

Graphing Practice

Characteristics of Rational Functions Worksheet

1. $f(x) = \frac{x^2 + x - 2}{x^2 - x - 6}$

① Factor $f(x) = \frac{(x+2)(x-1)}{(x-3)(x+2)}$

② Simplify $f(x) = \frac{x-1}{x-3}$

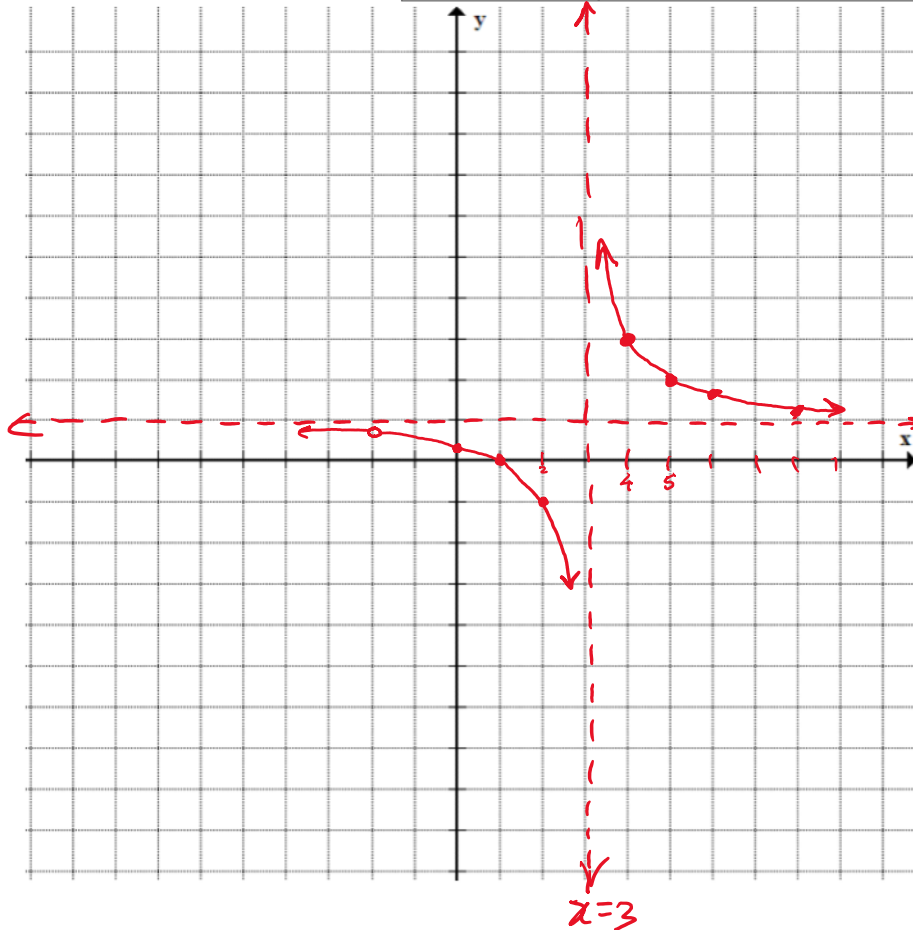
③ Characteristics.

Characteristic	Answer
a. simplified function	$f(x) = \frac{x-1}{x-3}$
b. POD (HAE) Point of Discontinuity	$(x+2) \rightarrow x \neq -2 \dots \dots (-2, \frac{3}{5})$
c. vertical asymptote(s)	$(x-3)$ VA $\rightarrow x=3$
d. horizontal asymptote	$\frac{1x^2}{1x^2} \rightarrow y=1$ HA
e. y-intercept	$x=0$ $y = \frac{0-1}{0-3} \rightarrow y = \frac{1}{3}$ $(0, \frac{1}{3})$
f. x-intercept (numerator)	$y=0 \rightarrow 0 = \frac{x-1}{x-3}$ $0 = x-1 \rightarrow (1, 0)$
g. domain	VA + POD $\sum x \mid x \neq 3, x \neq -2, x \in \mathbb{R}$
h. range	HA + POD $\sum y \mid y \neq 1, y \neq \frac{3}{5}, y \in \mathbb{R}$

$y = \frac{-2-1}{-2-3} = \frac{-3}{-5} = \frac{3}{5}$
when $x = -2$

$f(x) = \frac{x-1}{x-3}$

x	y
2	-1
4	3
5	2
6	$\frac{5}{3}$
8	$\frac{7}{5}$



$\frac{2-1}{2-3} = \frac{1}{-1} = -1$

$\frac{4-1}{4-3} = \frac{3}{1} = 3$

$\frac{5-1}{5-3} = \frac{4}{2} = 2$

$\frac{6-1}{6-3} = \frac{5}{3}$

$\frac{8-1}{8-3} = \frac{7}{5}$

$$2. f(x) = \frac{2x^2}{x^2 - 1}$$

① factor: $f(x) = \frac{2x^2}{(x+1)(x-1)}$

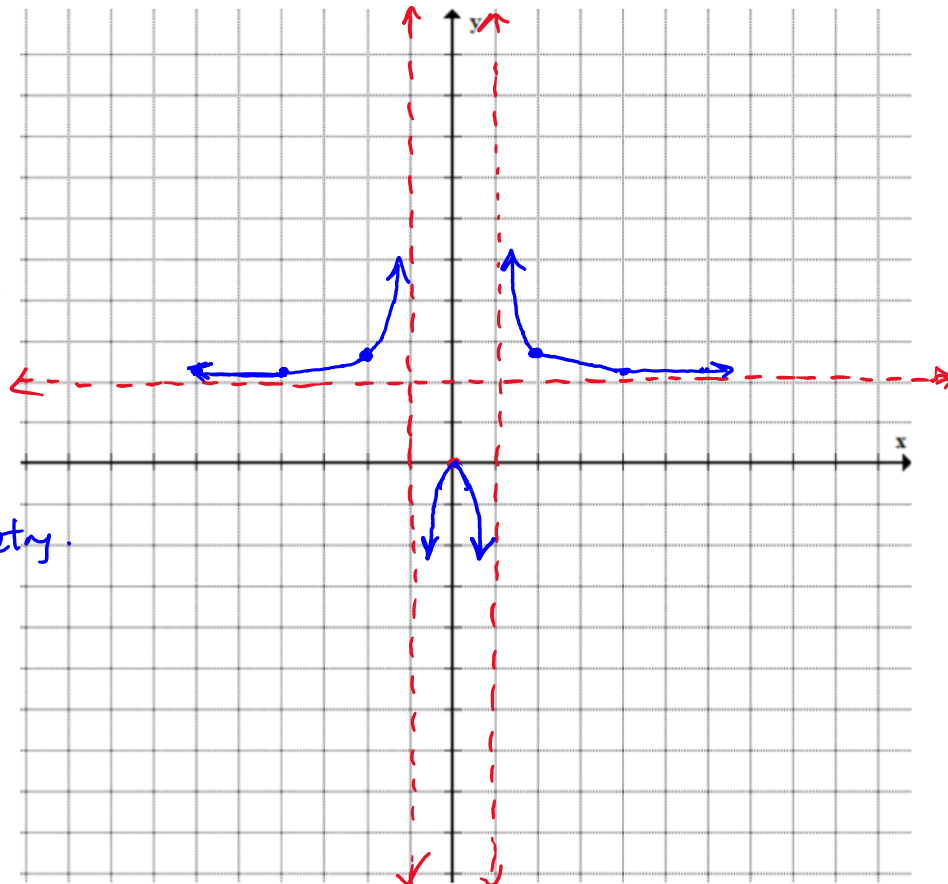
② Simplify: none

③ characteristics:

Characteristic	Answer
a. simplified function	$y = \frac{2x^2}{(x+1)(x-1)}$
b. POD (none)	no hole
c. vertical asymptote(s)	VA $x = -1 \quad x = 1$
d. horizontal asymptote	$\frac{2x^2}{x^2} \rightarrow$ HA $y = 2$
e. y-intercept	$y = \frac{2(0)^2}{(0-1)(0+1)} \rightarrow \frac{0}{-1} \rightarrow y = 0 (0,0)$
f. x-intercept	$0 = \frac{2x^2}{x^2-1} \rightarrow 0 = 2x^2$ $\leftarrow x^2-1 \quad 0 = x^2 \rightarrow x = 0$ $(0,0)$
g. domain VA	$\sum x \mid x \neq -1, x \neq 1, x \in \mathbb{R}$
h. range HA	$\sum y \mid y \neq 2, y \in \mathbb{R}$

x	y
-6	$\frac{72}{35} = 2.1$
-4	$\frac{32}{15} = 2.1$
-2	$\frac{8}{3} = 2.7$
2	
4	
6	

symmetry



$$\frac{2x^2}{x^2 - 1}$$

$$\frac{2(-6)^2}{(-6)^2 - 1} = \frac{72}{35}$$

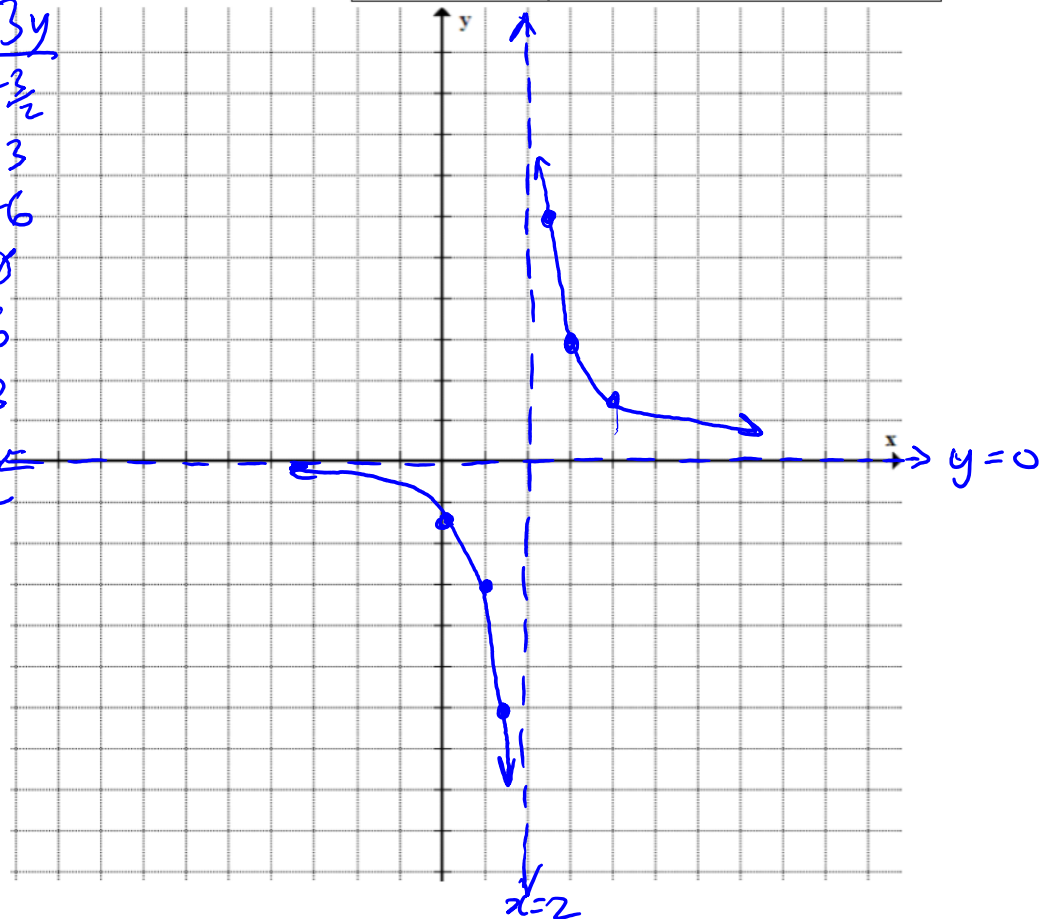
3. $f(x) = \frac{3}{x-2}$

↓
 $y = \frac{1}{x}$ transformed to
 $y = \frac{3}{x-2}$
 VE ↓ 2
 2 right

Characteristic	Answer
a. simplified function	$f(x) = \frac{3}{x-2}$
b. POD	none
c. vertical asymptote(s)	$x = 2$
d. horizontal asymptote	$y = 0$
e. y-intercept	$y = \frac{3}{0-2} \rightarrow y = -\frac{3}{2} (0, -\frac{3}{2})$
f. x-intercept	$0 = \frac{3}{x-2}$ none $0 = 3$
g. domain	$\{x \mid x \neq 2, x \in \mathbb{R}\}$
h. range	$\{y \mid y \neq 0, y \in \mathbb{R}\}$

x	y
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
0	∅
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$

x+2	3y
0	$-\frac{3}{2}$
1	-3
$\frac{3}{2}$	-6
2	∅
$\frac{5}{2}$	6
3	3
4	$\frac{3}{2}$

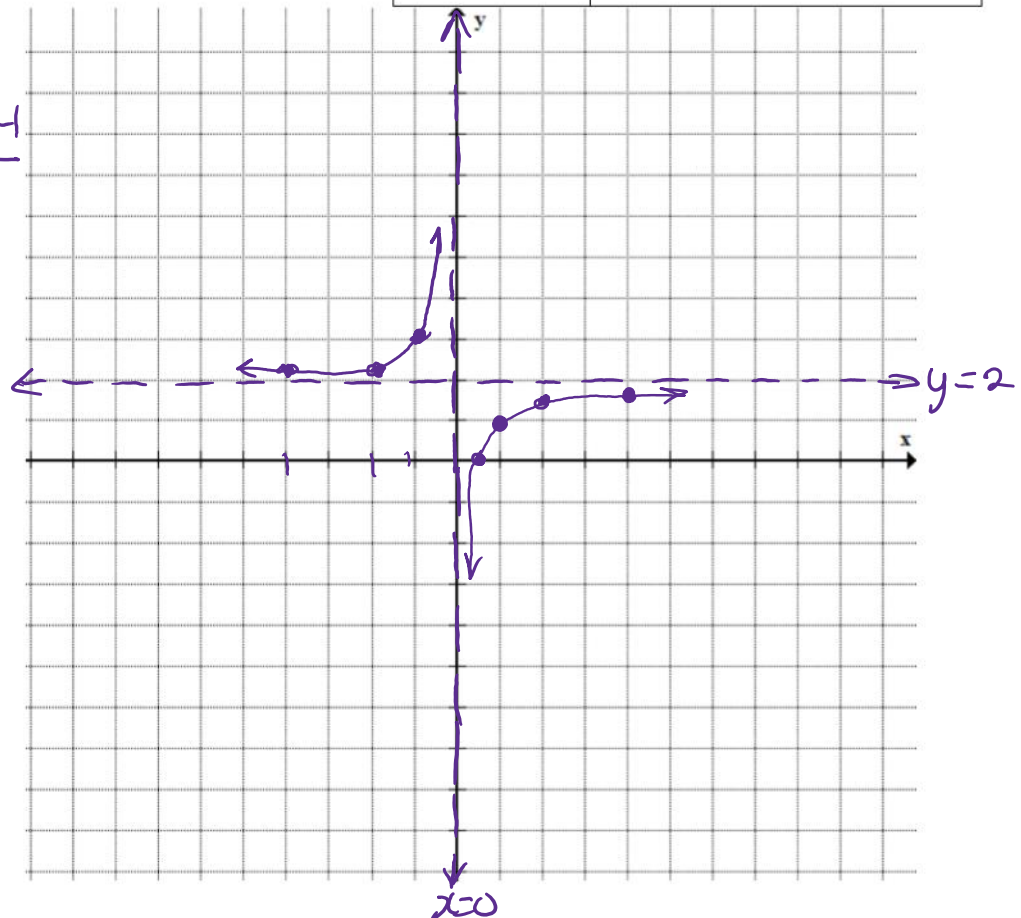


4. $f(x) = \frac{2x-1}{x}$

Characteristic	Answer
a. simplified function	$f(x) = \frac{2x-1}{x}$
b. POD	none
c. vertical asymptote(s)	$x=0$
d. horizontal asymptote	$\frac{2x}{1x}$ $y=2$
e. y-intercept	$y = \frac{2(0)-1}{0} \rightarrow \emptyset$ none
f. x-intercept	$0 = 2x-1$ $x = \frac{1}{2}$ $(\frac{1}{2}, 0)$
g. domain	$\{x \mid x \neq 0, x \in \mathbb{R}\}$
h. range	$\{y \mid y \neq 2, y \in \mathbb{R}\}$

x	y
-4	$\frac{9}{4}$
-2	$\frac{5}{4}$
-1	3
1	1
2	$-\frac{3}{2}$
4	$-\frac{7}{4}$

$$\frac{2(-4)-1}{-4}$$



5. $f(x) = \frac{x^2 + x - 12}{x^2 - 9}$

$$f(x) = \frac{(x+4)(x-3)}{(x+3)(x-3)}$$

$$f(x) = \frac{x+4}{x+3}$$

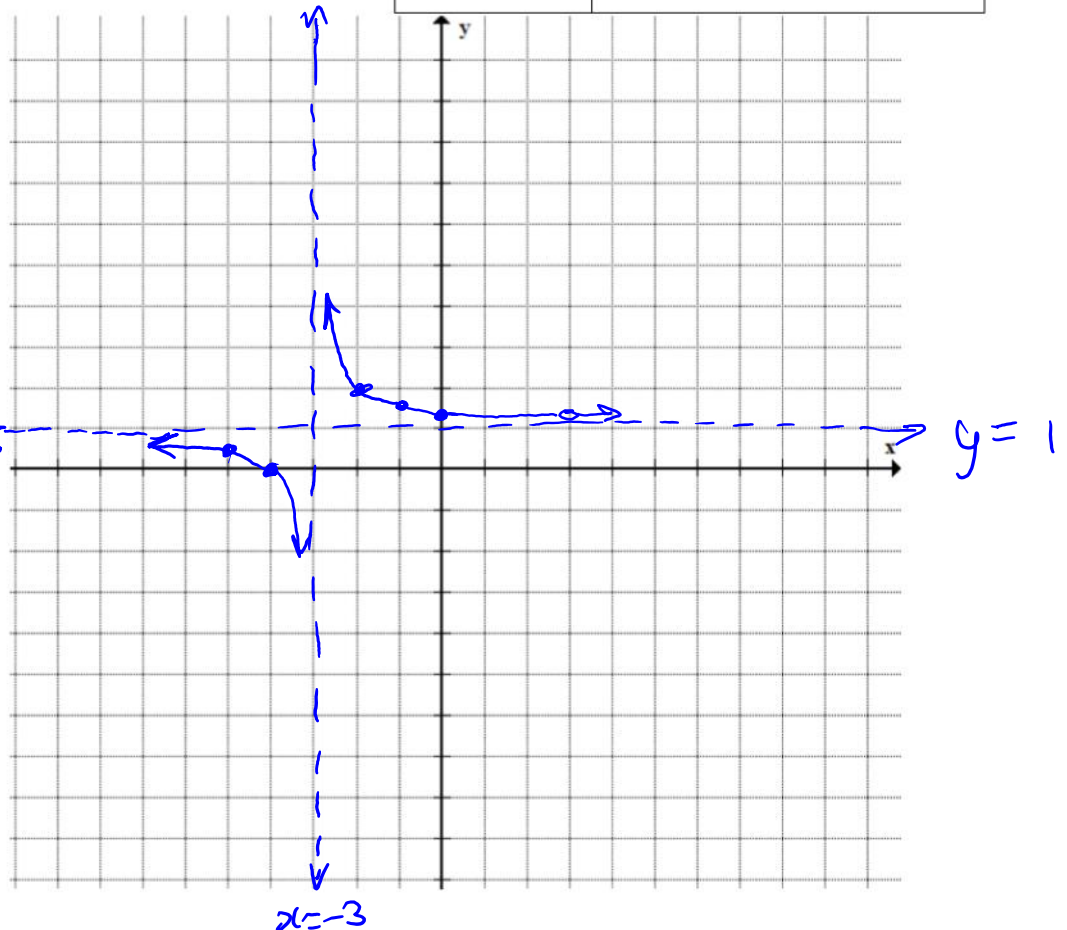
Characteristic	Answer
a. simplified function	$f(x) = \frac{x+4}{x+3}$
b. POD	$\textcircled{x=3} \quad f(-3) = \frac{3+4}{3+3} \Rightarrow \frac{7}{6} \left(3\frac{1}{6}\right)$
c. vertical asymptote(s)	$x = -3$
d. horizontal asymptote	$y = 1$
e. y-intercept	$y = \frac{0+4}{0+3} \rightarrow \frac{4}{3} \left(0\frac{4}{3}\right)$
f. x-intercept	$0 = x+4 \rightarrow x = -4 \quad (-4, 0)$
g. domain	$\{x \mid x \neq -3, 3, x \in \mathbb{R}\}$
h. range	$\{y \mid y \neq 1, \frac{7}{6}, y \in \mathbb{R}\}$

x	y
-5	$\frac{1}{2}$
-2	2
-1	$\frac{3}{2}$

$$\frac{-5+4}{-5+3}$$

$$\frac{-2+4}{-2+3}$$

$$\frac{-1+4}{-1+3}$$



omit
6. $f(x) = \frac{x^2 - 4}{x + 3}$

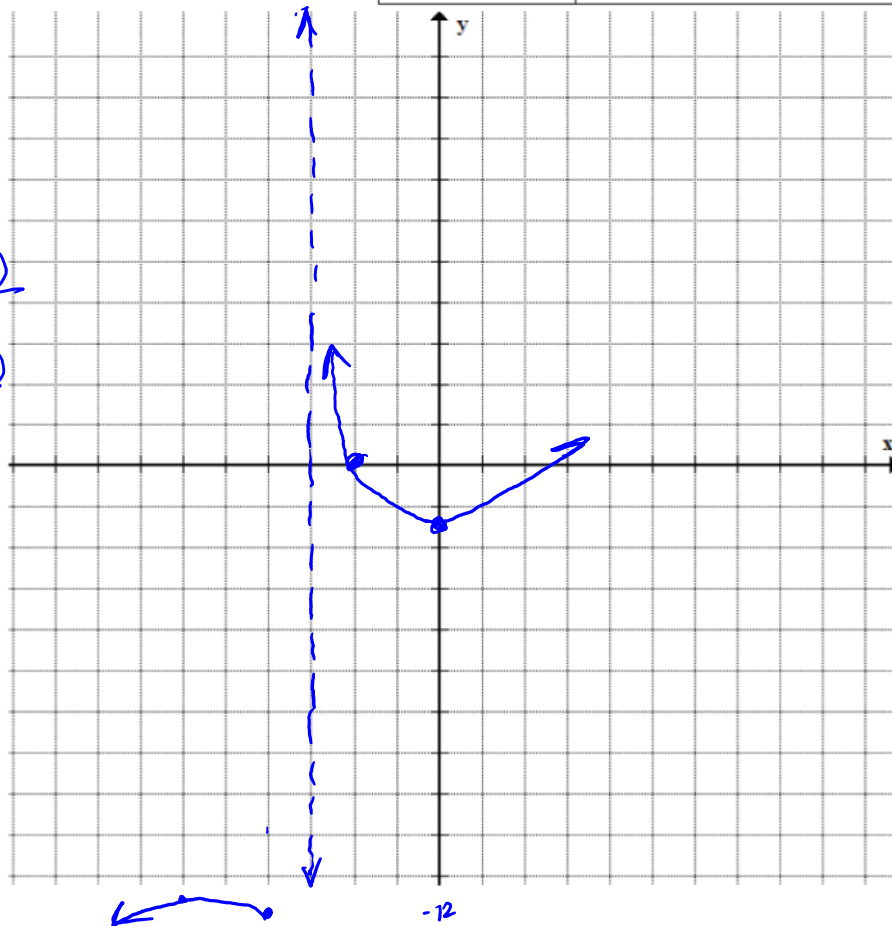
$$f(x) = \frac{(x+2)(x-2)}{x+3}$$

Slanted asymptote
= NOT ON TEST

Characteristic	Answer
a. simplified function	$f(x) = \frac{(x+2)(x-2)}{x+3}$
b. POD	none
c. vertical asymptote(s)	$x = -3$
d. horizontal asymptote	$y = \text{none}$
e. y-intercept	$0 = \frac{(0+2)(0-2)}{0+3} \rightarrow y = \frac{-4}{3}$
f. x-intercept	$0 = (x+2)(x-2) \rightarrow (2,0), (-2,0)$
g. domain	$\{x \mid x \neq -3, x \in \mathbb{R}\}$
h. range	$\{y \mid y \in \mathbb{R}\}$

$$y = \frac{(x+2)(x-2)}{x+3}$$

x	y
-6	$-\frac{32}{3}$
-4	-12



omit

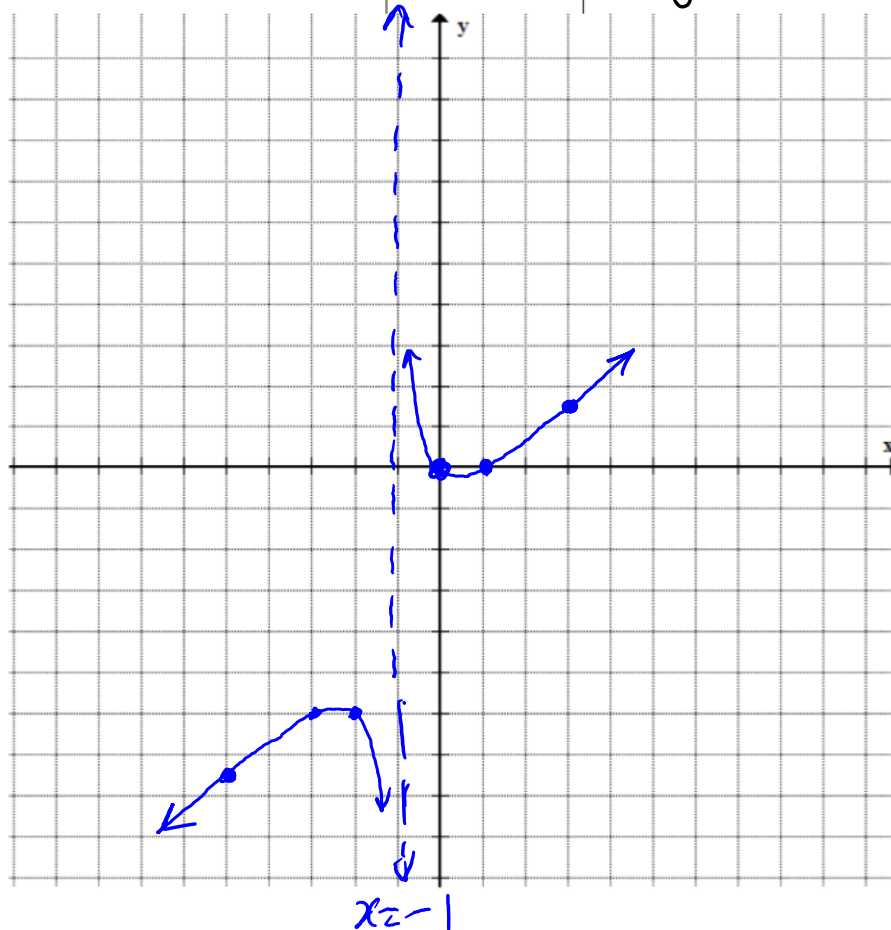
$$7. f(x) = \frac{x^2 - x}{x + 1}$$

$$f(x) = \frac{x(x-1)}{x+1}$$

slant asymptote
(not on test)

Characteristic	Answer
a. simplified function	$y = \frac{x(x-1)}{x+1}$
b. POD	none
c. vertical asymptote(s)	$x = -1$
d. horizontal asymptote	none
e. y-intercept	$(0, 0)$
f. x-intercept	$(0, 0) + (1, 0)$
g. domain	$\{x \mid x \neq -1, x \in \mathbb{R}\}$
h. range	$\{y \mid \text{none}, y \in \mathbb{R}\}$

x	y
-5	-7.5
-3	-6
-2	-6
3	1.5



$$8. f(x) = \frac{x^2 - x - 2}{x + 1}$$

① factor

② simplify

$$f(x) = \frac{(x-2)(x+1)}{x+1}$$

$$f(x) = x - 2$$

$$y = mx + b$$

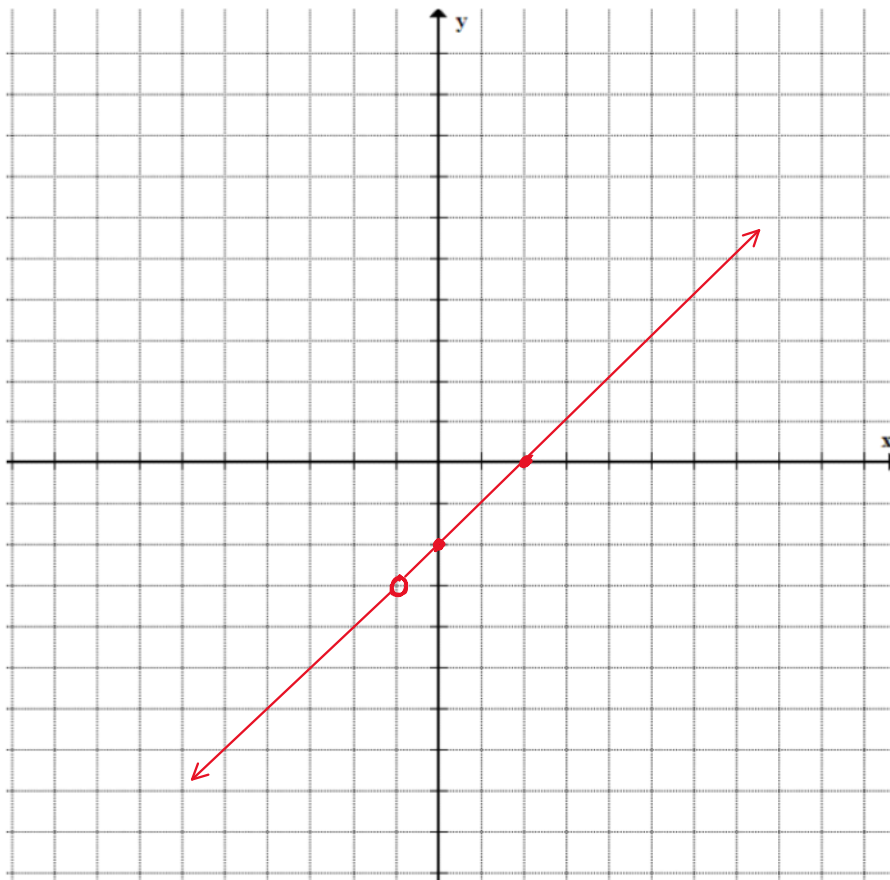
x	y
-1	-3
0	-2
2	0

hole

Characteristic	Answer
a. simplified function	$y = x - 2$
b. POD HOLE	$x + 1 \neq 0 \rightarrow x \neq -1$ $(-1, -3)$
c. vertical asymptote(s)	none
d. horizontal asymptote	none
e. y-intercept	$(0, -2)$
f. x-intercept	$(2, 0)$
g. domain POD	$\{x \mid x \neq -1, x \in \mathbb{R}\}$
h. range POD	$\{y \mid y \neq -3, y \in \mathbb{R}\}$

$y = -1 - 2$
 $y = -3$

$y = 0 - 2$
 $y = -2$
 $0 = x - 2$
 $x = 2$



$$9. f(x) = \frac{x+1}{x^2+3x+2}$$

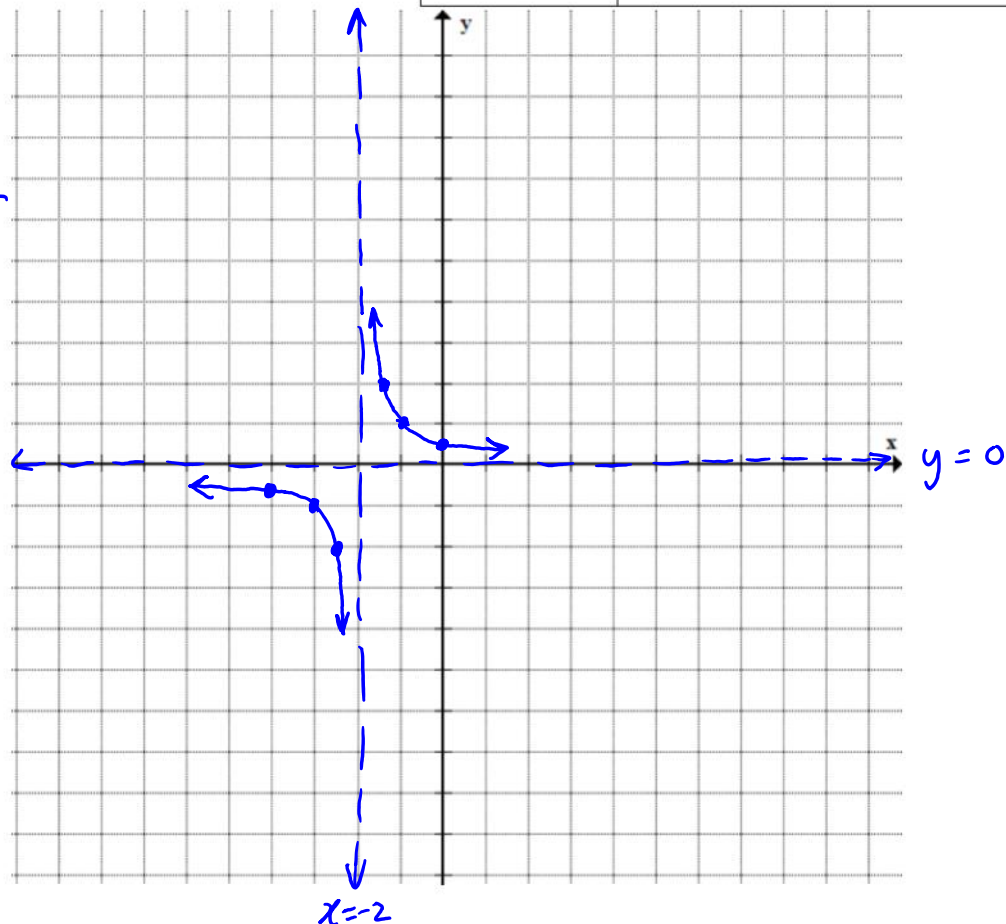
$$f(x) = \frac{\cancel{x+1}}{(x+2)\cancel{(x+1)}}$$

$$f(x) = \frac{1}{x+2}$$

Characteristic	Answer
a. simplified function	$f(x) = \frac{1}{x+2}$
b. POD	$\textcircled{0} x=-1$ $y = \frac{1}{-1+2} \rightarrow y=1 \quad (-1, 1)$
c. vertical asymptote(s)	$x = -2$
d. horizontal asymptote	$y = 0$
e. y-intercept	$y = \frac{1}{0+2} \rightarrow y = \frac{1}{2} \quad (0, \frac{1}{2})$
f. x-intercept	none
g. domain	$\{x \mid x \neq -1, x \neq -2, x \in \mathbb{R}\}$
h. range	$\{y \mid y \neq 0, y \neq 1, y \in \mathbb{R}\}$

$y = \frac{1}{x}$

x	y	x	y
-2	$-\frac{1}{2}$	-4	$-\frac{1}{4}$
-1	-1	-3	-1
$-\frac{1}{2}$	-2	$-\frac{3}{2}$	-2
0	\emptyset	-2	\emptyset
$\frac{1}{2}$	2	$-\frac{3}{2}$	2
1	1	-1	1
2	$\frac{1}{2}$	0	$\frac{1}{2}$



10. $f(x) = \frac{x^2 - 9}{x^2 - 2x - 3}$

$$f(x) = \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}(x+1)}$$

$$f(x) = \frac{x+3}{x+1}$$

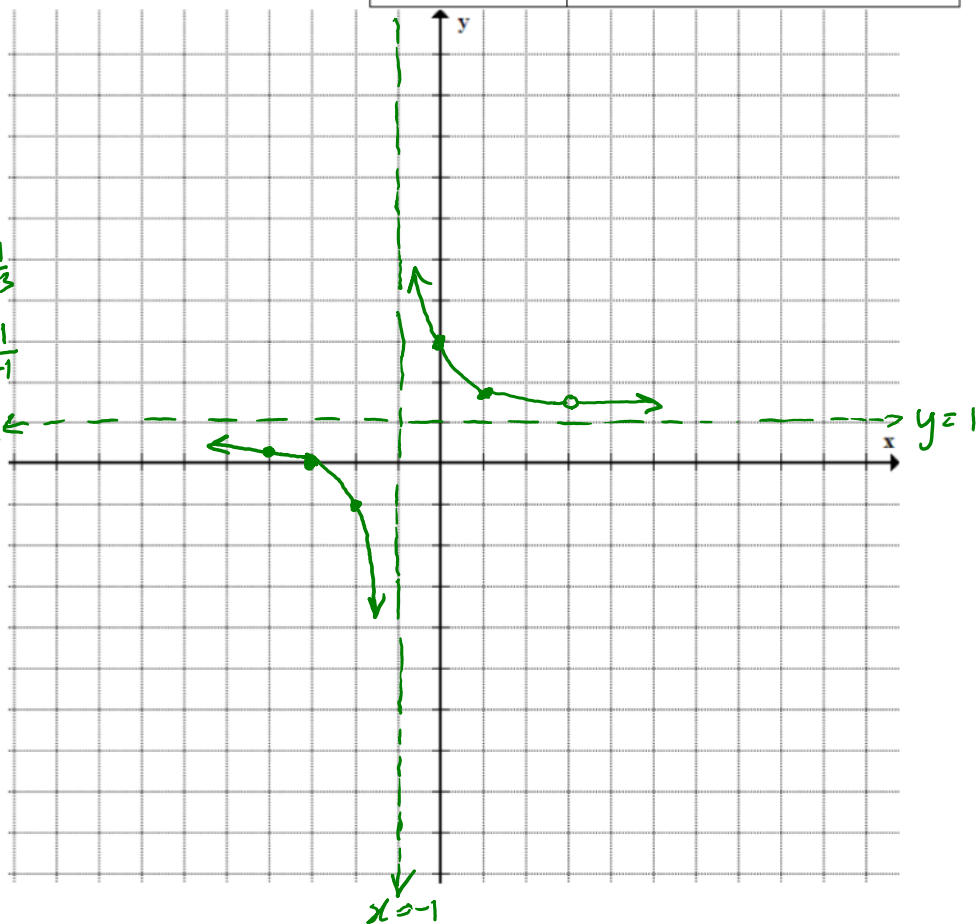
Characteristic	Answer
a. simplified function	$f(x) = \frac{x+3}{x+1}$
b. POD	$\textcircled{x=3}$ $y = \frac{3+3}{3+1} \rightarrow y = \frac{6}{4} \rightarrow y = \frac{3}{2} \quad (3, \frac{3}{2})$
c. vertical asymptote(s)	$x = -1$
d. horizontal asymptote	$y = 1$
e. y-intercept	$y = \frac{0+3}{0+1} \rightarrow y = 3 \quad (0, 3)$
f. x-intercept	$0 = x+3$ $x = -3 \quad (-3, 0)$
g. domain	$\{x \mid x \neq -1, x \neq 3, x \in \mathbb{R}\}$
h. range	$\{y \mid y \neq 1, y \neq \frac{3}{2}, y \in \mathbb{R}\}$

x	y
-4	$\frac{1}{3}$
-2	-1
2	$\frac{5}{3}$

$$\frac{-4+3}{-4+1} = \frac{-1}{-3} = \frac{1}{3}$$

$$\frac{-2+3}{-2+1} = \frac{1}{-1} = -1$$

$$\frac{2+3}{2+1} = \frac{5}{3}$$



$$\text{IF } f(x) = \frac{x^2 - 5x}{2x}$$

$$f(x) = \frac{\cancel{x}(x-5)}{\cancel{2x}} \rightarrow \begin{matrix} \text{POD} \\ \text{HOLE} \end{matrix} \quad x=0$$

$$f(x) = \frac{1}{2}(x-5)$$

$$f(x) = \frac{1}{2}x - \frac{5}{2} \quad y = mx + b$$