Chapter 6 Review Practice Questions: Simplifying, Verifying, and Proving Trig Identities, & Solving Trig Equations

1. Simplify the following:

a. $\dfrac{\sin x + \tan x}{1+\cos x}$

① $\dfrac{\sin x + \dfrac{\sin x}{\cos x}}{1+\cos x}$

common denom ② $\left(\dfrac{\cos x}{\cos x}\right)$

③ combine $\dfrac{\dfrac{\cos x \sin x}{\cos x} + \dfrac{\sin x}{\cos x}}{1+\cos x}$

④ multiply by reciprocal $\dfrac{\cos x \sin x + \sin x}{\cos x} \times \dfrac{1}{1+\cos x}$

⑤ factor $\dfrac{\sin x(\cancel{\cos x + 1})}{\cos x} \times \dfrac{1}{\cancel{\cos x + 1}} \Rightarrow \dfrac{\sin x}{\cos x} \Rightarrow \boxed{\tan x}$

b. $\dfrac{\tan^2 x}{\tan^2 x + 1} \Rightarrow \dfrac{\dfrac{\sin^2 x}{\cos^2 x}}{\sec^2 x}$ ① quotient ident. ② pythag. ident

$\dfrac{\dfrac{\sin^2 x}{\cos^2 x}}{\dfrac{1}{\cos^2 x}}$ ③ reciprocal ident.

$\dfrac{\sin^2 x}{\cancel{\cos^2 x}} \times \dfrac{\cancel{\cos^2 x}}{1}$ ④ multiply by reciprocal

$\boxed{\sin^2 x}$

2. Verify the following identity algebraically using $\theta = 45^\circ$ and $\theta = \dfrac{\pi}{6}$

$$\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ 45

$\dfrac{1-\sin 45^\circ}{\cos 45^\circ} = \dfrac{\cos 45^\circ}{1+\sin 45^\circ}$

common denom $\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$ $\dfrac{1-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \dfrac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$ $\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

conjugate $\left(\frac{\sqrt{2}+1}{\sqrt{2}+1}\right)$ $\dfrac{\sqrt{2}-1}{1} = \dfrac{1}{\sqrt{2}+1}$

$\dfrac{2-1}{\sqrt{2}+1}$

$\dfrac{1}{\sqrt{2}+1} = \dfrac{1}{\sqrt{2}+1}$ ✓verified.

$\dfrac{\left(\frac{2}{2}\right)1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \dfrac{\frac{\sqrt{2}}{2}}{\left(\frac{2}{2}\right)1+\frac{\sqrt{2}}{2}}$

$\dfrac{\frac{2-\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ $\dfrac{\frac{\sqrt{2}}{2}}{\frac{2+\sqrt{2}}{2}}$

$\dfrac{2-\sqrt{2}}{\cancel{2}} \times \dfrac{\cancel{2}}{\sqrt{2}}$ $\dfrac{\sqrt{2}}{\cancel{2}} \times \dfrac{\cancel{2}}{2+\sqrt{2}}$ conjugate

rationalize $\dfrac{2-\sqrt{2}}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{\sqrt{2}}\right)$ $\dfrac{\sqrt{2}}{2+\sqrt{2}}\left(\dfrac{2-\sqrt{2}}{2-\sqrt{2}}\right)$

$\dfrac{\sqrt{2}(2-\sqrt{2})}{4-2}$

$\dfrac{\cancel{2}\sqrt{2}-\cancel{2}}{\cancel{2}}$ $\dfrac{\cancel{2}\sqrt{2}-\cancel{2}}{\cancel{2}}$

$\sqrt{2}-1 = \sqrt{2}-1$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\dfrac{1-\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}} = \dfrac{\cos\frac{\pi}{6}}{1+\sin\frac{\pi}{6}}$

$\dfrac{1-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}}$

$\dfrac{\frac{2}{2}-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{\frac{\sqrt{3}}{2}}{\frac{2}{2}+\frac{1}{2}}$

$\dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}$

$\frac{1}{2} \times \frac{2}{\sqrt{3}}$ $= \frac{\sqrt{3}}{2} \times \frac{2}{3}$

rationalize $\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{\sqrt{3}}{3}$

$\dfrac{\sqrt{3}}{3} = \dfrac{\sqrt{3}}{3}$ verified ✓

3. Write the following as a single trigonometric function:

a. $\cos 27^{\circ} \cos 33^{\circ} - \sin 27^{\circ} \sin 33^{\circ}$

$$= \cos(27^{\circ} + 33^{\circ})$$

$$= \boxed{\cos 60^{\circ}}$$

b. $2\cos^2\left(\frac{\pi}{3}\right) - 1$

$$= \cos 2\left(\frac{\pi}{3}\right)$$

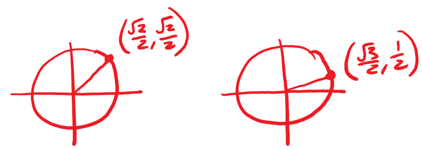
$$= \boxed{\cos \frac{2\pi}{3}}$$

c. $2\sin 3x \cos 3x$

$$= \sin 2(3x)$$

$$= \boxed{\sin 6x}$$

4. Determine the exact value of the following:



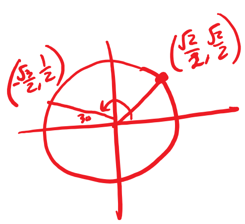
a. $\cos \frac{5\pi}{12} = \cos\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$

$$= \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \underbrace{\cos\frac{\pi}{4}}_{\left(\frac{\sqrt{2}}{2}\right)} \underbrace{\cos\frac{\pi}{6}}_{\left(\frac{\sqrt{3}}{2}\right)} - \sin\frac{\pi}{4} \sin\frac{\pi}{6}$$

$$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$\boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

b. $\sin 195^{\circ} = \sin(150^{\circ} + 45^{\circ})$



$$= \underbrace{\sin 150^{\circ}}_{\left(\frac{1}{2}\right)} \underbrace{\cos 45^{\circ}}_{\left(\frac{\sqrt{2}}{2}\right)} + \underbrace{\cos 150^{\circ}}_{\left(-\frac{\sqrt{3}}{2}\right)} \underbrace{\sin 45^{\circ}}_{\left(\frac{\sqrt{2}}{2}\right)}$$

$$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

5. Prove the following identities:

a. $\frac{1-\sin^2 x-2\cos x}{\cos^2 x-\cos x-2}=\frac{1}{1+\sec x}$

$$\frac{\cos^2 x-2\cos x}{(\cos x-2)(\cos x+1)} \quad\Big|\quad \frac{1}{1+\frac{1}{\cos x}}$$

$$\frac{\cos x(\cancel{\cos x-2})}{(\cancel{\cos x-2})(\cos x+1)} \quad\Big|\quad \frac{1}{\frac{\cos x}{\cos x}+\frac{1}{\cos x}}$$

$$\frac{\cos x}{\cos x+1} \quad\Big|\quad \frac{1}{\frac{\cos x+1}{\cos x}}$$

$$\frac{\cos x}{\cos x+1} = \frac{\cos x}{\cos x+1}$$

b. $\frac{\csc x\cos x}{\tan x+\cot x}=\cos^2 x$

reciprocal + quotient ident. sub.

$$\frac{\frac{1}{\sin x}\cdot\cos x}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}$$

common denom.

$$\frac{\frac{\cos x}{\sin x}}{\left(\frac{\sin x}{\sin x}\right)\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\left(\frac{\cos x}{\cos x}\right)}$$

$$\frac{\frac{\cos x}{\sin x}}{\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}}$$

multiply by reciprocal

pythag. ident.

simplify.

$$\frac{\cos x}{\cancel{\sin x}}\times\frac{\cancel{\sin x}\cos x}{1}$$

$$\cos^2 x = \cos^2 x$$

c. $\sec x + \tan x = \dfrac{\cos x}{1-\sin x}$

$$\frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

multiply by the conjugate

$$\left(\frac{1-\sin x}{1-\sin x}\right) \frac{1+\sin x}{\cos x}$$

$$\frac{1-\sin^2 x}{\cos x(1-\sin x)}$$

pythag ident.

$$\frac{\cos^{\cancel{2}} x}{\cancel{\cos x}(1-\sin x)}$$

$$\frac{\cos x}{1-\sin x} = \frac{\cos x}{1-\sin x}$$

6. Solving the following:

a. $\cos x \tan x - \sin^2 x = 0$ $\qquad 0 \le \theta \le 2\pi$

identity

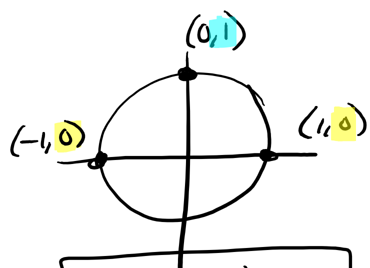
$$\cancel{\cos x}\left(\frac{\sin x}{\cancel{\cos x}}\right) - \sin^2 x = 0$$

factor

$$\sin x - \sin^2 x = 0$$

$$\sin x(1-\sin x) = 0$$

$$\sin x = 0 \qquad \sin x = 1$$



$$\boxed{x = 0, \frac{\pi}{2}, \pi, 2\pi}$$

b. $\sin^2 x = \cos x - \cos 2x$ $\qquad 0^\circ \le \theta \le 360^\circ$

$$1-\cos^2 x = \cos x - (2\cos^2 x - 1)$$

$$\cancel{1} - \cos^2 x = \cos x - 2\cos^2 x + 1$$

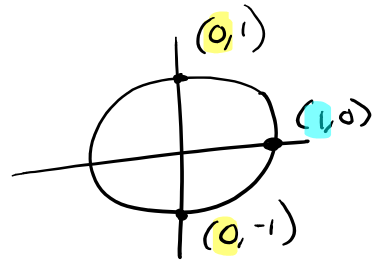
$$-\cos x + 2\cos^2 x$$

$$\cancel{-1}$$

$$\cos^2 x - \cos x = 0$$

$$\cos x(\cos x - 1) = 0$$

$$\cos x = 0 \qquad \cos x = 1$$



$$x = 0^\circ, 90^\circ, 270^\circ, 360^\circ$$

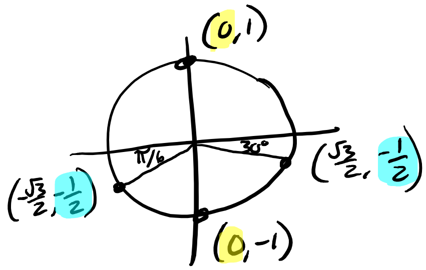
c. $\sin 2x + \cos x = 0$ Provide general solutions in radians AND degrees.

$2\sin x\cos x + \cos x = 0$

factor

$\cos x(2\sin x + 1) = 0$

$\cos x = 0 \qquad \sin x = \frac{-1}{2}$



$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

in general

$x = \frac{\pi}{2} + \pi n, n \in I$ (covers $\frac{3\pi}{2}$)

$x = \frac{7\pi}{6} + 2\pi n, n \in I$

$x = \frac{11\pi}{6} + 2\pi n, n \in I$

$x = 90°, 210°, 270°, 330°$

in general

$x = 90° + 180°n, n \in I$ (covers 270°)

$x = 210° + 360°n, n \in I$

$x = 330° + 360°n, n \in I$