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Logarithmic & Exponential Form

Express each equation in logarithmic form.

1) $2^3 = r$ <i>base</i> → 2, <i>exponent</i> → 3, <i>result</i> → r $\log_2 r = 3$	2) $a^7 = 128$
3) $u^{\frac{1}{4}} = 3$ $\log_u 3 = \frac{1}{4}$	4) $7^6 = s$
5) $4^z = 64$	6) $3^{-d} = \frac{1}{9}$
7) $x^{\frac{1}{6}} = 2$	8) $6^3 = r$

Express each equation in exponential form.

9) $\log_8 512 = h$ <i>But the base:</i> $512 = 8^h$	10) $\log_a 36 = 2$
11) $\log_4 \left(\frac{1}{16}\right) = -y$ $\frac{1}{16} = 4^{-y}$	12) $\log_{64} p = \frac{1}{6}$
13) $\log_x 81 = 4$	14) $\log_{49} 7 = f$
15) $\log_{11} n = 2$	16) $\log_8 k = \frac{1}{2}$

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Answer key**Express each equation in logarithmic form.**

1) $2^3 = r$

$\log_2 r = 3$

2) $a^7 = 128$

$\log_a 128 = 7$

3) $u^{\frac{1}{4}} = 3$

$\log_u 3 = \frac{1}{4}$

4) $7^6 = s$

$\log_7 s = 6$

5) $4^z = 64$

$\log_4 64 = z$

6) $3^{-d} = \frac{1}{9}$

$\log_3 \left(\frac{1}{9}\right) = -d$

7) $x^{\frac{1}{6}} = 2$

$\log_x 2 = \frac{1}{6}$

8) $6^3 = r$

$\log_6 r = 3$

Express each equation in exponential form.

9) $\log_8 512 = h$

$8^h = 512$

10) $\log_a 36 = 2$

$a^2 = 36$

11) $\log_4 \left(\frac{1}{16}\right) = -y$

$4^{-y} = \frac{1}{16}$

12) $\log_{64} p = \frac{1}{6}$

$64^{\frac{1}{6}} = p$

13) $\log_x 81 = 4$

$x^4 = 81$

14) $\log_{49} 7 = f$

$49^f = 7$

15) $\log_{11} n = 2$

$11^2 = n$

16) $\log_8 k = \frac{1}{2}$

$8^{\frac{1}{2}} = k$

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Single Logarithm and Expansion

Expand each expression :

1) $\log_a \left(\frac{x^2 y^3}{m n} \right) =$ _____

2) $\log_3 \sqrt{5a^7} =$ _____

3) $5 \log_4 \left(\frac{a^2 b}{n^3} \right) =$ _____

4) $\log_2 \left(\frac{b}{c} \right)^4 =$ _____

5) $4 \log_a \left(\frac{p^6 q^3}{r^2 s} \right) =$ _____

Rewrite each expression in single logarithm:

6) $(4 \log_5 x + 5 \log_5 y) - \log_5 z =$ _____

7) $(3 \log_7 m + 12 \log_7 n) - 3 \log_7 p =$ _____

8) $\frac{1}{3} (4 \log_2 s + \log_2 t) =$ _____

9) $40 \log_3 t - (8 \log_3 w + 16 \log_3 x) =$ _____

10) $6 (\log_8 5 - \log_8 m) =$ _____

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Answer key

Expand each expression :

$$1) \log_a \left(\frac{x^2 y^3}{m n} \right) = \underline{(2 \log_a x + 3 \log_a y) - (\log_a m + \log_a n)}$$

$$2) \log_3 \sqrt{5a^7} = \underline{\frac{1}{2} (\log_3 5 + 7 \log_3 a)}$$

$$3) 5 \log_4 \left(\frac{a^2 b}{n^3} \right) = \underline{(10 \log_4 a + 5 \log_4 b) - 15 \log_4 n}$$

$$4) \log_2 \left(\frac{b}{c} \right)^4 = \underline{4 (\log_2 b - \log_2 c)}$$

$$5) 4 \log_a \left(\frac{p^6 q^3}{r^2 s} \right) = \underline{(24 \log_a p + 12 \log_a q) - (8 \log_a r + 4 \log_a s)}$$

Rewrite each expression in single logarithm:

$$6) (4 \log_5 x + 5 \log_5 y) - \log_5 z = \underline{\log_5 \left(\frac{x^4 y^5}{z} \right)}$$

$$7) (3 \log_7 m + 12 \log_7 n) - 3 \log_7 p = \underline{3 \log_7 \left(\frac{m n^4}{p} \right)}$$

$$8) \frac{1}{3} (4 \log_2 s + \log_2 t) = \underline{\log_2 \sqrt[3]{s^4 t}}$$

$$9) 40 \log_3 t - (8 \log_3 w + 16 \log_3 x) = \underline{8 \log_3 \left(\frac{t^5}{w x^2} \right)}$$

$$10) 6 (\log_8 5 - \log_8 m) = \underline{\log_8 \left(\frac{5}{m} \right)^6}$$

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Logarithm - Solve

Solve for x.

Example 1:

$$\begin{aligned} \log_{64} 4 &= x \\ 64^x &= 4 \\ 4^{3x} &= 4 \rightarrow 3x=1 \\ x &= \frac{1}{3} \end{aligned}$$

Example 2:

$$\begin{aligned} \log_5 x^{\frac{1}{2}} &= 2 \\ (5^2)^2 &= (x^{\frac{1}{2}})^2 \rightarrow (5^2)^2 = (x)^2 \\ 5^4 &= x \\ x &= \mathbf{625} \end{aligned}$$

Solve for x.

$\log_4 2 = x$
 \downarrow
 $\log_4 \sqrt{4} = x$
 $\log_4 4^{\frac{1}{2}} = x$
 $\frac{1}{2} = x$
 $x = \frac{1}{2}$

1) $\log_4 2 = x$

$$\begin{aligned} 4^x &= 2 \\ (2^2)^x &= 2 \\ 2^{2x} &= 2^1 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

x =

2) $\log_x 64^{\frac{1}{3}} = 2$

$$\begin{aligned} x^2 &= 64^{\frac{1}{3}} \\ x^2 &= \sqrt[3]{64} \\ x^2 &= \sqrt[3]{2^3} \\ x^2 &= 2 \\ x &= \pm 2 \end{aligned}$$

Restrict: $x > 0$
 $x \neq -2$

x =

3) $\log_6 \left(\frac{1}{6}\right) = x$

$$\begin{aligned} \log_6 6^{-1} &= x \\ 6^x &= \frac{1}{6} \\ 6^x &= 6^{-1} \\ x &= -1 \end{aligned}$$

x =

4) $\log_2 \left(\frac{1}{16}\right) = x$

x =

5) $\log_{\frac{1}{2}} \left(\frac{1}{8}\right) = x$

x =

6) $\log_x 3 = \frac{1}{4}$

x =

7) $\log_x 6 = \frac{1}{2}$

x =

8) $\log_3 x^{\frac{1}{3}} = 2$

x =

9) $\log_4 x = 3$

x =

10) $\log_{125} 25 = x$

x =

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Logarithmic Equation

Solve for x.

Example 1:

$$\begin{aligned}\log_{64} 4 &= x \\ 64^x &= 4 \\ 4^{3x} &= 4 \\ x &= \frac{1}{3}\end{aligned}$$

Example 2:

$$\begin{aligned}\log_5 x^{\frac{1}{2}} &= 2 \\ 5^2 &= x^{\frac{1}{2}} \\ 5^4 &= x \\ x &= \mathbf{625}\end{aligned}$$

Solve for x.

1) $\log_4 2 = x$

x = $\frac{1}{2}$

2) $\log_x 64^{\frac{1}{3}} = 2$

x = 2

3) $\log_6 \left(\frac{1}{6}\right) = x$

x = -1

4) $\log_2 \left(\frac{1}{16}\right) = x$

x = -4

5) $\log_{\frac{1}{2}} \left(\frac{1}{8}\right) = x$

x = 3

6) $\log_x 3 = \frac{1}{4}$

x = 81

7) $\log_x 6 = \frac{1}{2}$

x = 36

8) $\log_3 x^3 = 2$

x = 729

9) $\log_4 x = 3$

x = 64

10) $\log_{125} 25 = x$

x = $\frac{2}{3}$

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Logarithm - Solve

Solve for x.

Example 1:

$$\begin{aligned} \log_3 \left(\frac{1}{3} \right) &= x-5 \\ (3)^{x-5} &= \left(\frac{1}{3} \right) \\ (3)^{x-5} &= 3^{-1} \\ x &= 4 \end{aligned}$$

Example 2:

$$\begin{aligned} \log_8 (2x)^3 &= 2 \\ 8^2 &= (2x)^3 \\ (8^2)^{\frac{1}{3}} &= 2x \\ 4 &= 2x \\ x &= 2 \end{aligned}$$

Solve for x.

no Restriction

Handwritten notes:
 $\log_{36} 6 = x+3$
 $\sqrt{36}$
 $\frac{1}{2} = x+3$
 -3
 $-\frac{3}{2} = x$
 $x = -\frac{3}{2}$

1) $\log_{36} 6 = x+3$
 $6 = 36^{x+3}$
 $6 = (6^2)^{x+3}$
 $6 = 6^{2x+6}$
 $1 = 2x+6$
 $-6 = 2x$
 $x = -\frac{3}{2}$

2) $\log_{3x} 64 = 2$
 $x = \text{_____}$

3) $\log_{32} \left(\frac{1}{4} \right) = x-1$
 $x = \text{_____}$

4) $\log_3 \left(\frac{1}{9} \right) = 2x$
 $x = \text{_____}$

5) $\log_{\frac{1}{32}} \left(\frac{x}{8} \right) = \frac{1}{5}$
 $x = \text{_____}$

6) $\log_{25} 625 = 2x+3$
 $x = \text{_____}$

7) $\log_{x+1} 16 = 4$
 $16 = (x+1)^4$
 $x = \text{_____}$

8) $\log_6 (4x)^{\frac{1}{2}} = 2$
 $x = \text{_____}$

9) $\log_9 (x-1) = 3$
 $x-1 = 9^3$
 $x = \text{_____}$

10) $\log_{2x} 2^{-4} = 2$
 $x = \text{_____}$

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Answer key

Solve for x.

Example 1:

$$\begin{aligned}\log_3 \left(\frac{1}{3}\right) &= x-5 \\ (3)^{x-5} &= \left(\frac{1}{3}\right) \\ (3)^{x-5} &= 3^{-1} \\ x &= 4\end{aligned}$$

Example 2:

$$\begin{aligned}\log_8 (2x)^3 &= 2 \\ 8^2 &= (2x)^3 \\ (8^2)^{\frac{1}{3}} &= 2x \\ 4 &= 2x \\ x &= 2\end{aligned}$$

Solve for x.

1) $\log_{36} 6 = x+3$

x = $-\frac{5}{2}$

2) $\log_{3x} 64 = 2$

x = $\frac{8}{3}$

3) $\log_{32} \left(\frac{1}{4}\right) = x-1$

x = $\frac{3}{5}$

4) $\log_3 \left(\frac{1}{9}\right) = 2x$

x = -1

5) $\log_{\frac{1}{32}} \left(\frac{x}{8}\right) = \frac{1}{5}$

x = 4

6) $\log_{25} 625 = 2x+3$

x = $-\frac{1}{2}$

7) $\log_{x+1} 16 = 4$

x = 1

8) $\log_6 (4x)^{\frac{1}{2}} = 2$

x = 324

9) $\log_9 (x-1) = 3$

x = 730

10) $\log_{2x} 2^{-4} = 2$

x = $\frac{1}{8}$

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Logarithm - Solve

Solve for x.

Example 1:

$$\log_{64} 4 = x+2$$

$$(64)^{x+2} = 4$$

$$4^{3x+6} = 4$$

$$3x+6 = 1$$

$$x = -\frac{5}{3}$$

Example 2:

$$\log_4 4x^{\frac{1}{2}} = 2$$

$$4^2 = (4x)^{\frac{1}{2}}$$

$$4^4 = 4x$$

$$x = 64$$

Solve for x.

1) $\log_4 2 = x-5$

x =

2) $\log_{x+2} (27) = 3$

x =

3) $\log_2 \left(\frac{1}{4}\right) = 2x+1$

x =

4) $\log_4 \left(\frac{1}{16}\right) = \frac{x}{2}$

x =

5) $\log_{\frac{1}{3}} \left(\frac{1}{9}\right) = 4x$

x =

6) $\log_{5x} 8 = 3$

x =

7) $\log_{x-1} (16) = \frac{1}{2}$

x =

8) $\log_3 (x+4)^{\frac{1}{3}} = 1$

x =

9) $2 \log_4 (x-2) = 4$

x =

10) $\log_{128} 2 = x+3$

x =

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Answer key

Solve for x.

Example 1:

$$\log_{64} 4 = x+2$$

$$(64)^{x+2} = 4$$

$$4^{3x+6} = 4$$

$$3x+6 = 1$$

$$x = -\frac{5}{3}$$

Example 2:

$$\log_4 4x^{\frac{1}{2}} = 2$$

$$4^2 = (4x)^{\frac{1}{2}}$$

$$4^4 = 4x$$

$$x = 64$$

Solve for x.

1) $\log_4 2 = x-5$

$$x = \frac{11}{2}$$

2) $\log_{x+2} (27) = 3$

$$x = 1$$

3) $\log_2 \left(\frac{1}{4}\right) = 2x+1$

$$x = -\frac{3}{2}$$

4) $\log_4 \left(\frac{1}{16}\right) = \frac{x}{2}$

$$x = -4$$

5) $\log_{\frac{1}{3}} \left(\frac{1}{9}\right) = 4x$

$$x = \frac{1}{2}$$

6) $\log_{5x} 8 = 3$

$$x = \frac{2}{5}$$

7) $\log_{x-1} (16) = \frac{1}{2}$

$$x = 257$$

8) $\log_3 (x+4)^{\frac{1}{3}} = 1$

$$x = 23$$

9) $2 \log_4 (x-2) = 4$

$$x = 18$$

10) $\log_{128} 2 = x+3$

$$x = -\frac{20}{7}$$

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Logarithmic Equation

Solve for x.

1) $2 \log_3 x = \log_3 (12x-36)$ R

$$\log_3 x^2 = \log_3 (12x-36)$$

$x > 0; 12x-36 > 0$
 $x > 3$

$$x^2 = 12x - 36$$

$$-12x + 36 -12x + 36$$

$$x^2 - 12x + 36 = 0 \rightarrow (x-6)(x-6) = 0$$

$x = 6$

2) $\log_2 (x-11) + \log_2 (x-2) = \log_2 10$ R

$$\log_2 (x-1)(x-2) = \log_2 10$$

$$(x-1)(x-2) = 10$$

$$x^2 - 13x + 22 - 10 = 0$$

$$x^2 - 13x + 12 = 0$$

$x-11 > 0$
 $x > 11$
 $x-2 > 0$
 $x > 2$
 $x=12, x=1$
extraneous

$x = 12$

3) $\log_7 \left(\frac{x+8}{x+6} \right) = 2$ R

$$\frac{x+8}{x+6} = 49$$

$$x+8 = 49x + 294$$

$$-48x = 286 \rightarrow x = -\frac{143}{24}$$

$\frac{x+8}{x+6} > 0$
 $x+8 > 0$
 $x > -8$
 $x+6 > 0$
 $x > -6$

$x = -\frac{143}{24}$

4) $\log_4 (x-5) + \log_4 (x+5) = \log_4 24$

$$\log_4 (x-5)(x+5) = \log_4 24$$

$$x^2 - 25 = 24$$

$$x^2 = 49$$

$$x = \pm 7$$

$x-5 > 0$
 $x > 5$
 $x+5 > 0$
 $x > -5$
 $x \neq -7$ extraneous

$x = 7$

5) $2 \log_3 (x-1) = \log_3 3$ ✓

$x =$

6) $2 \log_8 x = \log_8 (7x-12)$ ✓

$x =$

7) $\log_3 (x+2) + \log_3 (x-3) = \log_3 14$

$x =$

8) $2 = \log_4 \left(\frac{x+7}{x+5} \right)$ R

$$4^2 = \frac{x+7}{x+5}$$

$$16(x+5) = x+7$$

$$16x + 80 = x + 7$$

$$15x = -73$$

$$x = -\frac{73}{15}$$

$x+7 > 0$
 $x > -7$
 $x+5 > 0$
 $x > -5$

Name : _____

Score : _____

Answer key

Solve for x.

1) $2 \log_3 x = \log_3 (12x-36)$

x = **6**

2) $\log_2 (x-11) + \log_2 (x-2) = \log_2 10$

x = **12**

3) $\log_7 \left(\frac{x+8}{x+6} \right) = 2$

x = **-4**

4) $\log_4 (x-5) + \log_4 (x+5) = \log_4 24$

x = **7**

5) $2 \log_3 (x-1) = \log_3 3$

x = **2**

6) $2 \log_8 x = \log_8 (7x-12)$

x = **4, 3**

7) $\log_3 (x+2) + \log_3 (x-3) = \log_3 14$

x = **5**

8) $2 = \log_4 \left(\frac{x+7}{x+5} \right)$

x =