

Review of Algebra and Factoring

Common Factoring

Determine the greatest common factor by checking what the largest term divisible by all terms is (numbers and variables).

Ex. $2x^2 - 6x \rightarrow \boxed{2x(x-3)}$

Complete the following for practice:

a) $3x^3 - 9x^2$

b) $-8x^3 + 2x^2 - 22x$

Binomial Factoring with a Difference of Squares

When the 2 terms of the binomial are perfect squares and there is a subtraction between them, you can use this method for factoring. Form must be $(a^2 - b^2)$.

Ex. $x^2 - 9 \rightarrow \boxed{(x+3)(x-3)}$ $4x^2 - 25y^2 \rightarrow \boxed{(2x+5y)(2x-5y)}$

Here, you put the square root of x and the square root of 9 in each bracket with different signs between them: this is the difference of squares factoring.

NOTE: $x^2 + 9$ is a sum of squares and cannot be factored.

Complete the following:

a) $a^2 - 16$

b) $144 - 9y^2$

c) $36x^2 - 49$

Trinomial Factoring

A trinomial is in the form: $ax^2 + bx + c$. There are different methods for trinomial factoring; including decomposition, guess and check, short-cut factoring, box method. I will show you decomposition and short-cut factoring (I usually do short-cut factoring in class).

When a trinomial has a leading coefficient of 1, the method is simple:

$$x^2 - 4x - 5 \rightarrow C = -5.$$

Find two numbers that multiply to -5 but add to -4. Here the two numbers or factors are -5 and +1. Place these two factors in the brackets with x and you're done.

$$x^2 - 4x - 5 = \boxed{(x-5)(x+1)}$$

When the leading coefficient is not 1, use one of the following methods.

Decomposition	Short-cut Factoring	Another Short-cut Factoring	Box Method																			
<p>$3x^2 - 19x - 14$ A = 3, C = -14 \rightarrow AC = -42</p> <p>Two numbers that multiply to -42 but add to -19</p> <p>These numbers are: -21 and +2</p> <p>Replace the middle (b) term with these two factors written with an x</p> $3x^2 - 21x + 2x - 14$ <p>Split it in half and factor each half</p> $3x^2 - 21x \mid +2x - 14$ $3x(x-7) + 2(x-7)$ <p>Place each term in front of the brackets in its own bracket and write the other common binomial in one bracket.</p> $(3x+2)(x-7)$ <p>Done!</p>	<p>$3x^2 - 19x - 14$ A = 3, C = -14 \rightarrow AC = -42</p> <p>Two numbers that multiply to -42 but add to -19</p> <p>These numbers are: -21 and +2</p> <p>Place the first term of the trinomial without the squared in the first spot of each factored bracket</p> $(3x \quad)(3x \quad)$ <p>Place the two factors (-21 & +2) in the brackets to form the binomials</p> $(3x-21)(3x+2)$ <p>Reduce the terms in the binomials like you would fractions</p> $(\cancel{1}3x - \cancel{2}7)(3x+2)$ $= (x-7)(3x+2)$ <p>Done!</p>	<p>$3x^2 - 19x - 14$ A = 3, C = -14 \rightarrow AC = -42</p> <p>Two numbers that multiply to -42 but add to -19</p> <p>These numbers are: -21 and +2</p> <p>Rewrite the trinomial with the AC as the last term</p> $x^2 - 19x - 42$ <p>Place the two factors (-21 & +2) in the brackets to form the binomials</p> $(x-21)(x+2)$ <p>Divide and reduce the constant in each bracket by the A from original trinomial</p> $\left(x - \frac{21}{3}\right)\left(x + \frac{2}{3}\right) = (x-7)\left(x + \frac{2}{3}\right)$ <p>If there is still a denominator, write that number in front of the x in the same brackets to get final factored form.</p> $(x-7)\left(x + \frac{2}{3}\right) = (x-7)(3x+2)$	<p>$3x^2 - 19x - 14$ A = 3, C = -14 \rightarrow AC = -42</p> <p>Two numbers that multiply to -42 but add to -19</p> <p>These numbers are: -21 and +2</p> <p>Place the first and last term in the box in the first and last spot</p> <table border="1" data-bbox="1333 1266 1448 1325"> <tr> <td>$3x^2$</td> <td></td> </tr> <tr> <td></td> <td>-14</td> </tr> </table> <p>Place the two factors with x in the second and third box</p> <table border="1" data-bbox="1333 1497 1448 1556"> <tr> <td>$3x^2$</td> <td>-21x</td> <td></td> </tr> <tr> <td>2x</td> <td>-14</td> <td></td> </tr> </table> <p>Common factor each row and each column and collect the factors in two brackets for final factored form.</p> <table border="1" data-bbox="1308 1791 1472 1875"> <tr> <td>$3x^2$</td> <td>-21x</td> <td>3x</td> </tr> <tr> <td>2x</td> <td>-14</td> <td>2</td> </tr> <tr> <td>x</td> <td>-7</td> <td></td> </tr> </table> <p>$(3x+2)(x-7)$</p>	$3x^2$			-14	$3x^2$	-21x		2x	-14		$3x^2$	-21x	3x	2x	-14	2	x	-7	
$3x^2$																						
	-14																					
$3x^2$	-21x																					
2x	-14																					
$3x^2$	-21x	3x																				
2x	-14	2																				
x	-7																					

Complete the following using a method of your choice:

a) $6x^2 - 5x - 4$

b) $2x^2 + 11x + 5$

c) $2x^2 + x - 1$

d) $2x^2 - 3x - 2$

To solve, you make each binomial bracket equal zero and solve for x .

Solving the example from above:

$$(3x + 2)(x - 7)$$

$$3x + 2 = 0 \quad x - 7 = 0$$

$$3x = -2 \quad \boxed{x = 7}$$

$$\boxed{x = -\frac{2}{3}}$$

Factoring Trinomials (a = 1)

Factor each completely.

1) $b^2 + 8b + 7$

2) $n^2 - 11n + 10$

3) $m^2 + m - 90$

4) $n^2 + 4n - 12$

5) $n^2 - 10n + 9$

6) $b^2 + 16b + 64$

7) $m^2 + 2m - 24$

8) $x^2 - 4x + 24$

9) $k^2 - 13k + 40$

10) $a^2 + 11a + 18$

11) $n^2 - n - 56$

12) $n^2 - 5n + 6$

13) $b^2 - 6b + 8$

14) $n^2 + 6n + 8$

15) $2n^2 + 6n - 108$

16) $5n^2 + 10n + 20$

17) $2k^2 + 22k + 60$

18) $a^2 - a - 90$

19) $p^2 + 11p + 10$

20) $5v^2 - 30v + 40$

21) $2p^2 + 2p - 4$

22) $4v^2 - 4v - 8$

23) $x^2 - 15x + 50$

24) $v^2 - 7v + 10$

25) $p^2 + 3p - 18$

26) $6v^2 + 66v + 60$

Factoring Trinomials (a = 1)

Factor each completely.

1) $b^2 + 8b + 7$

$(b + 7)(b + 1)$

2) $n^2 - 11n + 10$

$(n - 10)(n - 1)$

3) $m^2 + m - 90$

$(m - 9)(m + 10)$

4) $n^2 + 4n - 12$

$(n - 2)(n + 6)$

5) $n^2 - 10n + 9$

$(n - 1)(n - 9)$

6) $b^2 + 16b + 64$

$(b + 8)^2$

7) $m^2 + 2m - 24$

$(m + 6)(m - 4)$

8) $x^2 - 4x + 24$

Not factorable

9) $k^2 - 13k + 40$

$(k - 5)(k - 8)$

10) $a^2 + 11a + 18$

$(a + 2)(a + 9)$

11) $n^2 - n - 56$

$(n + 7)(n - 8)$

12) $n^2 - 5n + 6$

$(n - 2)(n - 3)$

$$13) b^2 - 6b + 8$$
$$(b - 4)(b - 2)$$

$$14) n^2 + 6n + 8$$
$$(n + 2)(n + 4)$$

$$15) 2n^2 + 6n - 108$$
$$2(n + 9)(n - 6)$$

$$16) 5n^2 + 10n + 20$$
$$5(n^2 + 2n + 4)$$

$$17) 2k^2 + 22k + 60$$
$$2(k + 5)(k + 6)$$

$$18) a^2 - a - 90$$
$$(a - 10)(a + 9)$$

$$19) p^2 + 11p + 10$$
$$(p + 10)(p + 1)$$

$$20) 5v^2 - 30v + 40$$
$$5(v - 2)(v - 4)$$

$$21) 2p^2 + 2p - 4$$
$$2(p - 1)(p + 2)$$

$$22) 4v^2 - 4v - 8$$
$$4(v + 1)(v - 2)$$

$$23) x^2 - 15x + 50$$
$$(x - 10)(x - 5)$$

$$24) v^2 - 7v + 10$$
$$(v - 5)(v - 2)$$

$$25) p^2 + 3p - 18$$
$$(p - 3)(p + 6)$$

$$26) 6v^2 + 66v + 60$$
$$6(v + 10)(v + 1)$$

Factoring Trinomials ($a > 1$)**Factor each completely.**

1) $3p^2 - 2p - 5$

2) $2n^2 + 3n - 9$

3) $3n^2 - 8n + 4$

4) $5n^2 + 19n + 12$

5) $2v^2 + 11v + 5$

6) $2n^2 + 5n + 2$

7) $7a^2 + 53a + 28$

8) $9k^2 + 66k + 21$

9) $15n^2 - 27n - 6$

10) $5x^2 - 18x + 9$

11) $4n^2 - 15n - 25$

12) $4x^2 - 35x + 49$

13) $4n^2 - 17n + 4$

14) $6x^2 + 7x - 49$

15) $6x^2 + 37x + 6$

16) $-6a^2 - 25a - 25$

17) $6n^2 + 5n - 6$

18) $16b^2 + 60b - 100$

Factoring Trinomials ($a > 1$)**Factor each completely.**

1) $3p^2 - 2p - 5$

$(3p - 5)(p + 1)$

2) $2n^2 + 3n - 9$

$(2n - 3)(n + 3)$

3) $3n^2 - 8n + 4$

$(3n - 2)(n - 2)$

4) $5n^2 + 19n + 12$

$(5n + 4)(n + 3)$

5) $2v^2 + 11v + 5$

$(2v + 1)(v + 5)$

6) $2n^2 + 5n + 2$

$(2n + 1)(n + 2)$

7) $7a^2 + 53a + 28$

$(7a + 4)(a + 7)$

8) $9k^2 + 66k + 21$

$3(3k + 1)(k + 7)$

$$9) 15n^2 - 27n - 6$$
$$3(5n + 1)(n - 2)$$

$$10) 5x^2 - 18x + 9$$
$$(5x - 3)(x - 3)$$

$$11) 4n^2 - 15n - 25$$
$$(n - 5)(4n + 5)$$

$$12) 4x^2 - 35x + 49$$
$$(x - 7)(4x - 7)$$

$$13) 4n^2 - 17n + 4$$
$$(n - 4)(4n - 1)$$

$$14) 6x^2 + 7x - 49$$
$$(3x - 7)(2x + 7)$$

$$15) 6x^2 + 37x + 6$$
$$(x + 6)(6x + 1)$$

$$16) -6a^2 - 25a - 25$$
$$-(2a + 5)(3a + 5)$$

$$17) 6n^2 + 5n - 6$$
$$(2n + 3)(3n - 2)$$

$$18) 16b^2 + 60b - 100$$
$$4(b + 5)(4b - 5)$$