

Chapter 8 Homework to show for completion marks (5/5)
 Due KEY.

1. Describe the transformations and graph the base function AND the transformed function below – include mapping notation and the completed table of values.

a. $y = \log_3 x$ and $y = 2\log_3(x+3) - 4$

Describe the transformations:

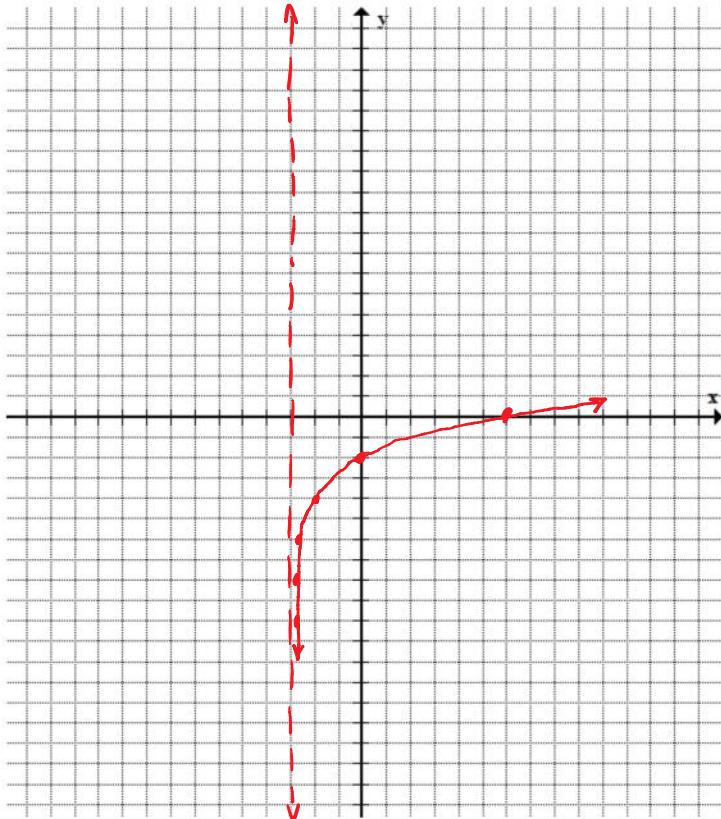
1) vertical stretch of 2
 2) 3 left, 4 down

$$y = \log_3 x$$

x	y
$\sqrt[3]{27}$	-3
$\sqrt[3]{9}$	-2
$\sqrt[3]{3}$	-1
1	0
3	1
9	2
27	3

SHOW ↓

x	2y	x-3	2y-4
$\sqrt[3]{27}$	-6	$\frac{81}{27} - 3$	-10
$\sqrt[3]{9}$	-4	$\frac{27}{9} - 3$	-8
$\sqrt[3]{3}$	-2	$\frac{9}{3} - 3$	-6
1	0	-2	-4
3	2	0	-2
9	4	6	0
27	6	24	2



State the domain and range: $\{x | x > -3, x \in \mathbb{R}\}$

$$\{y | y \in \mathbb{R}\}$$

State the equation of the asymptote:

$$x = -3$$

b. $y = \log_2 x$ and $y = -\log_2(-x+4)-1$ $\rightarrow y = -\log_2(-(x-4))-1$

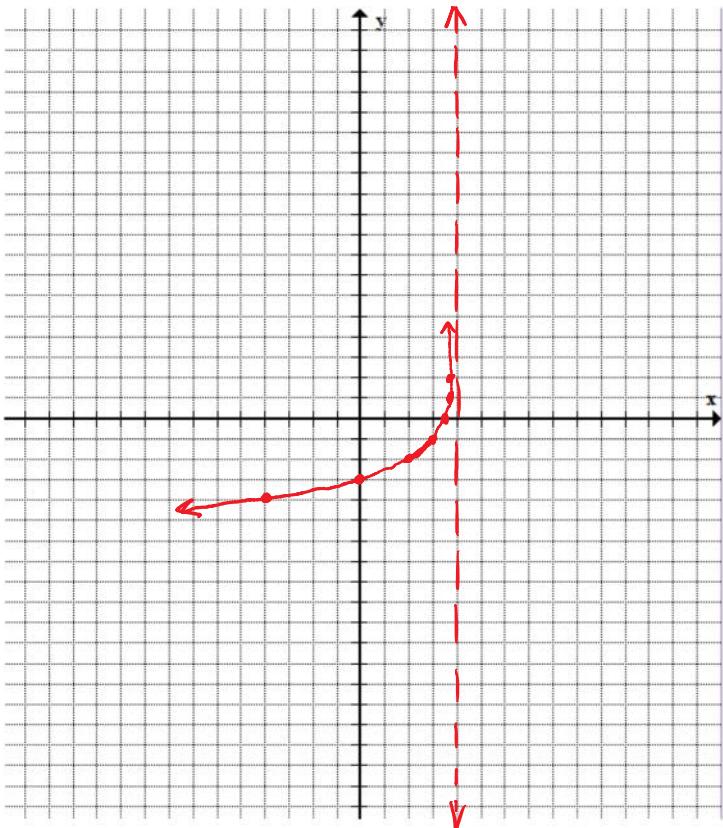
Describe the transformations:

- i) reflection over the x - and y -axis
- ii) 4 right, 1 down

$$y = \log_2 x$$

x	y
$\frac{1}{8}$	-3
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3

$-x$	$-y$	$-x+4$	$-y-1$
$-\frac{1}{8}$	3	$+\frac{32}{8}$	$\frac{3}{8}$ 2
$-\frac{1}{4}$	2	$+\frac{16}{4}$	$\frac{15}{4}$ 1
$-\frac{1}{2}$	1	$+\frac{8}{2}$	$\frac{7}{2}$ 0
-1	0	3	-1
-2	-1	2	-2
-4	-2	0	-3
-8	-3	-4	-4



State the domain and range:

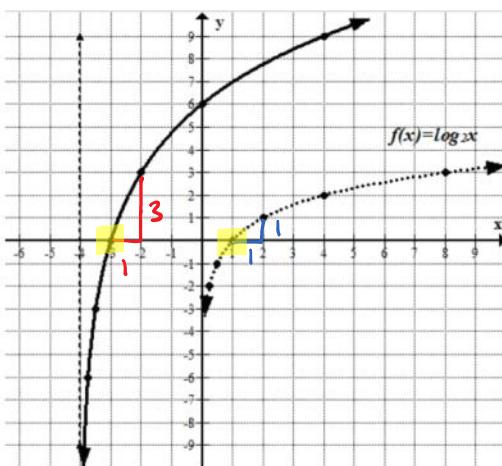
$$\begin{cases} x | x < 4, x \in \mathbb{R} \\ y | y \in \mathbb{R} \end{cases}$$

State the equation of the asymptote:

$$x = 4$$

2. Write an equation for each of the transformed graphs below. The base function is given.

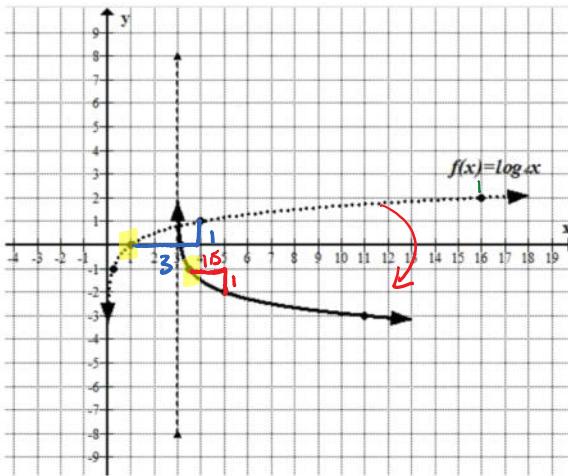
a.



- 1) vert. stretch of 3
- 2) 4 left

$$f(x) = 3 \log_2(x+4)$$

b.



- 1) horiz. stretch of $\frac{1}{2}$ + reflection over x-axis
- 2) 3 right, 1 down

$$H(x) = -\log_{\frac{1}{2}}(2(x-3))-1$$

3. Write the equation of each function with the given transformations:

- a. $f(x) = \log x$ is stretched vertically by a factor of 3, stretched horizontal by a factor of $\frac{1}{2}$, reflected in the y-axis, and translated 2 right and 1 down.

$$f(x) = 3 \log(-2(x-2))-1$$

- b. $y = \log_5 x$ is stretched vertically by a factor of $\frac{1}{2}$, reflected in the y-axis, and translated 5 left and 7 down.

$$y = \frac{1}{2} \log(-(x+5))-7$$

5. Write the following in exponential form:

a. $\log h = 2p$

$$q^{2p} = h$$

b. $\log t = r^2$

$$t^{r^2} = 5$$

6. Write the following in logarithmic form:

a. $2x = y^{1.5}$

$$\log_y 2x = 1.5$$

b. $99^{0.034} = w$

$$\log_9 w = 0.034$$

7. Expand the following logarithms:

$$a. \log_7\left(\frac{x^2\sqrt[3]{y}}{z^4}\right) = \log_7 x^2 + \log_7 y^{\frac{1}{3}} - \log_7 z^4$$

$$= \boxed{2\log_7 x + \frac{1}{3}\log_7 y - 4\log_7 z}$$

$$b. \log_2\left(\frac{8r^3}{t^2\sqrt{w}}\right) = \log_2 8 + \log_2 r^3 - \log_2 t^2 - \log_2 w^{\frac{1}{2}}$$

$$= \boxed{3 + 3\log_2 r - 2\log_2 t - \frac{1}{2}\log_2 w}$$

8. Write as a single logarithm and simplify:

$$a. \frac{1}{3}(\log_a x - \log_a \sqrt{x}) + \log_a 3x^2$$

$$\frac{1}{3} \log_a \left(\frac{x}{x^{\frac{1}{2}}} \right) + \log_a 3x^2$$

$$\log_a \left(x^{\frac{1}{2}} \right) + \log_a 3x^2$$

$$\log_a x^{\frac{1}{2}} + \log_a 3x^2$$

$$= \log_a (x^{\frac{1}{2}} \cdot 3x^2)$$

$$= \boxed{\log_a 3x^{\frac{13}{2}}}$$

$$x^{\frac{1}{2}} \cdot x^2 = x^{\frac{1}{2}+2} \\ x^{\frac{5}{2}} \\ = x^{\frac{13}{2}}$$

$$b. 2\log 8x - 3\log x - \log 5 + 2\log x$$

$$\log(8x)^2 - \log x^3 - \log 5 + \log x^2$$

$$= \log \left(\frac{64x^2 \cdot x^2}{x^3 \cdot 5} \right)$$

$$= \log \left(\frac{64x^4}{5x^3} \right) \rightarrow = \boxed{\log \left(\frac{64x}{5} \right)}$$

9. Solve the following (hint: change to exponential form):

$$a. \log_3(x-5) = 2$$

Root the Base

$$(x-5) = 3^2$$

$$x-5 = 9$$

$$\boxed{x = 14}$$

$$K: x-5 > 0 \\ \boxed{x > 5}$$

Restrictions
Required for
ALL log
equations
when solving

$$b. \log_5\left(\frac{49}{169}\right) = 2$$

Root the Base

$$\sqrt{\frac{49}{169}} = x^2$$

$$x = \pm \frac{7}{13}$$

$$K: x > 0 + \text{for base } x \neq 1$$

$$\leftarrow \text{reject } (-)$$

$$\boxed{x = \frac{7}{13}} \quad x \neq -\frac{7}{13}$$

10. Solve the following (algebraically using log laws):

a. $\log_2(x-4) = 4 - \log_2(x+2)$

$$\log_2(x-4) + \log_2(x+2) = 4$$

Product Law

$$\log_2(x-4)(x+2) = 4$$

Root the base

$$(x-4)(x+2) = 2^4$$

$$x^2 - 2x - 8 - 16 = 0$$

$$x^2 - 2x - 24 = 0$$

$$(x-6)(x+4) = 0$$

$$x=6 \quad x=-4$$

extraneous

$x=6$

check

Restrictions
 $x-4 > 0 \quad x+2 > 0$
 $x > 4 \quad x > -2$

b. $\log_3(x-4) - \log_3(x+2) = \log_3 5$

quotient Law

$$\log_3\left(\frac{x-4}{x+2}\right) = \log_3 5$$

$$\frac{x-4}{x+2} = 5$$

cross multiply

$$x-4 = 5(x+2)$$

$$x-4 = 5x+10$$

-5x

$$-4x = 14$$

$$x = -\frac{7}{2}$$

extraneous

$x = \text{no solution}$

check

Restrictions
 $x-4 > 0 \quad x+2 > 0$
 $x > 4 \quad x > -2$

11. Solve algebraically (use logs):

a. $5^{2x+1} = 11^{x-2}$

$$\log 5^{2x+1} = \log 11^{x-2}$$

$$(2x+1)\log 5 = (x-2)\log 11$$

$$2x\log 5 + \log 5 = x\log 11 - 2\log 11$$

$$2x\log 5 - x\log 11 = -2\log 11 - \log 5$$

$$\frac{x(2\log 5 - \log 11)}{2\log 5 - \log 11} = \frac{-2\log 11 - \log 5}{2\log 5 - \log 11}$$

$$x = \frac{-2\log 11 - \log 5}{2\log 5 - \log 11}$$

$$x \approx -7.80$$

$\frac{-2\log(11)-\log(5)}{2\log(5)-\log(11)}$
 -7.801924713

b. $4^{2x+1} = 9(4^{1-x})$

$$\log 4^{2x+1} = \log [9(4^{1-x})]$$

product Law

$$(2x+1)\log 4 = \log 9 + \log 4^{1-x}$$

$$2x\log 4 + \log 4 = \log 9 + (1-x)\log 4$$

$$2x\log 4 + \log 4 = \log 9 + \log 4 - x\log 4$$

$$2x\log 4 + x\log 4 = \log 9 + \log 4 - \log 4$$

$= 0$

$$3x\log 4 = \log 9$$

$$x = \frac{\log 9}{3\log 4}$$

$$x \approx 0.53$$

$\log(9)/[3\log(4)]$
 .5283208336

12. Simplify and solve (use log laws):

a. $7^{\log_7 x + \log_7 x^2} = 216$

$$\begin{aligned} 7^{\log_7 x + \log_7 x^2} &= 216 \\ \sqrt[3]{x^3} &= \sqrt[3]{216} \\ x = \pm 6 &\quad \boxed{x=6} \quad x \neq -6 \end{aligned}$$

Restrictions

$$x > 0$$

b. $\log_2(\log_{9x^2}(\log_4 16)) = -1$

boot the base

$$\begin{aligned} \log_{9x^2}(\log_4 4^2) &= 2^{-1} \\ \log_{9x^2} 2 &= \frac{1}{2} \end{aligned}$$

$$2 = (9x^2)^{\frac{1}{2}}$$

$$2 = \sqrt{9x^2} \rightarrow \sqrt{9} \cdot \sqrt{x^2} = 3x$$

$$\frac{2}{3} = \frac{3x}{3}$$

$$\boxed{x = \frac{2}{3}}$$

Restrictions

$$x^2 > 0$$

$$\therefore x \neq 0$$

$$\begin{aligned} 9x^2 &\neq 1 \\ x^2 &\neq \frac{1}{9} \\ x &\neq \pm \frac{1}{3} \end{aligned}$$

c. $x \log_{25} 5 = 8 - \log_6 36$

$$\begin{aligned} x \log_{25} \frac{5}{25} &= 8 - \log_6 36 \\ \frac{1}{2} x &= 8 - 2 \\ (2) \frac{1}{2} x &= 6 \quad (2) \\ \boxed{x = 12} & \end{aligned}$$

no Restrictions

13. The following function represents the blood pressure, in mmHg, of young adults related to the blood vessel volume, in microlitres.

$$V(p) = 0.23 + 0.35 \log(p - 56.1)$$

a. Determine the blood vessel volume if the blood pressure is 120mmHg.

$$V(p) = 0.23 + 0.35 \log(120 - 56.1)$$

$$= \boxed{0.86 \text{ } \mu\text{L}}$$

$$\begin{array}{r} 0.23 + 0.35 \log(120 - 56.1) \\ - 56.1 \\ \hline 0.8619253004 \end{array}$$

b. Determine the blood pressure if the vessel volume is 0.9 microlitres.

$$0.9 = 0.23 + 0.35 \log(p - 56.1)$$

$$-0.23 \leftarrow$$

$$\frac{0.67}{0.35} = \frac{0.35 \log(p - 56.1)}{0.35}$$

$$\frac{0.67}{0.35} = \log(p - 56.1)$$

$$\frac{(0.67)}{0.35} = p - 56.1$$

$$\text{boot base } 10$$

$$\begin{aligned} p &= 10^{\frac{(0.67)}{0.35}} + 56.1 \\ \boxed{p = 138.19 \text{ mmHg}} & \end{aligned}$$

$$\begin{array}{r} 10^{(0.67/0.35)+5} \\ 6.1 \\ \hline 138.1891416 \end{array}$$

$$A = A_0 (b)^{\frac{t}{h}}$$

14. Given the following information, determine an exponential equation that represents the growth or decay, then answer the question algebraically:

- a. Carbon-14 has a half-life of 5740 years. How long would it take a 5.3 gram sample to decay to 1.9 grams.

$$\begin{aligned} \frac{1.9}{5.3} &= \frac{5.3(0.5)^{\frac{t}{5740}}}{5.3} \\ \log \frac{1.9}{5.3} &= \log 0.5^{\frac{t}{5740}} \\ \log \left(\frac{1.9}{5.3}\right) &= \frac{t}{5740} \log 0.5 \\ \frac{5740 \log \left(\frac{1.9}{5.3}\right)}{\log 0.5} &= t \log 0.5 \end{aligned}$$

$t = \frac{5740 \log \left(\frac{1.9}{5.3}\right)}{\log 0.5}$

$t = 8495.16 \text{ yrs}$

$\frac{5740 \log(1.9/5.3)}{\log(0.5)}$
 8495.159482

- b. A certain kind of mold triples every 7 days. How long would it take a sample of 125 cells to grow to 100,000 cells?

$$\begin{aligned} \frac{100000}{125} &= \frac{125(3)^{\frac{t}{7}}}{125} \\ \log 800 &= \log 3^{\frac{t}{7}} \\ \log 800 &= \frac{t}{7} \log 3 \\ \frac{7 \log 800}{\log 3} &= t \log 3 \\ t = \frac{7 \log 800}{\log 3} &= 42.59 \text{ days} \end{aligned}$$

$7 \log(800) / \log(3)$
 42.59217067

- c. The population of Langley increases by 0.5% per year with a current population of 127,000 people. What would the populations be in 10 years?

$$\begin{aligned} A &= 127000 (1.005)^{10} \\ A &= 133494 \text{ people} \end{aligned}$$

$127000(1.005)^{10}$
 133494.7968

How long would it take for the populations to reach 200,000 people?

$$\begin{aligned} \frac{200000}{127000} &= \frac{127000(1.005)^t}{127000} \\ \log \frac{200}{127} &= \log 1.005^t \\ \frac{\log \left(\frac{200}{127}\right)}{\log 1.005} &= t \log 1.005 \\ t = \frac{\log \frac{200}{127}}{\log 1.005} &= 91.05 \text{ yrs} \end{aligned}$$

$\log(200/127) / \log(1.005)$
 91.05293241

$$I = 10^{R_A - R_L}$$

- d. If an earthquake in Town A is 3500 times more intense than an earthquake of magnitude 4.2 in Town B, what is the magnitude of the earthquake in Town A?

$$\log 3500 = \log 10^{R_A - 4.2}$$

$$\log 3500 = (R_A - 4.2) \log 10$$

$$\log 3500 = R_A - 4.2$$

$$R_A = \log 3500 + 4.2$$

$$R_A = 7.74 \approx 7.7$$

$$\log(3500) + 4.2 \\ 7.744068044$$

$$I = 10^{\text{pH}_u - \text{pH}_L}$$

- e. If the pH of stomach acid at 2.2 is 50,000 times more acidic than urine, what is the pH of urine?

$$\log 50000 = \log 10^{\text{pH}_u - 2.2}$$

$$\log 50000 = (\text{pH}_u - 2.2) \log 10$$

$$\log 50000 = \text{pH}_u - 2.2$$

$$\text{pH}_u = \log 50000 + 2.2$$

$$\text{pH} = 6.90 \approx 6.9$$

$$\log(50000) + 2.2 \\ 6.898970004$$