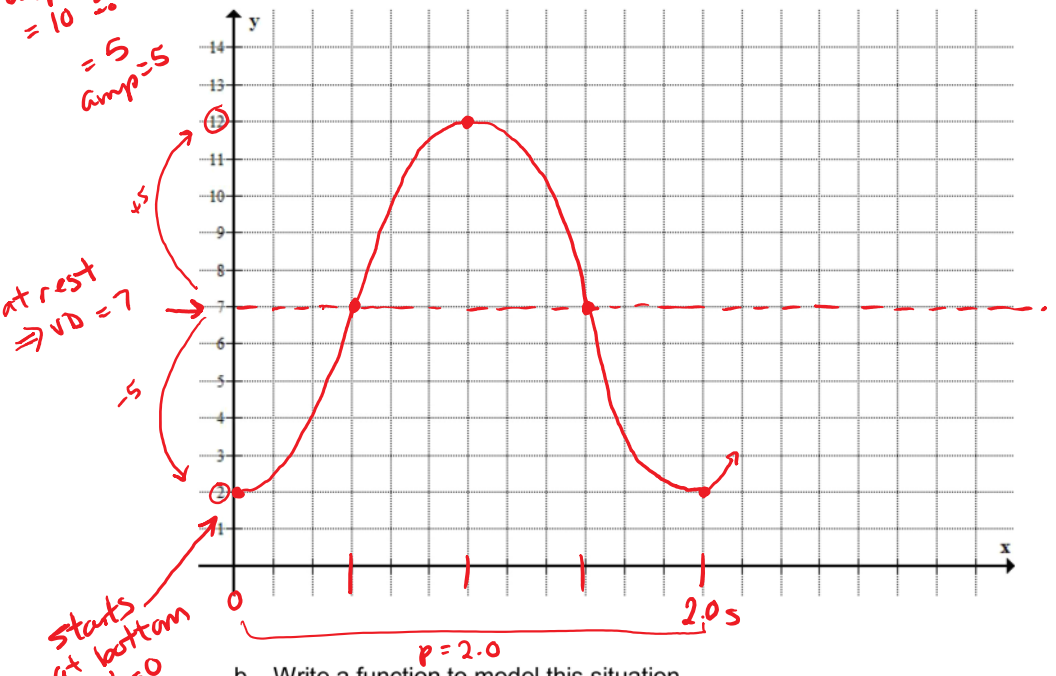
$t_1 = 1.04 + 0.0301 = 1.07$ sec

$t_2 = -0.0301 + 2.08 = 2.05$ sec

10. A mass on a spring is bouncing up and down. The mass's distance from the floor when it is at rest is 7 cm. The maximum displacement is 10cm as it bounces. It takes 2 s to complete one bounce or cycle. Suppose the mass is pulled down 5 cm first and let go at t=0.

a. Draw a sketch of the sinusoidal function that model's this situation.

displacement = 10 ÷ 2 = 5 amp = 5



b. Write a function to model this situation.

$a = 5$ $\quad VD = 7$ $\quad p = 2.0 \longrightarrow b = \frac{2\pi}{2.} = \pi$ $\quad$ p.s. = 0 for -cos

$$y = -5\cos(\pi x) + 7$$

$\cos(\pi(x))$

c. Algebraically determine the height after 0.8 s.

$$y = -5\cos(\pi \cdot 0.8) + 7$$

$$y = 11.05 \text{ cm}$$

d. Algebraically determine the time it takes to reach a height of 9cm.

$$9 = -5\cos(\pi x) + 7$$

$$-7 \quad -7$$

$$\frac{2}{-5} = \frac{-5\cos(\pi x)}{-5}$$

$$-\frac{2}{5} = \cos(\pi x)$$

$$\cos^{-1}\left(-\frac{2}{5}\right) = \pi x$$

$$\frac{\cos^{-1}\left(-\frac{2}{5}\right)}{\pi} = \frac{\pi x}{\pi}$$

$$x = \frac{\cos^{-1}\left(-\frac{2}{5}\right)}{\pi}$$

$$x = 0.63 \text{ sec.}$$