KEY

Chapter 6: Unit Circle, Trig Ratios, Special Angles & Graphing Practice

1. Convert each of the following to degrees:

a. $\frac{3\pi}{7} \times \frac{180}{\pi} = 77.14^\circ$

b. $-\frac{5\pi}{3} \times \frac{180}{\pi} = -300^\circ$

c. 2.47 *radians* $\times \frac{180}{\pi} = 141.52^\circ$

2. Covert each of the following to radians (leave in exact form):

a. $140^\circ \times \frac{\pi}{180} = \frac{7\pi}{9}$

b. $-625^\circ \times \frac{\pi}{180} = \frac{125\pi}{36}$

c. $540^\circ \times \frac{\pi}{180} = 3\pi$

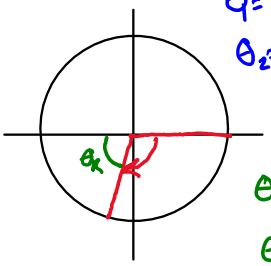
3. For each of the following angles, graph in standard position, find two coterminal angles and determine the reference angle:

a. -105°

$\theta = -105^\circ + 360^\circ = 255^\circ$

$\theta = -105^\circ - 360^\circ = -465^\circ$

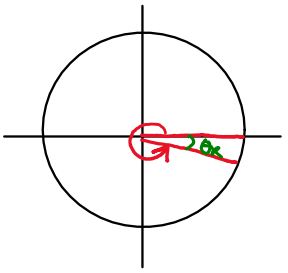
+other possible answers



$\theta_R = 180 - 105$

$\theta_R = 75^\circ$

b. $\frac{11\pi}{6}$



$\theta_1 = \frac{11\pi}{6} + \frac{12\pi}{6} = \frac{23\pi}{6}$ (2π)

$\theta_1 = \frac{11\pi}{6} - \frac{12\pi}{6} = -\frac{\pi}{6}$

$\theta_R = \frac{\pi}{6}$

4. Determine the arc length subtended by the angle, 85°, of a circle of radius 11cm.

$a = r\theta$

$85 \times \frac{\pi}{180} = \frac{17\pi}{36}$

$a = 11\left(\frac{17\pi}{36}\right)$

$\boxed{a = 16.32\text{ cm}}$

5. If a swing that is 4 meters long produces an arc of 9 meters, determine the angle (in degrees) that is produced by each swing.

$a = r\theta \longrightarrow \frac{9}{4} = \frac{4\theta}{4}$

$\theta = \frac{9}{4} \times \frac{180^\circ}{\pi}$

$\boxed{\theta = 128.92^\circ}$

6.

a. Determine the other trig ratios if $\cos\theta = -\frac{3}{4}$.

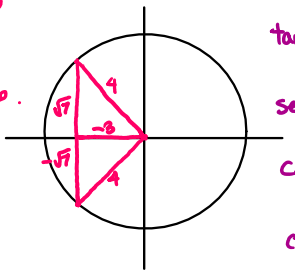
adj -3, hyp 4, QII + QIII

$y^2 = 4^2 - (-3)^2$

$y = \pm\sqrt{4^2 - (-3)^2}$

$y = \pm\sqrt{16 - 9}$

$y = \pm\sqrt{7}$



$\sin\theta = \pm\frac{\sqrt{7}}{4}$

$\tan\theta = \pm\frac{\sqrt{7}}{3}$

$\sec\theta = -\frac{4}{3}$

$\csc\theta = \pm\frac{4}{\sqrt{7}}$ or $\pm\frac{4\sqrt{7}}{7}$

$\cot\theta = \pm\frac{3}{\sqrt{7}}$ or $\pm\frac{3\sqrt{7}}{7}$

b. Determine the other trig ratios if $\sin\theta = -\frac{\sqrt{3}}{2}$.

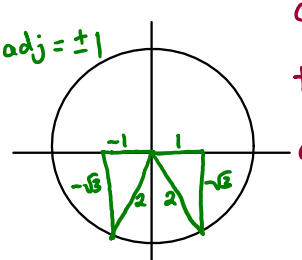
opp $-\sqrt{3}$, hyp 2, QIII + QIV

$x^2 = 2^2 - \sqrt{3}^2$

$x = \pm\sqrt{2^2 - \sqrt{3}^2}$

$= \pm\sqrt{4 - 3}$

$x = \pm 1$



$\cos\theta = \pm\frac{1}{2}$

$\tan\theta = \pm\sqrt{3}$

$\csc\theta = -\frac{2}{\sqrt{3}}$ or $-\frac{2\sqrt{3}}{3}$

$\sec\theta = \pm 2$

$\cot\theta = \pm\frac{1}{\sqrt{3}}$ or $\pm\frac{\sqrt{3}}{3}$

7. Solve for the approximate value of angle θ, within the given domain:

a. $\cos\theta = -0.287$ $\quad -\frac{3\pi}{2} \le \theta \le \pi$ (RADIAN MODE)

Calculator method = not on test

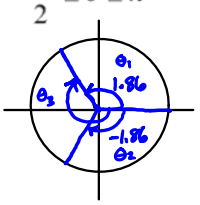
$\cos^{-1}(-0.287)$

$\theta_1 = 1.86$

$\theta_2 = -1.86$

$\theta_3 = -2\pi + 1.86$

$\theta_3 = -4.42$



b. $\sin\theta = 0.897$ $\quad 90^\circ \le \theta \le 450^\circ$ (DEGREE MODE)

$\sin^{-1}(0.897)$

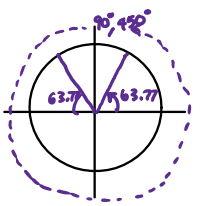
$\theta = 63.77^\circ$

$\theta_1 = 180 - 63.77$

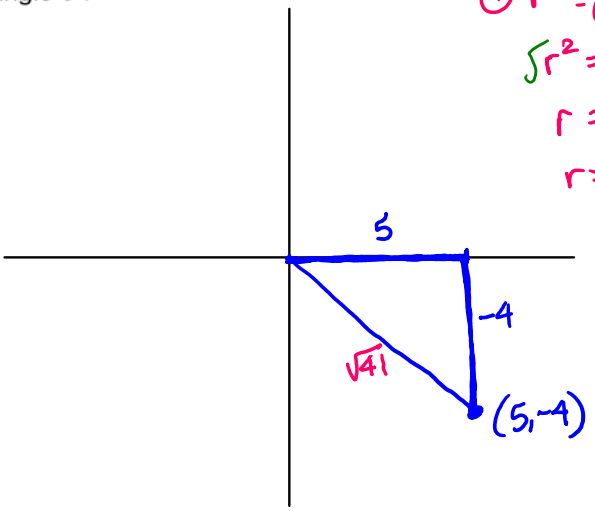
$\theta_1 = 116.23^\circ$

$\theta_2 = 360 + 63.77$

$\theta_2 = 423.77^\circ$



8. Given the point $(5,-4)$ on the terminal arm of a circle, determine the 6 trigonometric ratios for angle θ.



① $r^2 = (-4)^2 + 5^2$

$\sqrt{r^2} = \sqrt{16+25}$

$r = \pm\sqrt{41}$

$r = \sqrt{41}$

② $\cos\theta = \frac{5}{\sqrt{41}}$ or $\frac{5\sqrt{41}}{41}$

$\sin\theta = -\frac{4}{\sqrt{41}}$ or $-\frac{4\sqrt{41}}{41}$

$\tan\theta = -\frac{4}{5}$

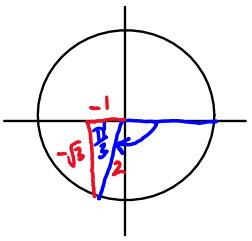
$\sec\theta = \frac{\sqrt{41}}{5}$

$\csc\theta = -\frac{\sqrt{41}}{4}$

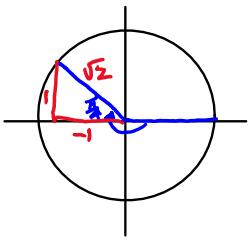
$\cot\theta = -\frac{5}{4}$

9. Determine the exact value of the following:

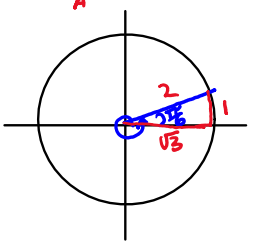
a. $\sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ (O/H)



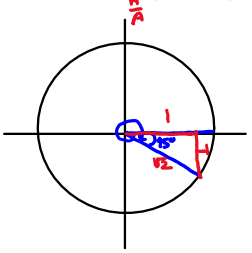
b. $\cos\left(-\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$ (A/H)



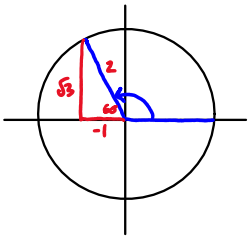
c. $\tan\left(\frac{13\pi}{6}\right) = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ (O/A)



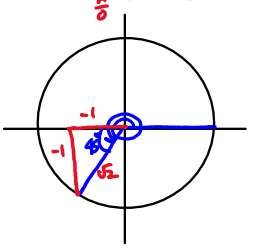
d. $\sec(-405^\circ) = \sqrt{2}$ (H/A)



e. $\csc(120^\circ) = \frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$ (H/O)



f. $\cot(585^\circ) = 1$ (A/O)



10. Determine all angles, θ, for each of the following and draw a diagram for each:

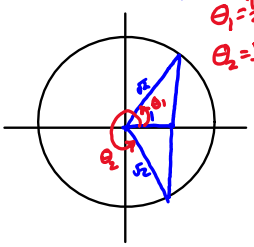
$0 \le \theta \le 2\pi$ and $0 \le \theta \le 360^\circ$

a. $\cos\theta = \frac{\sqrt{2}}{2}$ ← $\frac{\pi}{4}$ angle

+QI+QIV

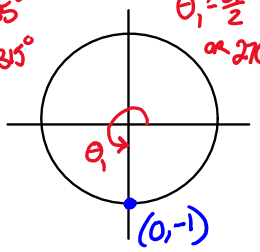
$\theta_1 = \frac{\pi}{4}$ or 45°

$\theta_2 = \frac{7\pi}{4}$ or 315°

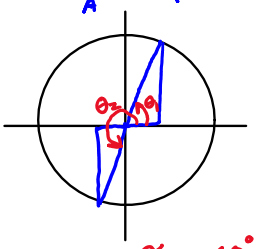


b. $\sin\theta = -1$ ← y-coord

$\theta_1 = \frac{3\pi}{2}$ or 270°



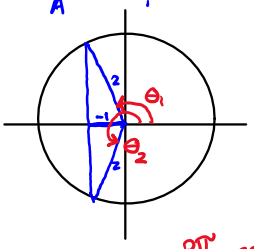
c. $\tan\theta = \sqrt{3}$ ($\frac{O}{A}$, $\frac{\sqrt{3}}{1}$) ← +QI+QIII ← $\frac{\pi}{3}$ or 60°



$\theta_1 = \frac{\pi}{3}$ or 60°

$\theta_2 = \frac{7\pi}{6}$ or 240°

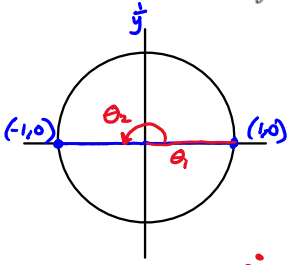
d. $\sec\theta = -2$ ($\frac{H}{A}$, $\frac{-2}{1}$) ← -QII+QIII ← $\frac{\pi}{3}$ or 60°



$\theta_1 = \frac{2\pi}{3}$ or 120°

$\theta_2 = \frac{4\pi}{3}$ or 240°

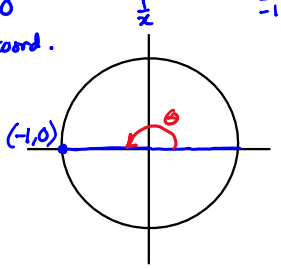
e. $\csc\theta = undefined = \frac{1}{0}$ ($\frac{1}{y}$) ← y-coord.



$\theta_1 = 0$ or 0°

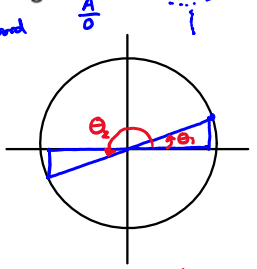
$\theta_2 = \pi$ or 180°

f. $\sec\theta = -1 = \frac{1}{-1}$ ($\frac{1}{x}$) ← x-coord



$\theta = \pi$ or 180°

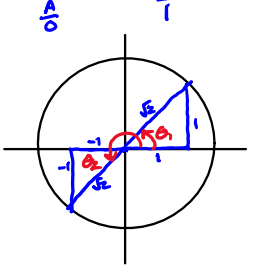
g. $\cot\theta = \sqrt{3}$ ($\frac{A}{O}$, $\frac{\sqrt{3}}{1}$) ← +QI+QIII ← $\frac{\pi}{6}$ or 30°



$\theta_1 = \frac{\pi}{6}$ or 30°

$\theta_2 = \frac{7\pi}{6}$ or 210°

h. $\cot\theta = 1 = \frac{1}{1}$ ($\frac{A}{O}$) ← +QI+QIII ← $\frac{\pi}{4}$ or 45°

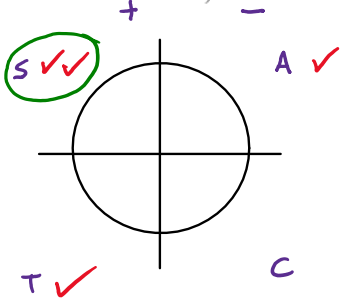


$\theta_1 = \frac{\pi}{4}$ or 45°

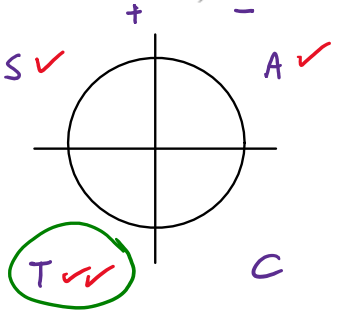
$\theta_2 = \frac{5\pi}{4}$ or 225°

11. Use a drawing of the quadrants to determine which quadrant the following angles, θ, are found ?

a. $\sin\theta > 0, \cos\theta < 0$ = QII

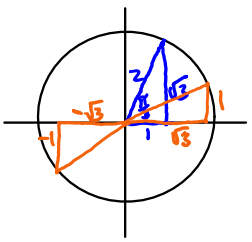


b. $\tan\theta > 0, \sec\theta < 0$ QIII



12. Find the value of θ in the indicated function:

a. $\cot\frac{\pi}{3} = \tan\theta$

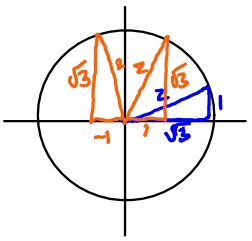


① $\cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$

② $\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$

b. $\cos\frac{\pi}{6} = \sin\theta$

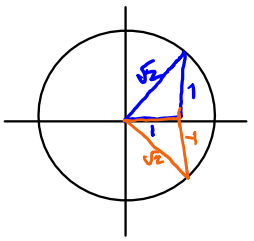


① $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

② $\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

c. $\sin\frac{\pi}{4} = \cos\theta$

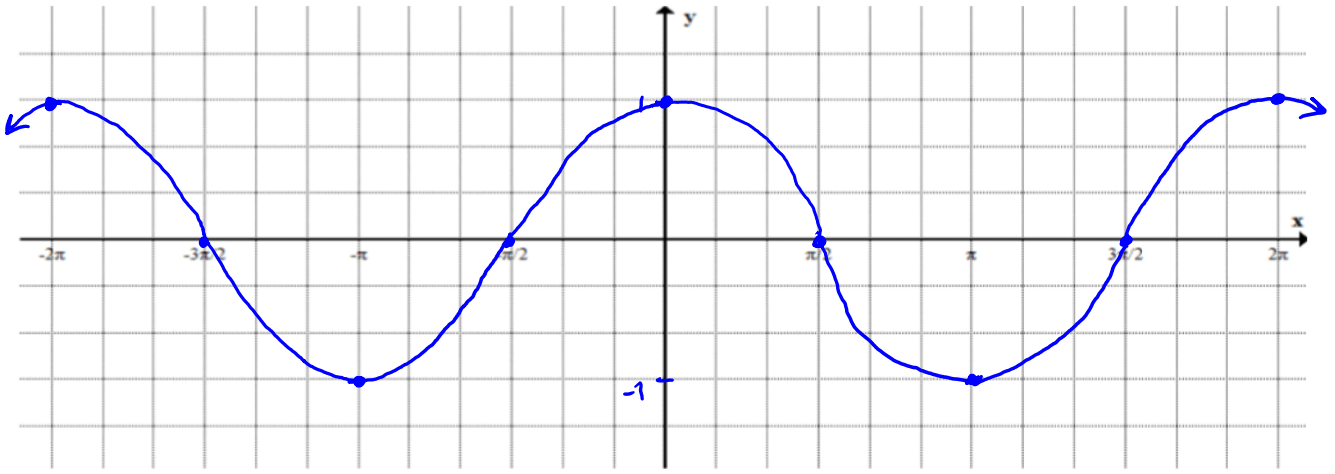


① $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

② $\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}, \frac{7\pi}{4}$

13. a. Graph $y = \cos x$ for $-2\pi \le \theta \le 2\pi$



b. State the domain and range

$\{x \mid x \in \mathbb{R}\}$ $\{y \mid -1 \le y \le 1, y \in \mathbb{R}\}$.

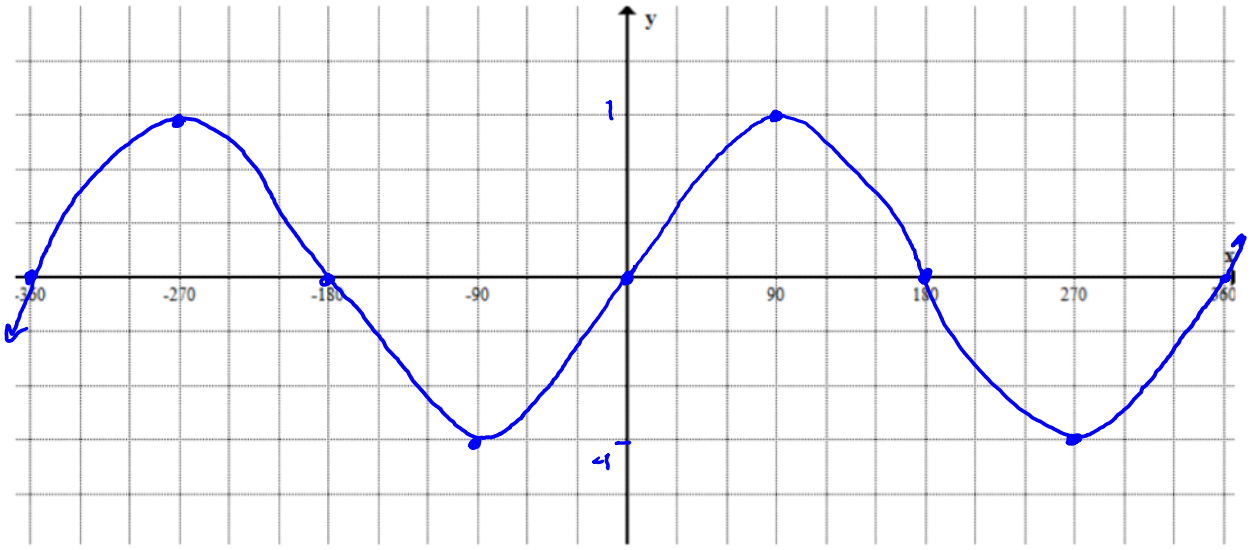
c. State the period and amplitude.

$p = 2\pi$ amp $= 2$

d. State the general equation for all x-intercepts.

$x = \frac{\pi}{2} + 2\pi n, n \in I$

14. a. Graph $y = \sin x$ for $-360^\circ \le \theta \le 360^\circ$



b. State the domain and range.

$\{x \mid x \in \mathbb{R}\}$ $\{y \mid -1 \le y \le 1, y \in \mathbb{R}\}$.

b. State the period and amplitude.

$p = 2\pi$ amp = 1

c. State the general equation for all x-intercepts.

$x = \frac{\pi}{2}n, n \in I$

15. Give the function $y = 5\sin\left(2\left(x - \frac{\pi}{4}\right)\right) - 3$, state the following characteristics:

a. Amplitude $= 5$

b. Period $= \frac{2\pi}{2} \rightarrow p = \pi$

c. Phase shift $= \frac{\pi}{4}$ (right)

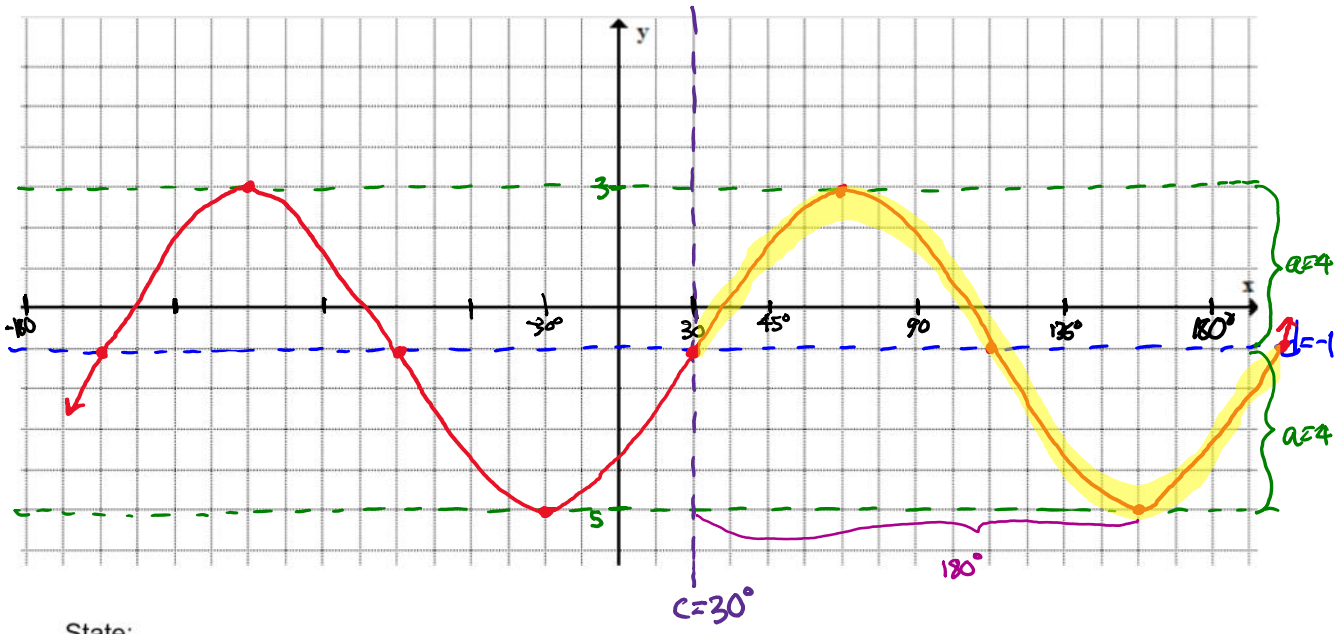
d. Vertical displacement $= -3$ (down 3)

e. Domain $\{x \mid x \in \mathbb{R}\}$.

f. Range $\{y \mid -8 \leq y \leq 2, y \in \mathbb{R}\}$.

16. Graph the following function $y = 4\sin(2x - 60^\circ) - 1$ Graph 2 complete cycles.

$y = 4\sin 2(x - 30^\circ) - 1$



State:

a. Amplitude $= 4$

b. Period $= \frac{360}{2} \rightarrow p = 180^\circ$

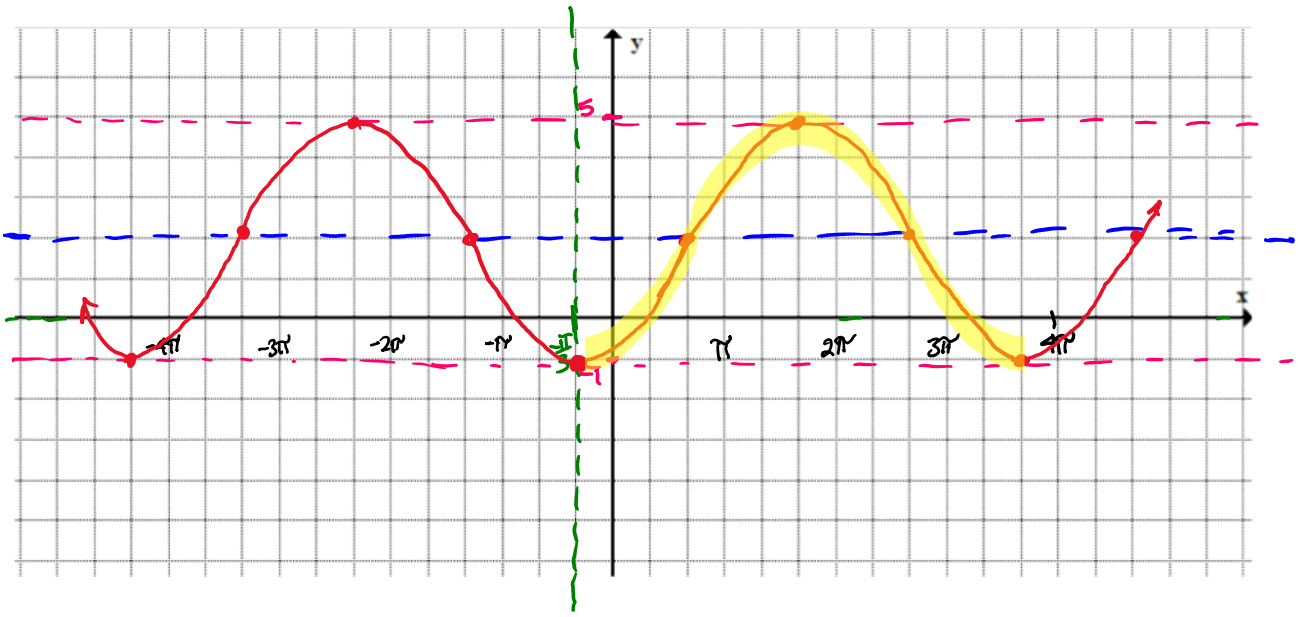
c. Phase shift $= 30^\circ$

d. Vertical displacement $= -1$ (down 1)

e. Domain $\{x \mid x \in \mathbb{R}\}$

f. Range $\{y \mid -5 \leq y \leq 3, y \in \mathbb{R}\}$

17. Graph the following function $y = -3\cos\left(\frac{1}{2}\left(x+\frac{\pi}{3}\right)\right)+2$ Graph 2 complete cycles. $2\pi \times 2$ $p = 4\pi$



State:

a. Amplitude $= 3$

b. Period $= 2\pi \times \frac{2}{1} \rightarrow p = 4\pi$

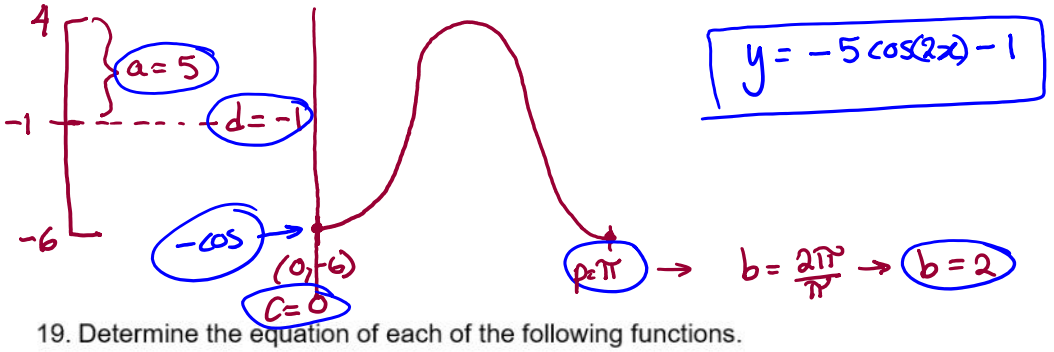
c. Phase shift $= -\frac{\pi}{3}$ (left)

d. Vertical displacement $= 2$ (up)

e. Domain $\{x \mid x \in \mathbb{R}\}$

f. Range $\{y \mid -1 \leq y \leq 5, y \in \mathbb{R}\}$.

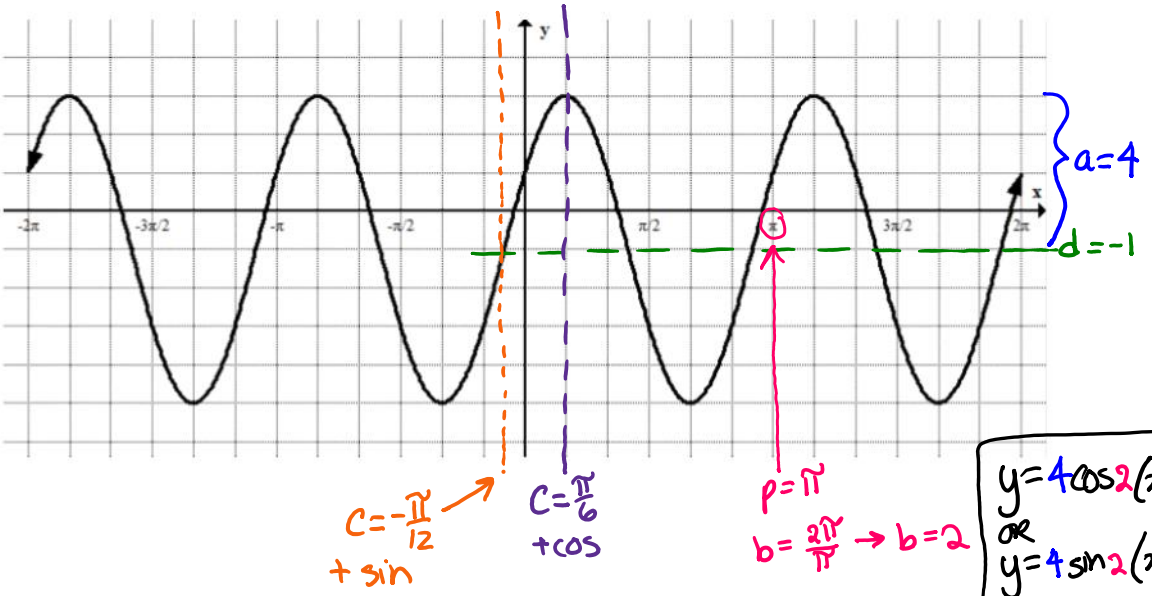
18. If the range of a function is $-6 \le y \le 4$ and the minimum occurs at $(0,-6)$ and the period of the function is π, determine the equation of the function.



$y = -5\cos(2x) - 1$

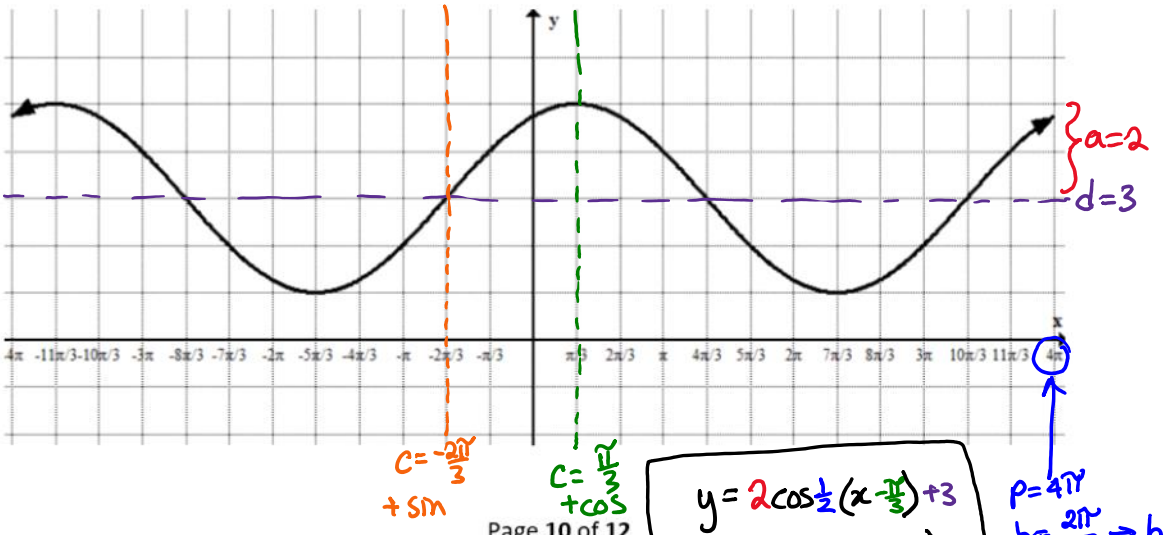
19. Determine the equation of each of the following functions.

a.



$y = 4\cos 2\left(x + \frac{\pi}{12}\right) - 1$
OR
$y = 4\sin 2\left(x - \frac{\pi}{6}\right) - 1$

b.



$y = 2\cos\frac{1}{2}\left(x - \frac{\pi}{3}\right) + 3$
OR
$y = 2\sin\frac{1}{2}\left(x + \frac{2\pi}{3}\right) + 3$

20. The rotation of the wheel of a bicycle is can be modelled by the equation $h(t) = 22\sin\left(\frac{\pi}{1.04}t\right) + 22$ where h is the height in cm, and t is the time in seconds.

a. Determine the height of a mark on the tire's tread after 12.3 seconds.

$t = 12.3$ sec

calc. in RADIAN MODE

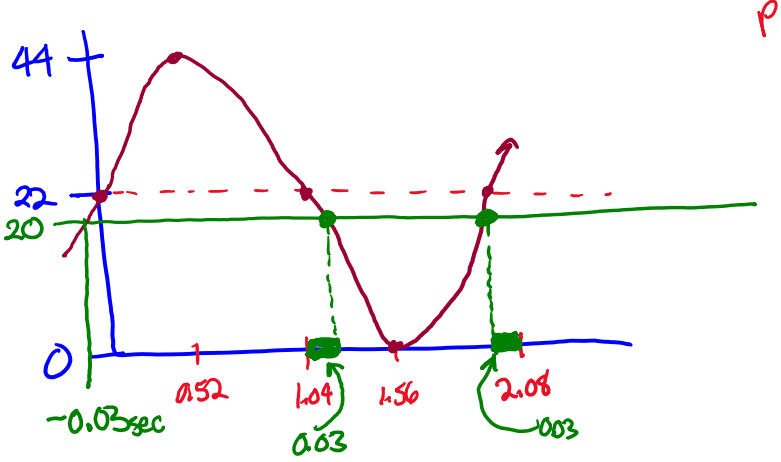
$$h(t) = 22\sin\left(\frac{\pi}{1.04} \cdot 12.3\right) + 22$$

$$\boxed{h(t) = 10.62 \text{ cm}}$$

b. Determine the first 3 times that the height of the mark in the tread is 20 cm above the ground.

$p = 2\pi \times \frac{1.04}{\pi}$

$p = 2.08$s



$$20 = 22\sin\left(\frac{\pi}{1.04}t\right) + 22$$

-22

$$-2 = 22\sin\left(\frac{\pi}{1.04}t\right)$$

$\div 22$ $\quad \div 22$

$$\sin^{-1}\left(-\frac{1}{11}\right) = \sin\left(\frac{\pi}{1.04}t\right)$$

$$\sin^{-1}\left(-\frac{1}{11}\right) = \frac{\pi}{1.04}t \quad \rightarrow \quad t = \frac{\sin^{-1}\left(-\frac{1}{11}\right)}{\frac{\pi}{1.04}}$$

$\div \frac{\pi}{1.04}$ $\quad \times \frac{1.04}{\pi}$

$t = -0.03$ sec.

$t = 1.04 + 0.03$

$t_1 = 1.07$ sec.

$t = 2.08 - 0.03$

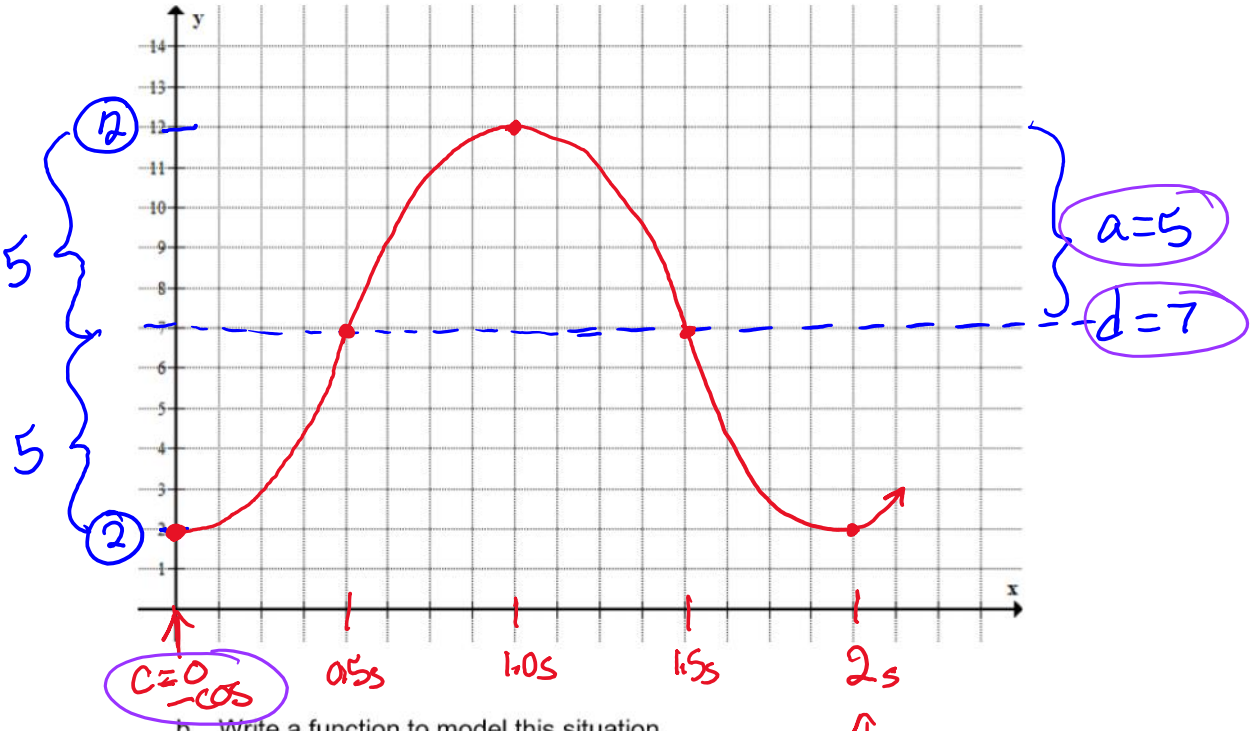
$t_2 = 2.05$ sec

$t_3 = 1.07 + 2.08$ (period)

$t_3 = 3.15$ sec

21. A mass on a spring is bouncing up and down. The mass's distance from the floor when it is at rest is 7 cm. The maximum displacement is 10cm as it bounces. It takes 2 s to complete one bounce or cycle. Suppose the mass is pulled down 5 cm first and let go at t=0.

a. Draw a sketch of the sinusoidal function that model's this situation.



b. Write a function to model this situation.

$h(t) = -5\cos 2t + 7$

period = 2sec → $b = \frac{2\pi}{2}$

$b = 2$

c. Algebraically determine the height after 0.8 s.

$h(t) = -5\cos 2(0.8) + 7$

$h(t) = 7.15$ cm

d. Algebraically determine the time it takes to reach a height of 9cm.

$9 = -5\cos 2t + 7$

$-7 \quad -7$

$\frac{2}{-5} = \frac{-5\cos 2t}{-5}$

$\cos^{-1}\left(-\frac{2}{5}\right) = \cos 2t$

$\frac{\cos^{-1}\left(-\frac{2}{5}\right)}{2} = \frac{2t}{2}$

$t = \frac{\cos^{-1}\left(-\frac{2}{5}\right)}{2}$

$t = 0.99$ sec