

Name: _____

Date: _____

BLM U3-2

Unit 3 Test

Multiple Choice

For #1 to 6, select the best answer.

1. The graph of the function $f(x) = 5^x$ is transformed by a vertical stretch by a factor of 3 about the x -axis, a reflection in the y -axis, and a horizontal translation 1 unit right. The range of the new function is

A $\{y \mid y > 0, y \in \mathbb{R}\}$
 B $\{y \mid y > 1, y \in \mathbb{R}\}$
 C $\{y \mid y > 2, y \in \mathbb{R}\}$
 D $\{y \mid y > 3, y \in \mathbb{R}\}$

A

2. The logarithmic form of $y = 2^{x+1} - 5$ is

A $\log_2(y+5) - 1 = x$
 B $\log_2(x+5) - 1 = y$
 C $\log_2(y+1) - 5 = x$
 D $\log_2(x+1) - 5 = y$

A

3. The equation of the asymptote of the inverse of $f(x) = 3 \log_2(x-2) + 1$ is

A $x = 1$
 B $y = 1$
 C $x = 2$
 D $y = 2$

D

4. If $5^{2x-3} = 3^x$, then x is

A $\frac{3 \log 5}{2 \log 5 + \log 3}$
 B $\frac{3 \log 5}{2 \log 5 - \log 3}$
 C $\frac{2 \log 5 - \log 3}{3 \log 5}$
 D $\frac{2 \log 5 + \log 3}{3 \log 5}$

B

5. If the graph of a logarithmic function has a vertical asymptote of $x = 1$ and an x -intercept equal to 5, a possible equation is

A $y = 5 \log_4(x-1) - 1$
 B $y = \log_4(5(x-1))$
 C $y = \log_4(x-1) - 1$
 D $y = \log_4(x-1) + 4$

C

6. The half-life of a certain radioactive substance is 8 days. There are 5 g present initially. The best approximation when there will be 1 g remaining is

A 10 days
 B 15 days
 C 16 days
 D 19 days

D

Numerical Response

7. The graph of the exponential function $f(x) = k(a^x)$ passes through the points (1, 4.5) and (-1, 0.5). Determine the values of a and k .

not on test

8. Determine the roots of the equation

$$5 \times 2^{x-1} = 10^{x+2} = 5. \text{ this type of equation, not on test}$$

9. Evaluate $b^{\log_b 9 + \log_b 3}$.

$$\rightarrow b^{\log_b(9 \times 3)} = b^{\log_b 27} \rightarrow \boxed{27} \#9$$

10. If $\log_2 x = 1$ and $\log_3 y = x$, what is the value of y ?

$$\rightarrow \text{root } x = 2^1 \rightarrow x = 2 \rightarrow \log_3 y = 2 \rightarrow \text{root } y = 3^2 = 9 \#10$$

11. If $\log_c x = 2$ and $\log_c y = 3$, what is the value of $\left(\log_c \frac{x}{y}\right)^2$?

$$\rightarrow (\log_c x - \log_c y)^2 = (2 - 3)^2 = (-1)^2 = 1 \#11$$

Written Response

12. The exponential function $f(x) = 2^x$ is transformed to $y = -3f(x+2)$.

- a) State the equation of the transformed function in exponential form.
 b) Sketch the graph of $y = -3f(x+2)$ and state the domain, range, and any intercepts.

13. Determine the coordinates of the points of intersection of the graphs of the functions $y = 3^{x-1}$ and $y = \log_3 x + 1$. Explain the significance of the points of intersection.

at $x=1, y=1$ $\boxed{(1,1)}$
 $y = 3^{1-1} = 3^0 = 1$
 $y = \log_3(1) + 1 = 0 + 1 = 1$

14. Solve for x . Verify your solution.

- a) $5^{x+6} = 625^{x-3}$
 b) $\log_3(3x+6) - \log_3(x-4) = 2$
 c) $2 \log(x-1) - \log(x+1) = \log(x-2)$



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(continued)

15. Express y as a function of x . State the domain. *= solve for y → y = _____*
- a) $\log(y + 1) = 2 \log x$
 - b) $\log_4(2y) = \log_4(x + 1) - \log_4(x - 1)$
 - c) $\log(y) + 1 = \log(x - 3)$
16. The population of Calgary was 906 000 on January 1, 1996 and 1 188 000 on January 1, 2006.
- a) If the growth rate of the city can be modelled as an exponential function, determine the equation of the function, $P(t) = ab^t$, where $P(t)$ is population, in thousands, and t is time, in years, since 1996. Express the value of a as a whole number and b to 3 decimal places.
 - b) State the average annual growth rate of the city, expressed to the nearest tenth of a percent.
 - c) Predict the year the population will first exceed 1.5 million people.
17. The pH of a solution is given by $\text{pH} = -\log[\text{H}^+]$, where $[\text{H}^+]$ is the concentration of hydrogen ions in moles per litre.
- a) Determine the concentration of hydrogen ions in a solution with $\text{pH} = 6.2$.
 - b) If the concentration of the hydrogen ions in a solution is 3.2×10^{-6} moles per litre, what is the pH of the solution?
18. An investment of \$2500 in a guaranteed investment certificate is paying interest at a rate of 3.25% per year, compounded monthly.
- a) Determine the equation of the exponential function.
 - b) Graph the function, stating the domain, range, and any intercepts.
 - c) How long will it take for the investment to double in value? Express your answer to the nearest year.



12. The exponential function $f(x) = 2^x$ is transformed to $y = -3/(x+2)$.

- a) State the equation of the transformed function in exponential form.
b) Sketch the graph of $y = -3/(x+2)$ and state the domain, range, and any intercepts.

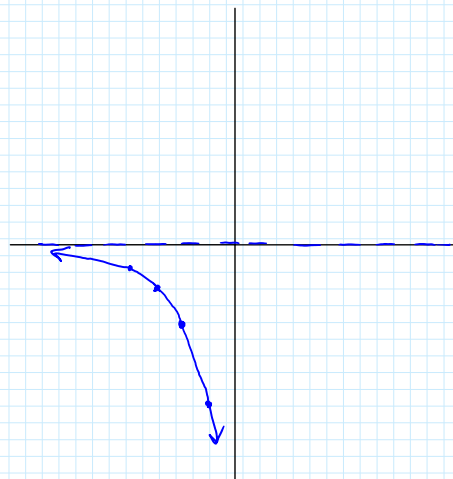
$$\{x | x \in \mathbb{R}\}$$

$$\{y | y < 0, y \in \mathbb{R}\}$$

$$HA: y = 0$$

$$y = -3(2)^{x+2}$$

x	y	$x-2$	$-3y$
-2	$1/4$	-4	$-3/4$
-1	$1/2$	-3	$-3/2$
0	1	-2	-3
1	2	-1	-6
2	4	0	-12
3	8	1	-24



14. Solve for x . Verify your solution.

a) $5^{x+6} = 625^{x-3}$

b) $\log_3(3x+6) - \log_3(x-4) = 2$

c) $2 \log(x-1) - \log(x+1) = \log(x-2)$

$$\begin{aligned} \log 5^{x+6} &= \log 625^{x-3} \\ (x+6) \log 5 &= (x-3) \log 625 \\ x \log 5 + 6 \log 5 &= x \log 625 - 3 \log 625 \\ x \log 5 - x \log 625 &= -3 \log 625 - 6 \log 5 \\ x(\log 5 - \log 625) &= -3 \log 625 - 6 \log 5 \\ x &= \frac{-3 \log 625 - 6 \log 5}{\log 5 - \log 625} \end{aligned}$$

$$b) \log_3 \left(\frac{3x+6}{x-4} \right) = 2$$

$$\frac{3x+6}{x-4} = 3^2(x-4)$$

$$3x+6 = 9(x-4)$$

$$3x+6 = 9x-36$$

$$-6x = -42$$

$$x = 7$$

$$R \text{ on NRVs}$$

$$3x+6 > 0 \quad x-4 > 0$$

$$x > -2 \quad x > 4$$

$$c) \log(x-1)^2 - \log(x+1) = \log(x-2)$$

$$\log \left(\frac{(x-1)^2}{x+1} \right) = \log(x-2)$$

$$\frac{(x-1)^2}{x+1} = (x-2)(x+1)$$

$$x^2 - 2x + 1 = x^2 - x - 2$$

$$-x = -3$$

$$x = 3$$

$$R \text{ (NRVs)}$$

$$x-1 > 0$$

$$x > 1$$

$$x+1 > 0$$

$$x > -1$$

$$x-2 > 0$$

$$x > 2$$

15. Express y as a function of x . State the domain. = solve for $y \rightarrow y =$

a) $\log(y+1) = 2 \log x$

b) $\log_2(2y) = \log_4(x+1) - \log_4(x-1)$

c) $\log(y) + 1 = \log(x-3)$

$$\begin{aligned} a) \log(y+1) &= \log x^2 \\ y+1 &= x^2 \\ y &= x^2 - 1 \end{aligned}$$

$$\begin{aligned} b) \log_2(2y) &= \log_4 \left(\frac{x+1}{x-1} \right) \\ 2y &= \frac{x+1}{x-1} \\ y &= \frac{x+1}{x-1} \times \frac{1}{2} \\ y &= \frac{x+1}{2(x-1)} \end{aligned}$$

$$\begin{aligned} c) \log y &= \log(x-3) - 1 \\ \log y &= \log(x-3) - \log 10 \\ \log y &= \log \left(\frac{x-3}{10} \right) \\ y &= \frac{x-3}{10} \end{aligned}$$

16. The population of Calgary was 906 000 on January 1, 1996 and 1 188 000 on January 1, 2006.

- If the growth rate of the city can be modelled as an exponential function, determine the equation of the function, $P(t) = ab^t$, where $P(t)$ is population, in thousands, and t is time, in years, since 1996. Express the value of a as a whole number and b to 3 decimal places.
- State the average annual growth rate of the city, expressed to the nearest tenth of a percent.
- Predict the year the population will first exceed 1.5 million people.

$P(t) = ab^t$ $a = 906$ (thousands)

$$\frac{1188000}{906000} = \frac{906000(b)^{10}}{906000}$$

$$\frac{1188}{906} = b^{10}$$

$$\log\left(\frac{1188}{906}\right) = 10 \log b$$

$$\log b = \frac{\log\left(\frac{1188}{906}\right)}{10}$$

$$b = 10^{\frac{\log\left(\frac{1188}{906}\right)}{10}}$$

$b = 1.028$

b) growth rate
 $b = 1 + r$
 $1.028 = 1 + r$
 $r = 0.028$
 $r = 2.8\%$

c) $1500000 = 906000(1.028)^t$

$$\frac{1500}{906} = 1.028^t$$

$$\log\left(\frac{1500}{906}\right) = t \log 1.028$$

$$t = \frac{\log\left(\frac{1500}{906}\right)}{\log 1.028}$$

$$t = 18.26 \text{ yr.}$$

$$t = 1996 + 18$$

$$t = 2014 \text{ the year 2014}$$

17. The pH of a solution is given by $\text{pH} = -\log [\text{H}^+]$, where $[\text{H}^+]$ is the concentration of hydrogen ions in moles per litre.

- Determine the concentration of hydrogen ions in a solution with $\text{pH} = 6.2$.
- If the concentration of the hydrogen ions in a solution is 3.2×10^{-6} moles per litre, what is the pH of the solution?

a) $\text{pH} = -\log [\text{H}^+]$
 $6.2 = -\log [\text{H}^+]$
 $-6.2 = \log [\text{H}^+]$
 $[\text{H}^+] = 10^{-6.2}$
 $[\text{H}^+] = 6.31 \times 10^{-7} \text{ mol/L}$

b) $\text{pH} = -\log (3.2 \times 10^{-6})$
 $\text{pH} = 5.5$

18. An investment of \$2500 in a guaranteed investment certificate is paying interest at a rate of 3.25% per year, compounded monthly.

- Determine the equation of the exponential function.
- Graph the function, stating the domain, range, and any intercepts.
- How long will it take for the investment to double in value? Express your answer to the nearest year.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 2500\left(1 + \frac{0.0325}{12}\right)^{12t}$$

c) $2500 \times 2 = 5000$
 or use $A_0 = 1, A = 2$

$$5000 = 2500\left(1 + \frac{0.0325}{12}\right)^{12t}$$

$$2 = \left(1 + \frac{0.0325}{12}\right)^{12t}$$

$$\log 2 = 12t \log\left(1 + \frac{0.0325}{12}\right)$$

$$t = \frac{\log 2}{12 \log\left(1 + \frac{0.0325}{12}\right)}$$

$$t = 21.36 \text{ yr.}$$
 $t = 21 \text{ yr}$